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THEORY AND METHODS OF SIGNAL PROCESSING

Application of the Dual-Tree Wavelet Transform for Digital Filtering of Noisy Audio Signals

A. S. Yasin[a, b], **A. N. Pavlov**[a, c, d, *], **and A. E. Hramov**[a, c, d]
[a] *Chernyshevskii State University, Saratov, 410012 Russia*
[b] *University of Technology, Baghdad, Iraq*
[c] *Yuri Gagarin State Technical University of Saratov, Saratov, 410054 Russia*
[d] *Kotel'nikov Institute of Radio Engineering and Electronics (Saratov Branch), Russian Academy of Sciences, Saratov, 410019 Russia*
**e-mail: pavlov.lesha@gmail.com*
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Abstract—The problem of refinement of the quality of filtering of noisy audio signals with the help of the methods based on a discrete wavelet transform with real bases and a dual-tree (complex) wavelet transform using analytical wavelets as basis functions is considered. Test examples and processing of experimental data have shown that, in the case of the optimum selection of the threshold level, the approach using the dual-tree wavelet transform ensures the minimum signal reconstruction error after correction of wavelet coefficients.

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INTRODUCTION

Currently, methods based on the wavelet transform [1–5] are widely used for solution of the problems of digital filtering of noisy signals. Numerous studies of variants of signal filtering with the help of the discrete wavelet transform (DWT) ensuring high-quality cleaning of experimental data from noise are discussed, in particular, in [6–10]. The main idea of wavelet filtering consists in the signal expansion at different resolution levels in which wavelet coefficients with small magnitudes predominantly correspond to fluctuations and coefficients with large magnitudes correspond to the noise-free signal. For this reason, an evident filtering variant consists in setting to zero small wavelet coefficients associated with noise and random distortions and subsequent reconstruction of the signal in the course of the inverse wavelet transform [2]. The quality of this filtering procedure depends on selection of the threshold function by which the wavelet coefficients are multiplied. Two variants, the "soft" variant and the "hard" variant, are traditionally used [6–8]. The first variant provides correction of all wavelet coefficients, while the second variant provides correction of only those coefficients that do not exceed a specified threshold value.

While approaches based on the DWT are considered as standard variants of wavelet filtering, they have several drawbacks among which oscillations of wavelet coefficients near singularities complicating the signal processing procedure, noninvariance with respect to shift causing unpredictable changes in the patterns of wavelet coefficients in the case of shifts of singularities, and the appearance of artifacts in the reconstructed signal after correction of wavelet coefficients. In order to eliminate these drawbacks, the method of the dual-tree complex wavelet transform (DCWT) was proposed and then refined in [11–15]. This approach is approximately invariant with respect to shift and operates with complex (analytic) wavelets constructed on the basis of real wavelet functions by adding the imaginary part calculated with the help of the Hilbert transform. The DCWT method provides independent calculation of two DWTs for determination of real and imaginary parts of wavelet coefficients.

In this paper, we compare wavelet filtering methods based on the DWT and the DCWT for test examples and audio signals representing voice messages recorded in the presence of noise of different intensity. We show that application of the DCWT method can substantially increase the quality of digital wavelet filtering by lowering the signal reconstruction error after correction of wavelet coefficients.

1. FILTERING BASED ON THE DISCRETE WAVELET TRANSFORM

The discrete wavelet transform provides signal expansion in the basis of localized functions, which represents a set of rescaled and shifted versions of parent wavelet $\psi(t)$ [1]. Implementation of the DWT provides feeding of the signal to the input of two conjugate quadrature mirror filters. Transmission of a signal through a low-pass filter can be considered as approximation of this signal at different resolution levels and transmission through a high-pass filter is interpreted as detailing at a specified resolution level [2]. After expansion of a signal in terms of scaling functions $\varphi(t)$ and wavelets $\psi(t)$, we obtain a set of coefficients containing information on the signal structure at different resolution levels [1, 4]:

$$f(t) = \sum_k c_k \varphi(t-k) + \sum_j \sum_k d_{j,k} 2^{j/2} \psi(2^j - k). \quad (1)$$

Expansion coefficients in terms of wavelet functions $d_{j,k}$ characterize amplitude components of the signal at different scales and different time instants. In the course of filtering, small wavelet coefficients are set to zero. In practice, two main variants of selection of the threshold function, the hard variant and the soft variant, are used. In the case of the hard variant of the threshold function,

$$p(x) = \begin{cases} x, & |x| \geq C \\ 0, & |x| < C \end{cases}, \quad (2)$$

only coefficients with small magnitudes are set to zero [6], which keeps the signal amplitude undistorted but is accompanied by the appearance of irregularities caused by the discontinuous nature of function (2). The soft variant of the threshold function,

$$p(x) = \begin{cases} x - C, & x \geq C \\ x + C, & x \leq -C \\ 0, & |x| \leq C \end{cases}, \quad (3)$$

eliminates irregularities but, in this case, all wavelet coefficients $d_{j,k}$ are corrected, by means of multiplication by function (3) and the signal amplitude becomes smaller [7]. It should be noted that correction of all coefficients is not an intrinsic drawback of the method. In particular, in signal reception in communication systems, it is important to ensure efficient suppression of interferences. Afterwards, the filtered signal can be amplified.

The key problem in both variants of the threshold function is selection of the value of parameter C. As follows from Fig. 1 presented for the test example (noisy harmonic oscillations), parameter C takes different optimum values for soft and hard setting of the threshold function. Application of threshold function (3) allows one to attain lesser value of the signal reconstruction error. The value of parameter C determines the noise level that should be filtered out. One of widely used methods of selection of the value of parameter C is the universal threshold level proposed in [6]:

$$C = \sigma\sqrt{2\ln N}, \quad (4)$$

where σ is the standard deviation of noise and N is the number of wavelet coefficients ($N \geq 4$). The number of coefficients in the discrete wavelet transform

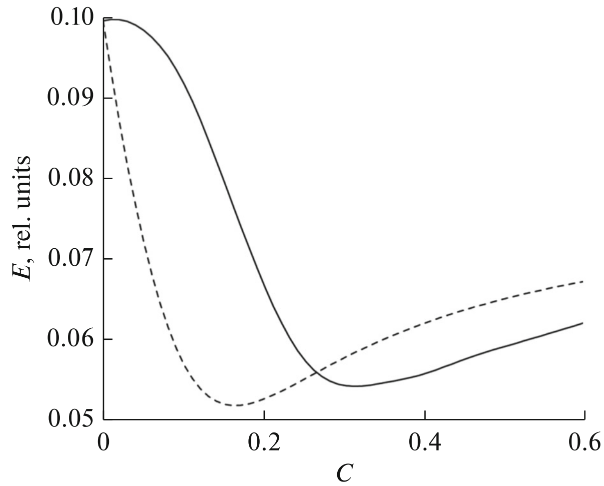

Fig. 1. Dependences of the rms reconstruction error on parameter C for the test signal (harmonic oscillations). The reconstruction method is based on the DWT with (solid curve) hard and (dashed curve) soft thresold functions. Calculations were performed for the Daubechies wavelet D8 and a noise level of 20 dB.

changes by a factor of two after passage from one resolution level to another. As a result, both approaches based on global setting of the threshold value (C takes a fixed value independent of the resolution level) and approaches providing setting of different threshold levels C_j depending on resolution j can be applied. Estimate σ is calculated by the following formula [10]:

$$\frac{M(|d_{J-1,k} - M(d_{J-1,k})|)}{0.6745}, \quad (5)$$

where M is the median and J is the maximum resolution level of the DWT. Selection of the preceding level $J-1$ stems from the fact that, at this level, wavelet coefficients predominantly correspond to noise. The probability of signal distortions during threshold filtering or the filtering error [7], which can be lowered with the help of methods using minimum threshold values causing lesser signal distortions, is often used as a quantitative criterion.

For this purpose, an alternative threshold setting method, the so-called SURE method [16], is widely used. According to this method, the estimated value of threshold level $\hat{C}$ is calculated as follows:

$$\hat{C} = \arg\min_C \sum_{m=0}^{N-1} P(D_m),$$
$$P(D_m) = \begin{cases} D_m^2 - \sigma^2, & D_m \leq C \\ \sigma^2 + C^2, & D_m > C \end{cases}. \quad (6)$$

According to the performed investigations, the value of quantity $\hat{C}$ is close to the optimum threshold value. Nevertheless, this approach may also result in errors at

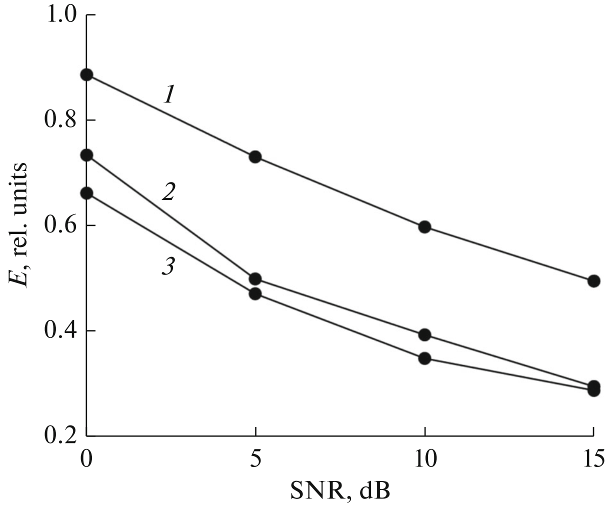

Fig. 2. Dependences of the rms reconstruction error on the singnal-to-noise ratio for the test signal (harmonic oscillations) in the case of reconstruction based on the DWT method using the threshold function (formula (3)) and Daubechies wavelets D20: (*1*) the universal threshold level, (*2*) combined application of the universal threshold level and the SURE method, and (*3*) the SURE method.

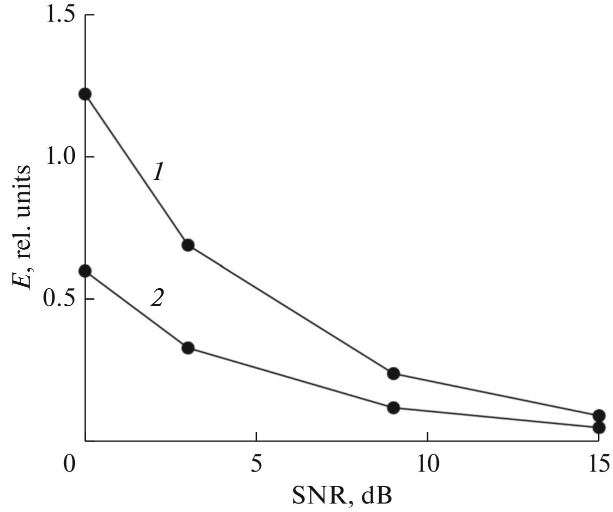

Fig. 3. Dependences of the optimum threshold level on the signal-to-noise ratio for the case of wavelet filtering of the test signal (harmonic oscillations) with the use of the (*1*) DWT and (*2*) DCWT methods.

large intensities of noise. In such situations, it is preferable to combine the universal threshold level and the SURE method: if wavelet coefficients are small, the former approach is used; otherwise, the latter method is applied. In order to determine selection of a particular method, the minimum energy level $\varepsilon_N = \sigma^2 N^{1/2} (\ln N)^{3/2}$ is specified and the threshold value is determined as follows:

$$C = \begin{cases} \sigma\sqrt{2\ln N}, & \|D_m\|^2 - N\sigma^2 \le \varepsilon_N \\ \hat{C}, & \|D_m\|^2 - N\sigma^2 > \varepsilon_N \end{cases}. \quad (7)$$

Different variants of selection of threshold level C in the case of soft setting of the threshold function are compared in Fig. 2 for the test example (harmonic function with additively admixed noise). According to the obtained results, the approach based on setting the universal threshold level results in the maximum rms error. The SURE method ensures the minimum error.

2. FILTERING BASED ON THE DUAL-TREE (COMPLEX) WAVELET TRANSFORM

Approaches based on the standard DWT have several substantial drawbacks, which were mentioned in Introduction. Besides, they cannot give information on phase relationships, which is often required in solution of some practical problems, for example, problems of interaction of self-oscillating systems. In order to improve methods based on the wavelet transform, the approach applying the complex wavelet transform was proposed [11, 12]. Since term “complex wavelet transform” is usually associated with application of a continuous wavelet transform with complex basis functions, the method described in [11, 12] will be hereinafter called the dual-tree wavelet transform. The basic idea of this approach consists in addition of imaginary parts obtained with the help of the Hilbert transform to real scaling functions and wavelets, which results in complex (analytic) low-pass and high-pass mirror filters.

In accordance with the DCWT method, we consider complex wavelets $\psi^c(t) = \psi^r(t) + j\psi^i(t)$ and form two orthonormal bases using functions $\psi^r(t)$ and $\psi^i(t)$. The wavelet transform is calculated independently with the use of each basis. As a result, we obtain complex wavelet coefficients $d^c_{j,k} = d^r_{j,k} + jd^i_{j,k}$. Algorithmically, the DCWT method is reduced to two independent pyramidal expansions of the signal. Unlike the standard DWT, an additional requirement is imposed: scaling functions and wavelets $\psi^c(t)$ must be analytic functions. In order to satisfy this requirement, special algorithms for construction of mirror filters are used [15]. In the performed studies, filters proposed in [11] and their MATLAB computer implementation developed in [17] were used.

The results of comparison of wavelet filtering of the test signal (harmonic function with additively admixed noise) based on the DWT and DCWT methods are presented in Fig. 3. As shows the analysis of the obtained data, the optimum threshold level for the DCWT method is lower than for the DWT method for all considered values of the signal-to-noise ratio (SNR). This means that application of the DCWT method ensures lowering of the probability of possible signal distortions resulting from threshold filtering (the lesser the value of parameter C, the lower the probability of removal of informative wavelet coeffi-

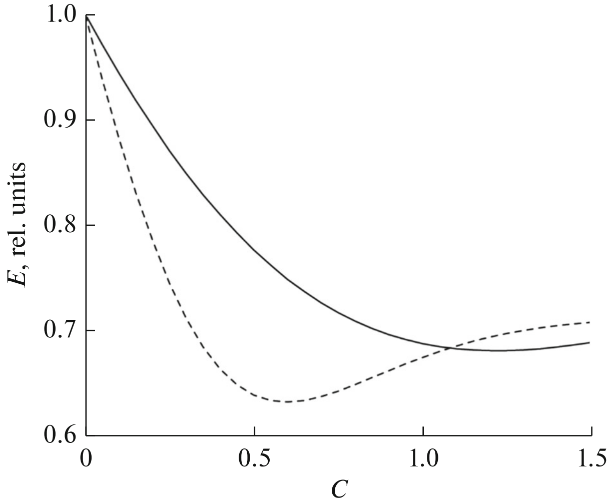

Fig. 4. Dependences of rms reconstruction error E on the threshold level for the case of reconstruction of the test signal (harmonic oscillations) with the use of the (solid curve) DWT and (dashed curve) DCWT methods. Calculations were performed for the Daubechies wavelet D20 and a noise level of 0 dB.

cients in the course of filtering), which is one of obvious advantages of this approach.

Examples of dependences of the rms filtering error for the DWT and DCWT methods are presented in Fig. 4. According to these examples, the DCWT method ensures the minimum value of the rms filtering error in the case of appropriate selection of the threshold value.

3. WAVELET FILTERING OF AUDIO SIGNALS

After comparison of wavelet filtering methods on the test example, let us consider application of these methods to experimental data. For this purpose, we consider various audio signals, predominantly voice messages, with additively admixed white noise of different intensity. An example of a fragment of the voice message and the results of wavelet filtering of different signals are presented in Fig. 5. Since spectral bands of signal and noise overlap, wavelet filtering cannot completely remove existing fluctuations and setting of large values of threshold levels for more efficient noise suppression results in distortions of the information message. For this reason, selection of optimum parameters of performed filtering is an important problem.

Visual analysis of signals shown in Figs. 5c, 5d, and 5e indicates that the DCWT method ensures minimum distortions. The results of calculation of the minimum error attained in the course of wavelet filtering based on the DWT method (variant of soft setting of the threshold function, Daubechies wavelet bases from D3 to D20) as well as the results of calculation of the minimum filtering error in the case of application of the DCWT method are presented in Fig. 6.

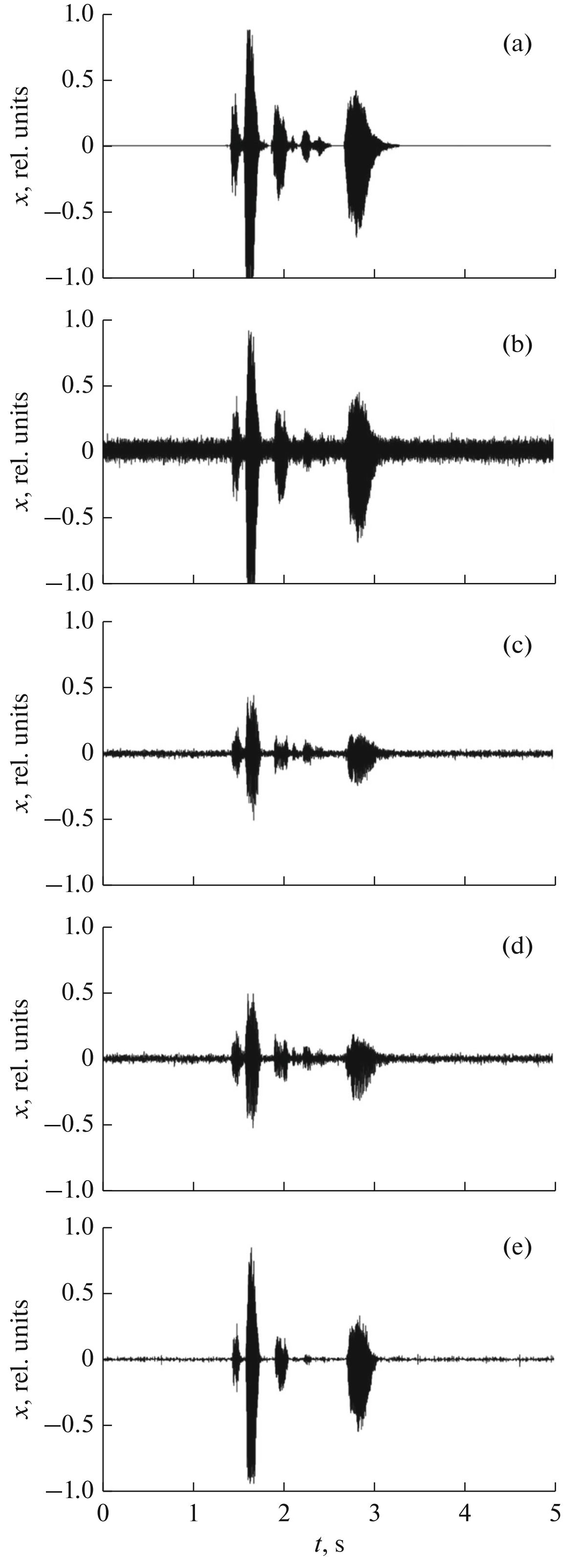

Fig. 5. Results of wavelet filtering of a fragment of voice message: (a) original signal, (b) signal with admixed additive noise, (c) signel after filering by the DWT method with function (3), (d) signal after filtering by the DWT method with function (2), and (e) signal after filtering by the DCWT method.

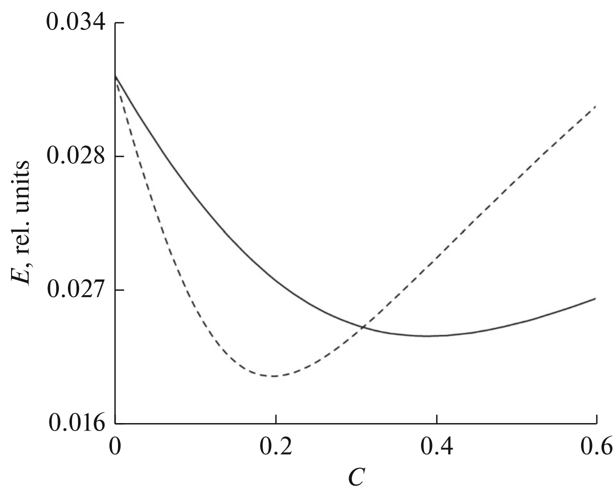

Fig. 6. Dependences of rms reconstruction error E on parameter C for reconstruction of the audio signal by the (solid curve) DWT and (dashed curve) DCWT methods. Calculations were performed for the Daubechies wavelet D8 (which ensured the minimum filtering error) and a noise level of 30 dB.

Note that these methods have different optimum values of the threshold level: the optimum threshold level is 0.020 for the DCWT method and is approximately two times higher ($C = 0.039$) for the DWT method. In the case of selection of the optimum value of parameter C, the rms error for the DCWT method is approximately 9% less than for the DWT-based method, which is a substantial improvement of the quality of digital filtering. Comparable results (lowering of the error by an average of 8%) were also obtained for other examples of audio signals and different values of SNR. Thus, we can assert that the DCWT method has apparent advantages over the standard variant of wavelet filtering using the DWT method.

CONCLUSIONS

Methods of wavelet filtering of noisy signals using real and complex basis functions have been compared. A method based on the discrete wavelet transform with basis functions of the Daubechies family has been tested for cases of soft and hard introduction of the threshold function and three variants of setting the threshold level. The method of the dual-tree (complex) wavelet transform, which use analytic functions as wavelets and provides addition of imaginary parts calculated on the basis of the Hilbert transform to real basis functions, has been considered. This approach can eliminate such substantial drawbacks of the standard DWT as noninvariance with respect to shift and the appearance of artifacts in the reconstructed signal.

As has been shown by the test example of a harmonic function with additively admixed white noise, the standard DWT method ensures the minimum signal reconstruction error in the course of wavelet filtering in the case of selection of the soft variant of setting the threshold function and introduction of the threshold level according to the SURE method. This deduction has been confirmed for different SNR values in the analyzed data. The DCWT method can lower the digital filtering error as compared to that of the standard approach based on the DWT method and results in additional lowering of the threshold level, which corresponds to lowering of the risk of possible distortions as a result of threshold filtering.

These conclusions have been confirmed during the analysis of audio signals representing fragments of voice messages with additively admixed white noise of different intensity. The analysis of experimental data indicates the superiority of the DCWT-based wavelet filtering method. This approach has ensured an average of 8-% lowering of the wavelet filtering error in the case of optimal selection of the threshold level. In practice, the variant of setting the threshold level according to the SURE method can be used to set a near-optimum threshold level.

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REFERENCES

1. Y. Meyer, *Wavelets: Algorithms and Applications* (SIAM, Philadelphia, 1993).
2. M. Vetterli and J. Kovacevic, *Wavelets and Subband Coding* (Prentice Hall, New Jersey, 1995).
3. I. Daubechies, *Ten Lectures on Wavelets* (SIAM, Philadelphia, 1992; NITS RKHD, Izhevsk–Moscow, 2001).
4. I. M. Dremin, O. V. Ivanov, and V. A. Nechitailo, Usp. Fiz. Nauk. **171**, 465 (2001).
5. A. N. Pavlov, A. E. Hramov, A. A. Koronovskii, et al., Usp. Fiz. Nauk. **182**, 905 (2012).
6. D. L. Donoho and I. M. Johnstone, Biometrika **81**, 425 (1994).
7. D. L. Donoho, IEEE Trans. Inf. Theory **41**, 613 (1995).
8. M. Jansen, *Noise Reduction by Wavelet Thresholding* (Springer-Verlag, New York, 2001).
9. H. Zhang, T. R. Blackburn, B. T. Phung, and D. Sen, IEEE Trans. Dielectr. Electr. Insul. **14**, 3 (2007).
10. S. G. Chang, B. Yu, and M. Vetterli, IEEE Trans. Inf. Theory **9**, 1532 (2000).
11. N. G. Kingsbury, Philos. Trans. R. Soc. London A **357**, 2543 (1999).
12. N. G. Kingsbury, Appl. Comput. Harmon. Anal. **10**, 234 (2001).
13. I. W. Selesnick, IEEE Signal Process. Lett. **8**, 170 (2001).
14. I. W. Selesnick, IEEE Trans. Signal Process. **52**, 1304 (2004).
15. I. W. Selesnick, R. G. Baraniuk, and N. G. Kingsbury, IEEE Signal Process. Mag. **22** (6), 123 (2005).
16. C. M. Stein, Ann. Stat. **9**, 1317 (1981).
17. http://eeweb.poly.edu/iselesni/WaveletSoftware/.

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