Numerical analysis of the chimera states in the multilayered network model

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ABSTRACT

We numerically study the interaction between the ensembles of the Hindmarsh-Rose (HR) neuron systems, arranged in the multilayer network model. We have shown that the fully identical layers, demonstrated individually different chimera due to the initial mismatch, come to the identical chimera state with the increase of inter-layer coupling. Within the multilayer model we also consider the case, when the one layer demonstrates chimera state, while another layer exhibits coherent or incoherent dynamics. It has been shown that the interactions chimera-coherent state and chimera-incoherent state leads to the both excitation of chimera as from the ensemble of fully coherent or incoherent oscillators, and suppression of initially stable chimera state

Keywords: Chimera state, multilayered network, Hindmarsh-Rose neuron system

1. INTRODUCTION

Currently, the world scientific community demonstrates the great interest in the study of chimera states,[1,3,4] arising in the ensembles of nonlinear oscillators of the different nature.

Such states, characterized by the coexistence of the groups of coherent and incoherent elements in the networks of coupled oscillators, were first discovered in 2002 by Kuramoto and Battogtokh[5] in the network of non-locally coupled nonlinear elements described by the equation of the Ginzburg-Landau.

In the last decade a large number of papers was published to describe the implementation of the chimera state in the networks of nonlinear elements, characterized by different types of the nodes. In this context, the formation of the chimera and chimera-like[6,7] states was considered in one-dimensional systems (chains of coupled oscillators), as well as in distributed systems consisting of equidistantly arranged Rossler,[8] Fitzhugh-Nagumo oscillators,[13] etc.

Together with the consideration of the networks, characterized by the different nodes, the chimera state were also demonstrated for the different properties of the network topology. In particular, in,[10,11] the chimera states were demonstrated to exist in the network of Hindmarsh-Rose neurons in the case of global, non-local, and, even, the local coupling. In addition to symmetric links, chimera was also observed in scale-free network.[12]

Along with the analysis of model systems, the interest in a chimera state is also induced by the experimental evidences of the detection of the states in the the real systems, whose properties like the chimera. Among these systems one can find the objects of biological, chemical and electronic nature.[13–15]

Among the large number of effects that are associated with the occurrence of chimera, an important and less studied topic is the stability of the latter,[16] including, the cases of the interactions between the system, demonstrating chimera, and the system, which is characterized by the coherent behavior of all nodes or the

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Dynamics and Fluctuations in Biomedical Photonics XIV, edited by Valery V. Tuchin, Kirill V. Larin, Martin J. Leahy, Ruikang K. Wang, Proc. of SPIE Vol. 10063, 100631J · © 2017 SPIE
CCC code: 1605-7422/17/$18 · doi: 10.1117/12.2251872

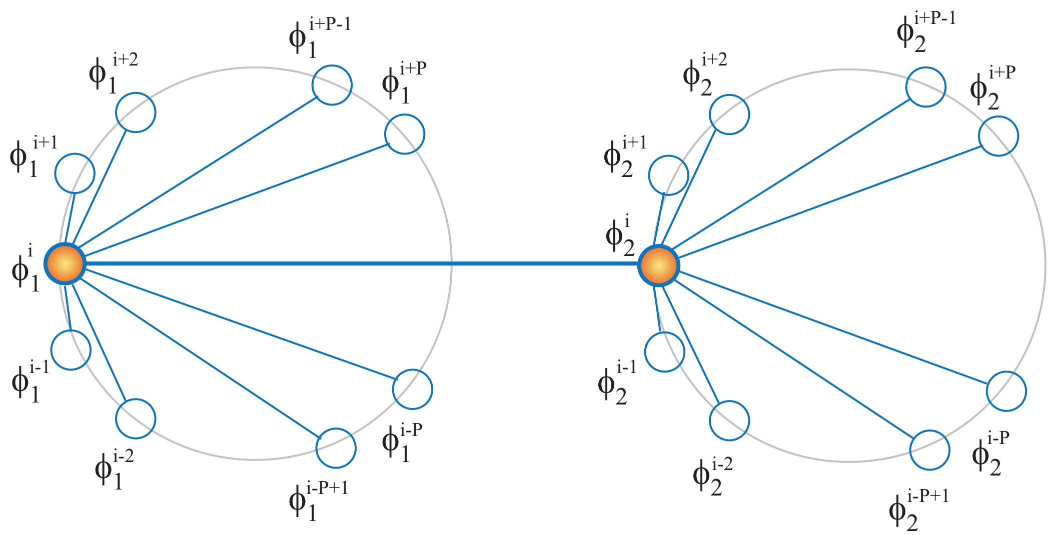

Figure 1. Schematic representation of multiplex structure.

complete incoherence. Obviously, this situation is common in real systems, related to the relevant areas of science (eg, neuroscience[17,18]) and its consideration, along with a theoretical interest, attracting the attention due to the prospect of the practical use.[19]

In the current research we analyze the interaction between the networks of nonlocally coupled oscillators which can demonstrate the chimera state, as well as exhibit coherent/incoherent behaviour with the help of multilayer network model. Such model is widely used in the last years both for the analysis of real data sets and mathematical modeling due to multilayer character of real-world network.[20,21] The analysis of multilayer networks started with a reformulation of classical topological parameters, such as the shortest path length, clustering coefficient, centrality or robustness of the nodes.[22–25] From the dynamical perspective, the multilayer formulation has been applied both to networks whose layers coexist or alternate in time.[21] In both cases, the multilayer formulation allows to identify synchronization regions that arise as a consequence of the interplay between the layers topologies,[26–28] as well as to define new types of synchronization based on the coordination between layers.[29]

Generally, the multilayer model is characterized by the nodes, which have the two types of the links (see Fig. 1). The first type is characterized by the interaction between the nodes, located in the same layer. The second type determines the coupling of this element with the elements belonging to different layers. Depending on the specific objectives of the multilayer configuration the relations between the elements within the network may be quite different. In this work we focus on the network, which inter-layer relations match the model, described in a recent paper.[30]

2. NUMERICAL MODEL UNDER STUDY

We study the realistic two-layered multiplex network of the Hindmarsh-Rose (HR) neuron systems. This HR model is realistic because depending upon the parameter values, individual oscillator exhibits different types of behaviors, such as square-wave bursting (chaotic and periodic), mixed mode bursting, spiking and plateau bursting etc. Recently, existence of chimera states was reported in a single layer network of HR-neuronal models using non-local and local coupling interaction. Here we show how multiplex interaction between layers of Hindmarsh-Rose neurons influences on chimera states in the system, and examine the effect of delay in this coupling.

We consider a network consisting of two multiplexing layers of 100 HR systems with realistic chemical synaptic

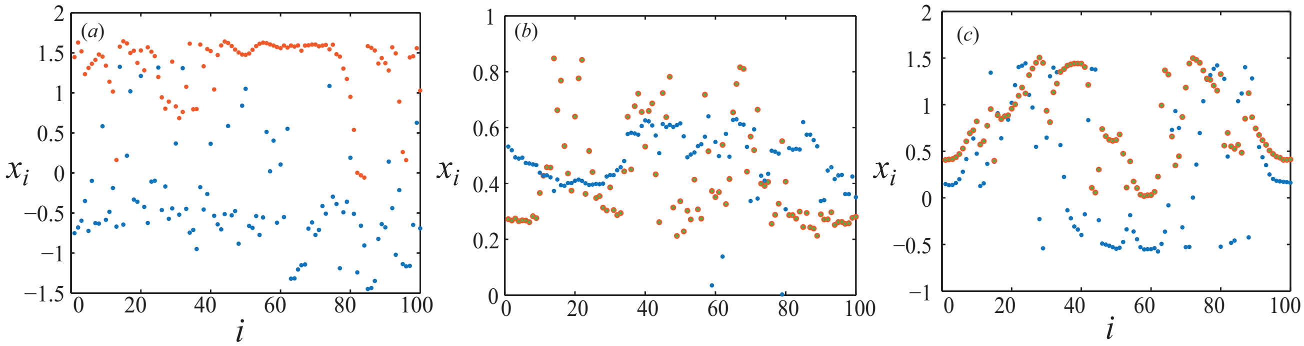

Figure 2. (Color online) Snapshots of x_i of the HR layers for (a) incoherent state in one layer (red dots) at $\lambda_1^1 = 0.2$ and chimera state in second layer (green dots) at $\lambda_1^2 = 1.0$ and $\lambda_3 = 0$, (b) asynchronous (ASC) chimera state at $\lambda_3 = 0.16$, and (c) synchronous chimera (SC) state at $\lambda_3 = 0.22$. Number of oscillators in each layer, $N = 100$.

nonlocal intra-layer interaction and inter-layer synaptic coupling with the presence of delay τ,

$$\begin{aligned} \dot{x}_i^j &= a(x_i^j)^2 - (x_i^j)^3 - y_i^j - z_i^j \\ &+ \tfrac{\lambda_1^j}{2P^j}(v_s - x_i^j)\textstyle\sum_{k=i-P^j}^{k=i+P^j} c_{ik}^j \Gamma(x_k^j) \\ &+ \lambda_3(v_s - x_i^j)\textstyle\sum_{l \neq j} \Gamma(x_i^l(t-\tau)) \\ \dot{y}_i^j &= (a+\alpha)(x_i^j)^2 - y_i^j, \\ \dot{z}_i^j &= c(bx_i^j - z_i^j + e), \quad i = 1, 2, \cdots, N \end{aligned} \tag{1}$$

where the state variable x represents the membrane potentials and, y and z correspond to the transport of ions across the membrane through the fast and slow channels respectively. The parameter c represents the ratio of slow-fast time scale. The synaptic coupling function $\Gamma(x)$ is a nonlinear input-output function as

$$\Gamma(x) = \frac{1}{1 + e^{-\lambda(x-\Theta_s)}}, \tag{2}$$

where $\lambda > 0$ determine the slope of the function and Θ_s is the firing threshold. We take the reversal potential $v_s = 2.0$ so that $v_s > x_i$ for all time t for which the synapses are excitatory and the input from other neurons to the i-th neurons can enhance the activity. We choose the synaptic threshold $\Theta_s = -0.25$ and slope of the sigmoidal function $\lambda = 10$. Here $\lambda_1^j (j = 1, 2)$ is the intra-layer coupling strength for the j^{th} layer. λ_3 is the inter-layer synaptic coupling strength and τ is the time-delay in transferring the information between the layers. The connectivity matrix $C = (c_{ik}^j)_{(n \times n)}$ is such that $c_{ik}^j = 1$ if the i-th neuron is connected with k-th neuron in the layer j and zero otherwise. P^j $(j = 1, 2)$ is the number of neighboring oscillators in both directions connected with each oscillator in each layer. The individual oscillator exhibits square-wave bursting for a choice of parameters, $a = 2.8$, $\alpha = 1.6$, $c = 0.001$, $b = 9$, $e = 5$.

At first, we examine how inter-layer coupling effects the dynamical state of the system in absence of time delay between layers ($\tau = 0$). For the first layer we set the value of intra-layer coupling strength corresponding to incoherent dynamics $\lambda_1^1 = 0.2$, while another one exhibit the chimera state $\lambda_1^1 = 1.0$. This is illustrated in Fig. 2(a), where the snapshots of $_i$ are depicted for both layers in absence of inter-layer coupling ($\lambda_3 = 0$). Unlike the KS model, this time the activation of inter-layer coupling ($\lambda_3 = 0.16$) leads to emergence of independent (i.e. asynchronous between layers) chimera states in both interconnected networks, that is depicted in Fig. 2(b). By turn, the increasing of inter-layer coupling ($\lambda_3 = 0.2$) results in transition to identical regime in both layers as clearly seen from Fig. 2(c). This illustrates the fact, that in the network of Hindmarsh-Rose neurons the multiplex interaction can excite not only synchronous inter-layer chimera state, but also non-identical chimera states in layers.

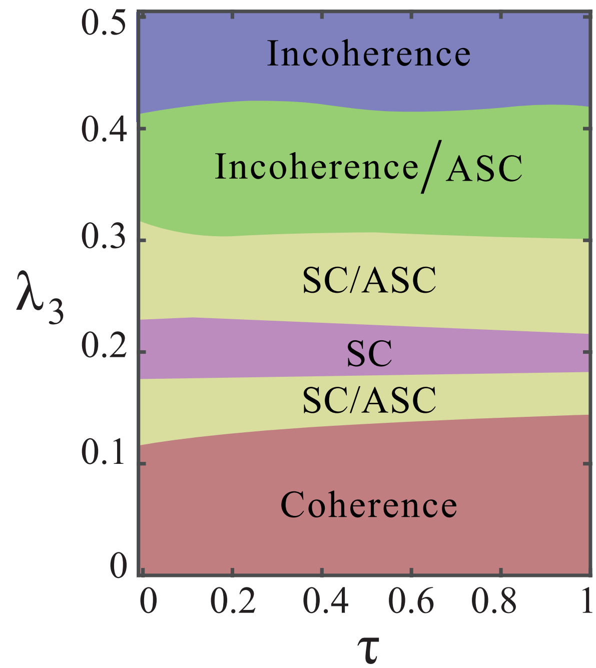

Figure 3. Dynamical regimes emerging in the network in a parameter space (λ_3, τ). The intra-layer couplings are fixed as $\lambda_1^1 = 0.2$ and $\lambda_1^2 = 1.0$. Number of oscillators in each layer, $N = 100$.

Next we consider the presence of inter-layer synaptic coupling delay ($\tau \neq 0$) and check the interaction between the layers by changing the inter-layer coupling strength λ_3. To get a complete overview of different spatiotemporal patterns realizing in the system, we depict the observing regime in the (τ, λ_3)–parameter space, shown in Fig. 3. The latter shows separate regions of incoherence, coherence and synchronous chimera states, supplemented by areas of multistable dynamics of incoherence/asynchronous chimera and synchronous/asynchronous chimera. As mentioned above, the increasing of λ_3 leads to transition from incoherence to independent chimera states in layers (ASC), and then to identical chimera (SC). Surprisingly, the region of synchronous chimera is divided from coherent area by ASC/SC multistability, i.e. some increasing of λ_3 from SC region induces break of synchrony between layers. At the same time, any strong dependence from synaptic coupling delay (τ) cannot be observed.

3. CONCLUSION

In conclusion, we have numerically studied the interaction between the ensembles of nonlocally coupled oscillators with the help of multilayer network model. It has been shown that the identical phase oscillators, arranged in two coupled layers and demonstrated individually the different chimera states come to the synchronous mode with the increase of inter-layer coupling. For the relatively small values of inter-layer coupling the layers demonstrate chimera states, where the shapes of the phase-distributions, corresponded to different layers remains different and depend on the degree of inter-layer interaction. We show, that variation of inter-layer coupling enables us to excite both identical (synchronous) and individual (asynchronous) chimera states in the interacting populations of model neurons. Therefore, in this system inter-layer coupling delay does not affect the dynamical features reported above. We suppose, that described phenomena could take place in real-world networks, which are usually studied, in recent times, using the framework of multilayered model.[20–29, 33]

4. ACKNOWLEDGMENTS

This work has been supported by the Russian Foundation of Basic Research (grant 15-52-45003) and Ministry of Education and Science of Russian Federation in the framework of the implementation of state assignment 3.861.2017/PCh. A.E.H. also acknowledges support from the Ministry of Education and Science of Russian Federation in the framework of the providing researches (project 3.4593.2017/BY).

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