

# Multifractal spectrum of physiological signals: a mechanism-related approach

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## ABSTRACT

In this paper we discuss an approach for mechanism-related analysis of physiological signals performed with the wavelet-based multifractal formalism. This approach assumes estimation of the singularity spectrum for the band-pass filtered processes at different physiological conditions in order to provide explanation of the occurred changes in the Hölder exponents and the multifractality degree. We illustrate the considered approach using two examples, namely, the dynamics of the cerebral blood flow (CBF) and the electrical activity of the brain.

**Keywords:** multifractality, singularity spectrum, cerebral blood flow, electroencephalogram

## 1. INTRODUCTION

Complex scaling phenomena in the dynamics of physiological systems are the subject of many studies aimed at the development of effective diagnostic measures that could provide informative characterization of the dynamics of living systems when analyzing essentially inhomogeneous and nonstationary experimental data.<sup>1-5</sup> Such measures are required, e.g., when studying responses of physiological systems to short-term changes in environmental conditions. In general, two important items should be taken into account: (i) the possibility of providing a reliable characterization of the system's state even in the case of time-varying processes, and (ii) the ability of such characterization based on relatively short signals. The recently proposed wavelet-transform modulus maxima (WTMM) method<sup>6,7</sup> satisfies to these conditions: it can be performed using a reduced amount of data for correlation analysis as compared with the standard correlation function.<sup>8</sup> Besides, this tool can be applied to nonstationary and inhomogeneous time series because the wavelet transform used at the first stage of this method allows ignoring a polynomial trend presented in the acquired physiological processes.

During the last decades, the WTMM-approach has demonstrated its essential potential in studying many types of complex systems. However, despite of its ability to reveal important markers of early pathological changes at transformations of normal physiological processes into the pathological dynamics, a disadvantage of this tool is the absence of a clear relation between the estimated numerical measures such as, e.g., the multifractality degree and the physiological control mechanisms responsible for the occurred changes.

In order to extend the method's abilities for a deeper understanding of the structure of physiological processes, we proposed an approach for mechanisms-related analysis<sup>9</sup> that assumes an application of WTMM-method to the band-pass filtered signals reflecting information about the dynamics associated with individual physiological control mechanisms. Such analysis can be done, e.g., in the case of cardiovascular or cerebrovascular dynamics where regulatory mechanisms are separated in the frequency domain. It can also be used in the study of the brain electrical activity since main rhythmic components detected in the electroencephalogram (EEG) are also separated and can be analyzed individually.

The paper is organized as follows. In Sec. 2 we consider the performed experiments in rats and in humans. Description of the WTMM-method and its modification used in our study is discussed in Sec. 3. Sec. 4 describes application of this technique and discusses the obtained results. Main concluding remarks are presented in Sec. 5.

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## 2. EXPERIMENTS

### 2.1 Cerebrovascular dynamics in rats

All experimental procedures were done in mongrel normotensive male rats ( $n=12$ ) in accordance with the Guide for the Care and Use of Laboratory Animals. One day before the experiment, rats were instrumented with polyethylene catheters for monitoring mean arterial pressure (MAP). The polyethylene catheter (PE-50 with a PE-10 tip, Scientific Commodities INC., Lake Havasu City, Arizona) was inserted into the femoral artery. The femoral vein was catheterized with PE-50 tubing fused PE-10.

Blood pressure signals were acquired with the PowerLab system (ADInstruments, Australia) using a pressure transducer. We studied responses of the cerebrovascular dynamics to abrupt changes in the arterial blood pressure (the acute peripheral hypertension) caused by the phenylephrine injections ( $0.25 \mu\text{g}/\text{kg}$ , iv). Typically, such injections increased MAP by about 10%.

Monitoring of CBF was provided with a home-made system for laser speckle contrast imaging (LSCI).<sup>10</sup> During the experiment, the exposed rat cortex was illuminated by the HeNe laser (Thorlabs HNL210L, 632.8 nm). Raw laser speckle images were acquired with the monochromatic CMOS camera Basler acA2500-14 gm and Computar M1614-MP2 lens. Images were recorded with the rate 40 frames/second. A moving window ( $55 \times 55$  pixels) was used to reduce noise presented in images by time averaging over 50 images. The speckle contrast data were converted into the flow velocity signals using the Gaussian approach.

In each experiment, recordings of the cerebral blood flow in small vessels of the micro-circulatory cerebral network were acquired and band-pass filtering of experimental data was performed in three frequency ranges<sup>11-13</sup> related to distinct mechanisms of physiological regulation:

Range I: 0.05–0.1 Hz. This range is typically associated with the NO-related endothelial function.

Range II: 0.1–0.25 Hz. The corresponding rhythm is associated with the neurogenic regulation.

Range III: 0.25–0.75 Hz. This range is related to the myogenic dynamics of smooth muscle cells.

Using these recordings, changes of CBF caused by the acute peripheral hypertension were analyzed.

### 2.2 Electrical activity of the brain

Experiments were done in healthy young people (mens, 20-25 years). Electrical activity of the brain was recorded using the standard setup for the electroencephalography (32-channels with the discretization 250 Hz). Experimental procedure was performed as follows. After the base-line measurements, EEG was recorded during movements of the right hand. The beginning of each movement was started after a short-term audio-signal. In each experiment, two recordings from the leads C4-A2 and C3-A1 were selected for the further analysis. EEG-recordings of the duration 5 sec were considered immediately after the audio-signal (including the movement) and before the new signal. The corresponding data were compared with the base-line EEG.

In order to reveal changes in the signal structure, band-pass filtering of EEG was performed with the wavelet-based approaches.<sup>14</sup> We analyzed dynamics related to two main rhythmic contributions, namely,  $\alpha$ - and  $\beta$ -rhythms associated with the frequency ranges I (8-13 Hz) and II (14-30 Hz), respectively.

## 3. METHOD

Band-pass filtered experimental process  $f(t)$  was analyzed using the WTMM-method.<sup>6,7</sup> This approach assumes a transition into the space of wavelet coefficients by performing the continuous wavelet-transform<sup>15</sup>

$$W(a, u) = \frac{1}{a} \int_{-\infty}^{\infty} f(t) \psi \left( \frac{t-u}{a} \right) dt, \quad (1)$$

where  $\psi$  is the basic wavelet,  $a$  and  $u$  are the dilation and the translation parameters. In the theory, main characteristics of the multifractal analysis (the Hölder exponents and the singularity spectrum) do not depend

on the used wavelet. In practice, however, there is such a dependence especially for short data series. We used here the MHAT wavelet

$$\psi = (1 - t^2) \exp\left(-\frac{t^2}{2}\right) \quad (2)$$

being the second derivative of the Gaussian function.

Near the singularity  $u^*$ , the wavelet-transform coefficients show the following power-law behavior

$$W(a, u^*) \sim a^{h(u^*)}, \quad a \rightarrow 0^+ \quad (3)$$

characterized by the Hölder exponent  $h(u^*)$ .

Regular behavior of the function  $f(t)$  at the time moment  $u^*$  provides a faster convergence of wavelet-transform coefficients

$$W(a, u^*) \sim a^m, \quad a \rightarrow 0^+ \quad (4)$$

where  $m$  is the number of vanishing moments of the wavelet  $\psi$ . Thus, irregularities produce skeleton lines being the lines of local maxima or minima of the wavelet-transform coefficients.

All skeleton lines detected at the fixed scale parameter  $a$  are extracted from a “surface” of wavelet-coefficients  $W(a, u)$  and used to determine the singularity spectrum within a global approach that assumes statistical analysis of singularities with the partition functions

$$Z(q, a) = \sum_{l \in L(a)} |W(a, u_l(a))|^q, \quad (5)$$

where  $L(a)$  is a full set of skeleton lines,  $u_l(a)$  is the current position of the line  $l$ . This definition does not accounts for the case when  $W(a, u)$  takes zero value, that is why the function  $Z(q, a)$  is typically estimated as

$$Z(q, a) = \sum_{l \in L(a)} \left( \sup_{a' \leq a} |W(a', u_l(s'))| \right)^q \sim a^{\tau(q)}. \quad (6)$$

Scaling exponents  $\tau(q)$  are the intermediate characteristics used to estimate the Hölder exponents and the singularity spectrum

$$h(q) = \frac{d\tau(q)}{dq} \quad (7)$$

$$D(h) = qh - \tau(q). \quad (8)$$

The width of the singularity spectrum

$$\Delta = h_{max} - h_{min} \quad (9)$$

is related to the most important characteristics of the multifractal analysis that can be interpreted as a complexity measure of analyzed data series. Simple processes ( $1/f$ -noise, normal Brownian motion, etc.) are quantified by the function  $D(h)$  consisting of a single point, while more complex processes are described by a broad range of  $h(q)$  and, therefore, a higher complexity is related to larger  $\Delta$ .

## 4. RESULTS AND DISCUSSION

### 4.1 Cerebrovascular dynamics in rats

In this study we analyzed complexity of micro-cerebral dynamics related to individual mechanisms of CBF-regulation in order to reveal responses to abrupt changes in the peripheral blood pressure. For this purpose, we estimated the width of the singularity spectrum for CBF-dynamics in the frequency ranges I, II, and III. Taking into account individual variability of the estimated measure, statistical analysis was performed for relative changes in the width of the singularity spectrum.

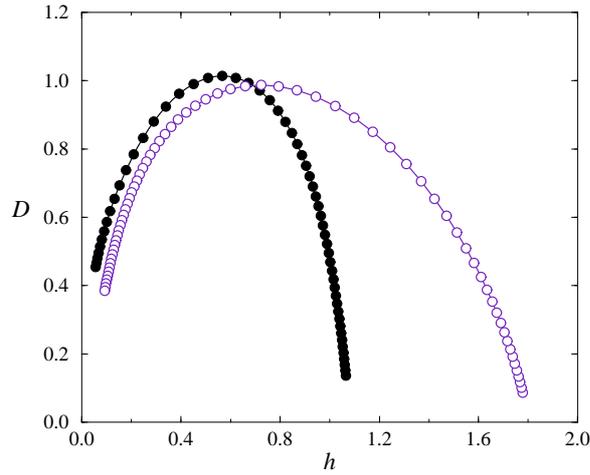


Figure 1. Singularity spectra characterizing the micro-cerebral CBF-dynamics in the frequency range 1 in the normal conditions (filled circles) and during the phenylephrine-related peripheral hypertension (open circles)

Figure 1 shows the occurred changes in the singularity spectrum associated with the frequency range I. The phenylephrine-related peripheral hypertension is accompanied by increased measure  $\Delta$  confirming a transition to a higher complexity of the micro-cerebral blood flow dynamics. Let us note that this effect is mainly observed for the frequency range I, i.e., it is associated with the NO-related endothelial function. The dynamics in other frequency ranges (Figure 2) and the dynamics of CBF in large cerebral vessels demonstrate significantly smaller variations.

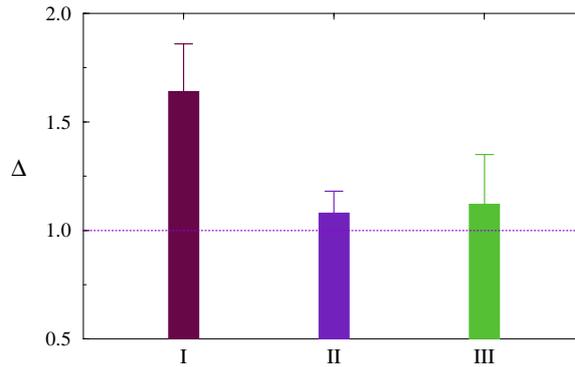


Figure 2. Statistical analysis of relative changes of the measure  $\Delta$  characterizing the microscopic CBF-dynamics in the frequency ranges I, II, and III. Results are shown as mean value  $\pm$  SE. Numerical value of the mean  $\Delta$  is normalized to the value obtained in the case of base-line dynamics related to normal peripheral arterial pressure.

## 4.2 Electrical activity of the brain

Multifractal analysis of EEGs confirms the multiscale structure of the analyzed signals for the original electroencephalograms (Figure 3). Analogous structure is observed for the filtered signals related to the ranges of  $\alpha$  and  $\beta$  rhythms.

At the movement of the hand, changes in the singularity spectrum were detected. Immediately after the movements, the width of the singularity spectrum was essentially reduced as compared with the base-line dynamics. A reduced measure  $\Delta$  was also observed during about 10-15 sec after the movement was stopped. A statistical analysis of complexity measure is given in Figure 4 where the base-line dynamics, the dynamics during the hand's movements and after the movements are shown.

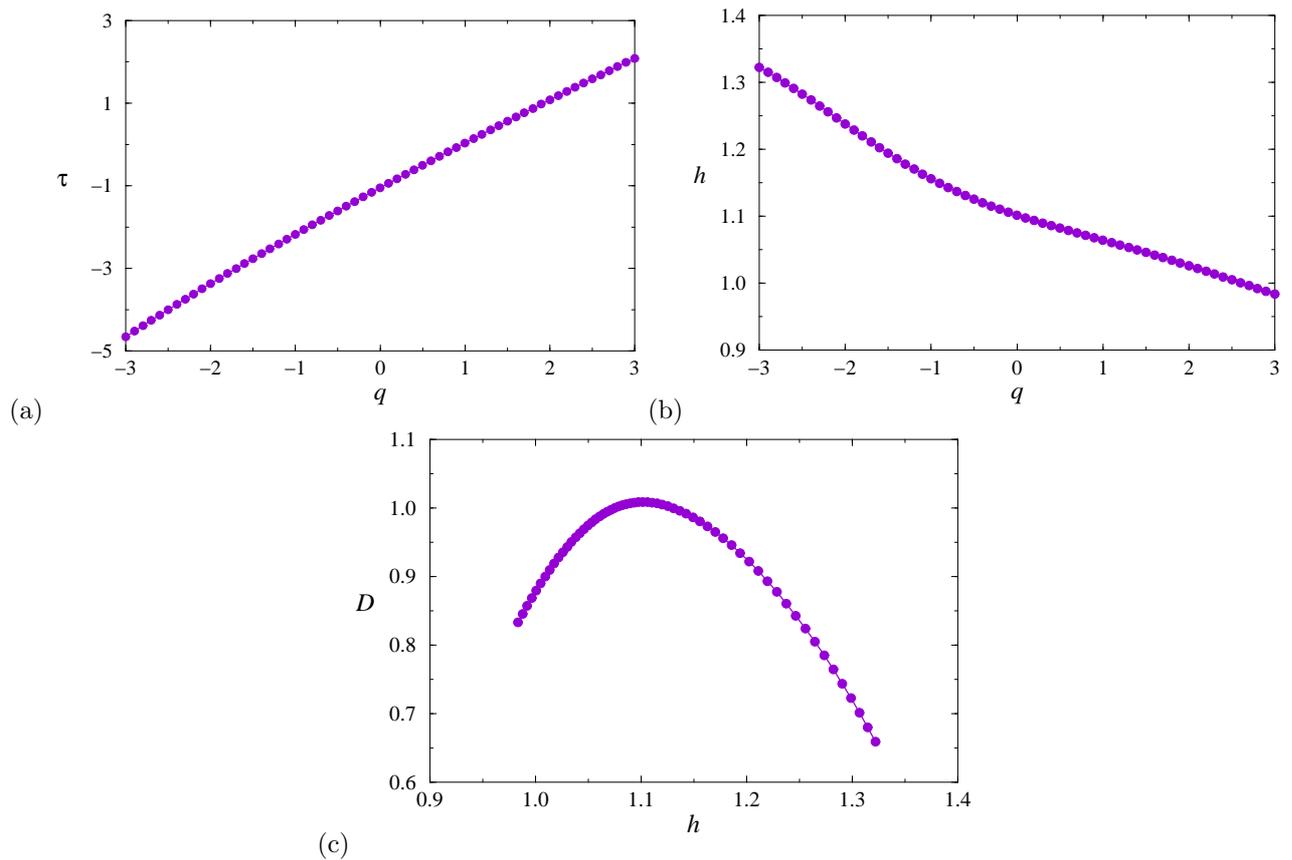


Figure 3. Main characteristics of the multifractal analysis for the base-line EEG-recording: (a) the scaling exponents, (b) the Hölder exponents, and (c) the singularity spectrum.

The results shown in Figure 4 are related to the original EEG-dynamics. Analogously, the dynamics in the frequency ranges of  $\alpha$  and  $\beta$  rhythms can be analyzed. Although a reduced  $\Delta$  is obtained for both these frequency ranges, the strongest changes are associated with the range of  $\alpha$ -rhythm. These changes are well observed at the beginning of the experiments, and they become less expressed after 10-20 similar movements of the hand. In Figure 4, we show statistical results for the first 30 movements (mean value  $\pm$  SE). In general, distinctions between the states 1, 2 and 3 in Figure 4 are reduced with an increased number of movements.

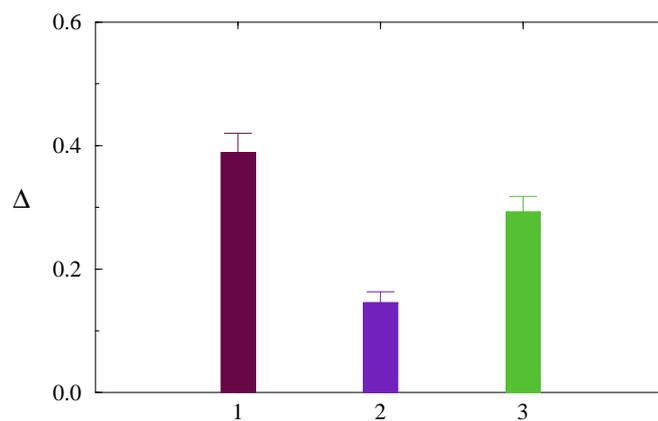


Figure 4. Statistical analysis of the complexity measure  $\Delta$ . The numbers 1, 2, and 3 mark the base-line dynamics, the dynamics during the hand's movements and about 10 sec after the movements, respectively

## 5. CONCLUSION

In this work we discussed the mechanism-related multifractal analysis that assumes a study of band-pass filtered physiological signals in order to characterize changes in the signals structure associated with individual regulatory mechanisms. We analyzed two types of experimental data, namely, the cerebrovascular dynamics in rats characterized by the micro-cerebral blood flow, and the electrical activity of the brain. In both examples, application of the considered approach provides a way of a clearer description of the occurred changes in distinct frequency ranges.

In the case of CBF-data we revealed changes in the micro-cerebral circulation caused by the phenylephrine-related increase in the mean arterial pressure. A separate analysis of several frequency ranges provides an opportunity to associate the occurred changes with the NO-related endothelial function. For EEG-data, the wavelet-based multifractal formalism reveals changes in the multifractal structure of the electrical brain dynamics during hand's movements in the ranges, associated with the frequency areas of  $\alpha$  and  $\beta$  rhythms.

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