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Determining the Degree of Synchronism for Intermittent Phase Synchronization in Human Electroencephalography Data

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Abstract—The phenomenon of intermittent phase synchronization during development of epileptic activity in human beings has been discovered based on EEG data. The presence of synchronous behavior phases has been detected both during spike-wave discharges and in the regions of background activity of the brain. The degree of synchronism in the intermittent phase-synchronization regime in both cases has been determined, and it has been established that spike-wave discharges are characterized by a higher degree of synchronism than exists in the regions of background activity of the brain. To determine the degree of synchronism, a modified method of evaluating zero conditional Lyapunov exponents from time series is proposed.

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Real physical systems frequently exhibit the phenomenon of chaotic phase synchronization [1, 2]. This regime is typical of both non-autonomous and coupled chaotic dynamical systems and is manifested by phase trapping between their states in the absence of correlation between amplitudes [3, 4]. The boundary of chaotic phase synchronization can exhibit a regime of intermittent phase synchronism [5, 6], in which the phase trapping only takes place in certain intervals of time. These intervals are referred to as "periods of laminar (synchronous) behavior." Synchronous intervals are separated by turbulent outbursts characterized by short-time growth in phase difference. This behavior has been also observed in nonautonomous periodic oscillators under the action of external noise, as well as in real neurophysiological systems, e.g., during the development of epileptic activity in both human beings and animals [2, 7].

The notions of "synchronization" and "phase trapping" are closely related to the behavior of the so-called "zero conditional Lyapunov exponent" [8]. In particular, it is known that the passage of the zero conditional Lyapunov exponent to the region of negative values for nonautonomous systems (exhibiting periodic dynamics) in the presence of noise and for coupled chaotic systems precedes the establishment of a phase-synchronization regime. It should be noted that a difference between the critical values of coupling parameters corresponding to the phase synchronization threshold and the moment of Lyapunov exponent passage to the negative region can be rather large. In other words, the zero conditional Lyapunov exponent turns out to be negative long before the onset of phase synchronization and, hence, its value can be treated as a measure of the degree of synchronism for intermittent phase synchronization that takes place at the boundary of a synchronous regime [8–10].

This characteristic value can be readily determined provided that the operator of system evolution is set in an explicit form [11]. For some cases, methods have been developed for solving the problem (by using time series) even in cases where the evolution operator is unknown. However, all the proposed methods have disadvantages (primarily, being highly sensitive to noises and errors), and new methods and algorithms have been proposed in order to overcome these difficulties [12, 13].

The present work was aimed at (i) development of a new, improved method of determining the zero conditional Lyapunov exponent for interacting systems from their time series and (ii) application of this method to estimating the degree of synchronism for an intermittent phase-synchronization regime in a real neurophysiological system. Once made available, this method will ensure more effective data processing and provide detection of the presence of synchronism and determination of its degree on the basis of neurophysiological data. This, in turn, will be useful for detecting and using this effect in practical applications, in particular, for medical diagnostics. On the other hand, this method can be used as an independent tool in investigations, since it can be applied to relatively short experimental time series of any nature, making possible diagnostics of the presence and degree of synchronism in the observed regime where this can be done by other methods.

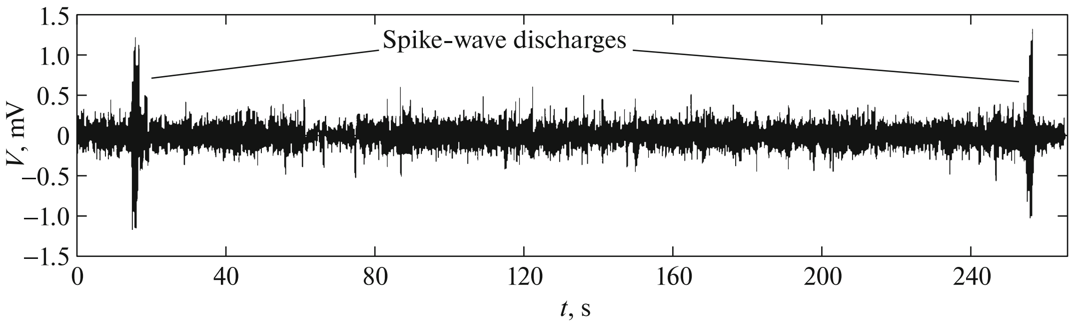

Fig. 1. Typical EEG signal taken from the C3 region of the human brain.

Signals for investigation were selected from real experimental data of neurophysiological nature, namely, electroencelophalograms (EEGs) taken from various regions of the brain of a patient suffering from epilepsy. Figure 1 shows an example of this signal.[1] Epileptic EEGs comprise intermittent time series involving spike-wave discharges alternating with the regions of background activity of the brain. As is known, spike-wave discharges are characterized by a high degree of synchronism [7, 14]. At the same time, investigations showed that the regions of background activity of the brain can also exhibit phases of synchronous behavior.

Figure 2 illustrates temporal variation of the phase difference $x(n) = \varphi_1 - \varphi_2$ of signals taken from C3 and Cz regions (for EEG electrode arrangement see [15, Fig. 24]). Here and below, only phases of synchronous behavior are presented. Figure 2a concerns spike-wave discharges, while Fig. 2b corresponds to synchronous regions of background activity of the brain. Phases $\varphi_{1,2}$ of the EEG signals were defined in terms of a continuous wavelet transform using the Morlet wavelet basis set [7, 16].

Synchronous regions with dynamics of each type were processed using the following method of estimation of the zero conditional Lyapunov exponent. By analogy with [9, 17], the Lyapunov exponent was calculated using the following formula:

$$\Lambda = \int_{x_1}^{x_2} \rho(x) \ln|1 + 2\Omega x| dx, \quad (1)$$

where $\rho(x)$ is the probability density of variable x. The probability-density distribution was numerically calculated and approximated by the following analytical expression:

$$\rho(x) = A \exp\left[-\frac{2}{D}\left(\varepsilon x - \frac{\Omega x^3}{3}\right)\right], \quad (2)$$

[1]EEG signals from other parts of the human brain are analogous to those presented in Fig. 1.

where A is the normalization coefficient defined as

$$\int_{x_1}^{x_2} \rho(x) dx = 1, \quad (3)$$

D is the noise intensity, and ε and D are control parameters. Relation (2) is characteristic of the region of supercritical values of control parameter ε that corresponds to a regime of intermittent phase synchronization [8, 9].

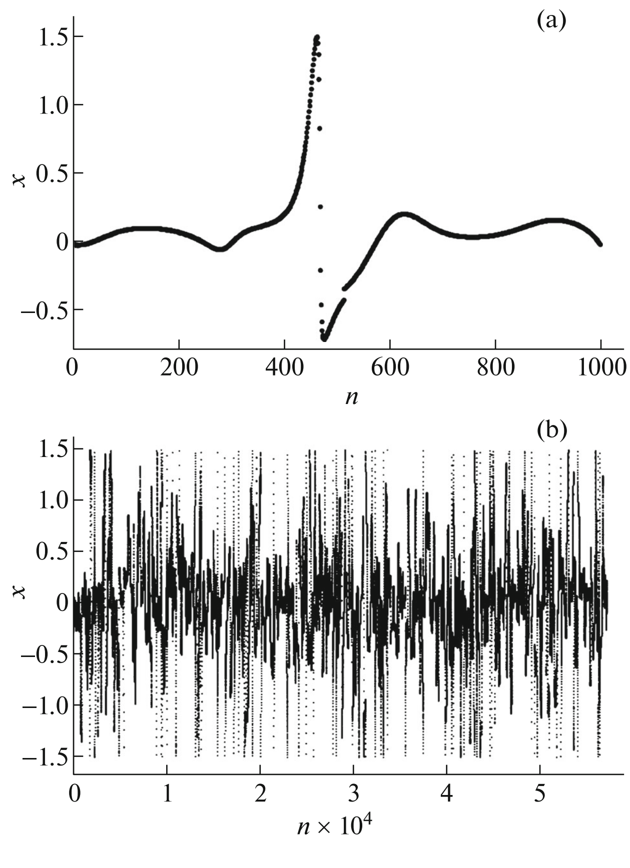

Fig. 2. Temporal variation of phase difference $x(n)$ between signals taken from the C3 and Cz regions during (a) spike-wave discharges and (b) synchronous phase of background activity of the brain (n is the discrete time).

The search for parameters of the approximation was carried out as follows. For determining D, it was assumed that the distribution of probability density $\rho(x)$ resembles the Gaussian distribution and can be written in a simplified form as

$$\rho_G(x) = A_G \exp[-2B(x-K)^2], \quad (4)$$

where K and B are analogs of the mathematical expectation and variance, respectively, and A_G is the normalization factor. Parameter K corresponds to the maximum of distribution (4), and, hence, it is related to parameters Ω and ε of distribution (2) as

$$K = -\sqrt{\frac{\varepsilon}{\Omega}}. \quad (5)$$

Upon expanding the right parts of Eqs. (2) and (4) in Taylor series at point (5), restricting expansion to the second order of smallness, and comparing the expansion coefficients at equal powers, we arrive at the following relation between parameters B and D:

$$D = \frac{\sqrt{\varepsilon\Omega}}{B}. \quad (6)$$

Parameters A_G and K can be estimated from comparison of the maxima of the numerically calculated probability distribution and its approximation by formula (4), while parameter B is determined using the method of least squares. Then, the relation between D, ε, and Ω is given by Eq. (6) with known B, which makes it possible to determine the other parameters of approximation. The relation between parameters A, ε, and Ω is also provided by comparison of the maxima of analytically (formula (2)) and numerically calculated distributions; parameter Ω is determined using the method of least squares; and parameters x_1 and x_2 in formula (1) are estimated empirically from the form of distribution $\rho(x)$.

The proposed method has been applied to both spike-wave discharges and the regions of background activity of the brain. Figures 3a and 3b show the obtained distributions of the probability density of phase difference in comparison to their approximation by formula (2) for spike-wave discharges and for the synchronous regions of background activity, respectively. the parameters of approximation were $B \approx 56.70$, $D \approx 0.182$, $\varepsilon \approx 0.468$, $A \approx 1.917 \times 10^{-38}$, and $\Omega \approx 0.4$ in the former case and $B \approx 2.30$, $D \approx 0.182$, $\varepsilon \approx 0.441$, $A \approx 0.384$, and $\Omega \approx 0.4$ in the latter case. The corresponding Lyapunov exponents in both cases are negative and their ratio $\Lambda_1/\Lambda_2 \approx 1.12$ is indicative of a higher degree of synchronism in spike-wave discharges than that in the regions of background activity of the human brain.

Thus, we have developed a method of estimating the degree of synchronism in the intermittent phase-synchronization regime, which employs time series and is based on calculations of the zero conditional Lyapunov exponents. The proposed method was applied to neurophysiological data representing EEGs of a patient suffering of epilepsy. It has been established that spike-wave discharges are characterized by a higher degree of synchronism than that in the regions of background activity of the brain.

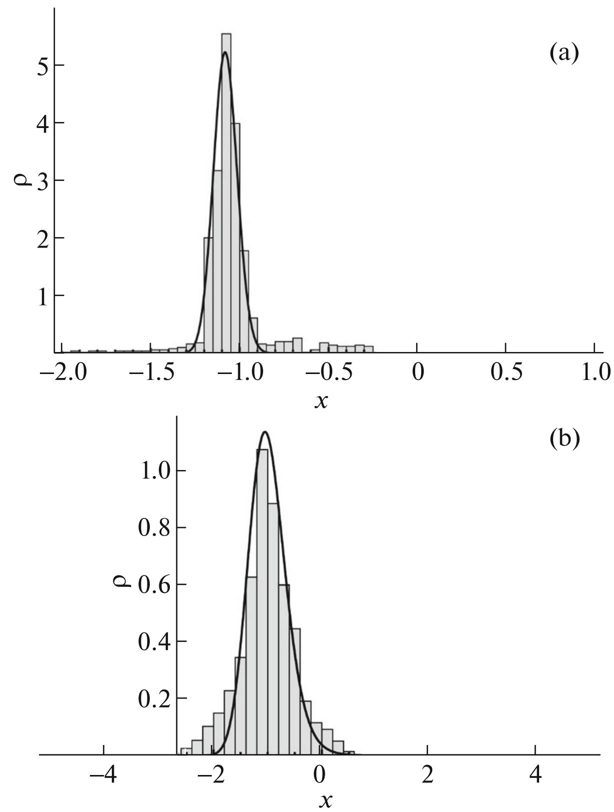

Fig. 3. Distributions (histograms) of probability density $\rho(x)$ for (a) spike-wave discharges and (b) regions of background activity of the human brain and their approximation (curves) by relation (2).

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