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RADIOPHYSICS

Recuperation in Superpower Cherenkov Generators with a Nonuniform Magnetic Field

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Abstract—We consider an effective method for increasing the total efficiency of relativistic Cherenkov generators, viz., recuperation. It has been found that, for a generator with an efficiency smaller than 15%, recuperation makes it possible to obtain an increase in the efficiency that exceeds 50% (from 12% to 68%). In generators with initially high efficiency (more than 50%), recuperation may turn out to be ineffective.

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INTRODUCTION

In recent years, powerful generators and amplifiers of microwave radiation have been widely used in various branches of science and technology [1–5]. High-power microwave radiation is used in radio location, counterradio measures, acceleration of ions, data transmission and processing in long-range communication systems, the investigation of radiation with matter, in biological studies, etc. [1–7]. Radiation sources with operating principles based on the longitudinal interaction of an electron beam with an electromagnetic field in periodic electrodynamic structures with positive or negative dispersions of the fundamental wave that belong to the most extensive class of powerful microwave electronic devices. The strongest amplification and most stable generation in this class of devices are observed in the case of synchronism between the beam and the field at frequencies close to the transmission band of waveguide systems. In these frequency ranges, resonance properties are manifested in all periodic systems, and forward and backward waves can be excited simultaneously in them. In superpower electronics that operate with relativistic high-current electron beams, Cherenkov and diffraction generators based on resonant periodic structures with positive dispersion of the fundamental wave and with transverse sizes that considerably exceed the radiation wavelength were found to be perspective devices. The development of high-current electron accelerators with currents of 1–35 kA and voltages of 0.3–2.0 MV makes it possible to design microwave Cherenkov generators with an electrodynamic system in the form of a segment of a periodic corrugated hollow waveguide, which have an output power of 1–30 GW in the centimeter and millimeter wavelength ranges with efficiency of 10–50%. These results were obtained using the simplest mathematical models applicable for a shallow periodic corrugation of the waveguide. The efficiency and output characteristics of generators can be increased by optimizing all parameters. For example, the use of an irregular profile in relativistic Cherenkov generators (RCGs) makes it possible to reach an efficiency of about 40–50% [5–8], and a further increase in the efficiency of the device can be achieved using recuperation.

1. DESCRIPTION OF THE MODEL

Here, we consider RCGs shown schematically in Fig. 1. These generators and the electrostatic field in them are axisymmetric; the problem is solved in cylindrical coordinates, and the field is calculated in the 2D approximation. The electrostatic field in the system is calculated using the finite-element method. The Laplace equation in cylindrical coordinates has the form

$$\nabla^2\varphi = \mathrm{div}\,\mathrm{grad}\,\varphi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) + \frac{\partial^2\varphi}{\partial z^2} = 0,$$

$$\mathbf{E} = -\mathrm{grad}\,\varphi,$$

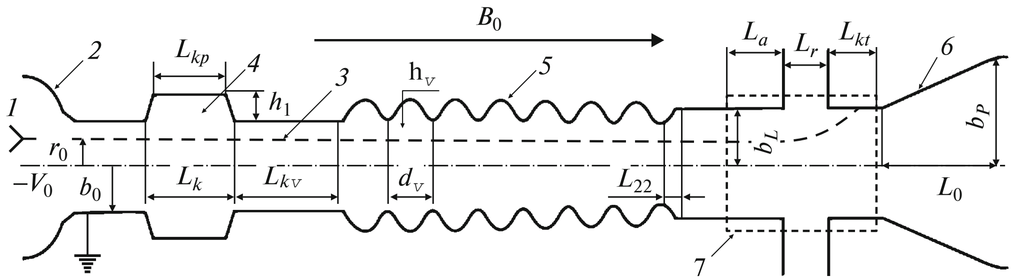

Fig. 1. Diagram of a Cherenkov generator: (*1*, *2*) cathode and anode forming a tubular electron beam; (*3*, *4*) modulating grooves; (*5*) periodic slow-wave comb; (*6*) outlet horn; (*7*) recuperation region.

where r and z are the transverse and longitudinal coordinates, φ is the electric field potential, and $\mathbf{E}$ is the electric field vector.

Electrons were deposited on the collector using nonuniform magnetic field B_0 directed along the z axis, which is defined as

$$B_0(z) = B_{0\max} + (B_{0\min} - B_{0\max})\sin^2\left(\frac{\pi z}{2L_m}\right).$$

Here, L_m is the domain of variation of the magnetic field.

The expression for components of vector $\mathbf{B}$ in terms of $B_0(z)$ has the form

$$\begin{cases} B_r = -\frac{1}{2}r_l\frac{\partial B_0(z)}{\partial z} + \frac{1}{16}r_l^3\frac{\partial^3 B_0(z)}{\partial z^3} \\ B_z = B_0(z) - \frac{1}{4}r_l^2\frac{\partial^2 B_0(z)}{\partial z^2}. \end{cases}$$

The equation of motion for a relativistic electron [9] is given by

$$\frac{d\mathbf{v}}{dt} = \frac{e}{m_0}\sqrt{1-\frac{v^2}{c^2}}\left(\mathbf{E} + [\mathbf{v},\mathbf{B}] - \frac{\mathbf{v}}{c^2}(\mathbf{v},\mathbf{E})\right),$$

where e and m_0 are the electron charge and the rest mass and c is the velocity of light in vacuum.

For numerical integration of the equations of motion, we used a special algorithm for integrating second-order differential equations [10]:

$$x_{t+\Delta t} = x_t + v_t\Delta t + \frac{1}{8}(5a_t - a_{t-\Delta t})\Delta t^2 + O(\Delta t^3),$$

$$v_{t+\Delta t} = v_t + \frac{1}{2}(3a_t - a_{t-\Delta t})\Delta t + O(\Delta t^3)$$

according to predictions, and corrected expressions are given by

$$x_{t+\Delta t} = x_t + v_t\Delta t + \frac{1}{8}(a_{t+\Delta t} + 3a_t)\Delta t^2 + O(\Delta t^3),$$

$$v_{t+\Delta t} = v_t + \frac{1}{8}(3a_{t+\Delta t} + 6a_t - a_{t-\Delta t})\Delta t + O(\Delta t^3),$$

where $x(t)$ is the particle coordinate; $v(t)$ and $a(t)$ are the particle velocity and acceleration, respectively; and Δt is the step of integration with respect to time.

For the initial conditions for electrons, we used the positions and velocities of particles, obtained in simulating the Cherenkov generators [5–8].

The total efficiency was defined in terms of the energy loss by the electron beam,

$$\eta = \frac{1}{N}\sum_{l=1}^{N}\frac{\gamma_0 - \gamma_l}{\gamma_0 - 1},$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{\mathbf{v}}{c}\right)^2}},$$

where N is the number of particles; l is the particle number; and γ_0 and γ are the relativistic factor of particles at the input and output of the device, respectively.

2. RESULTS OF SIMULATION

Using the mathematical theory developed in [5–8], we determined the optimal variants of RCGs with various parameters and efficiencies. These variants were used to estimate the recuperation efficiency in this type of device.

The parameters of the variants considered here are as follows.

Variant 1 (3-cm one-wave generator). Working frequency f = 10 GHz; the chosen reference frequency ω_0 corresponds to λ_0 = 3 cm. The voltage across the beam is V_0 = 520 kV (β_0 = 0.869), the current I_0 = 2500 A, and the focusing magnetic field is B_0 = 0.357 T. The regular corrugated segment contains n_v = 10 periods, d_v = 2.62, h_v = 1.45, b_0 = 3.75 (1.26 cm, 0.69 cm, 1.79 cm). The width and height of the modulating groove and its distance from the beginning of the comb are L_1 = 2.22, h_1 = 1.092, and L_{1v} = 5.79 (1.06 cm, 0.52 cm, and 2.76 cm), Δ_{pc} = 0.8. Beam radius is r_0 = 3.1 (1.48 cm). The number of electron layers is two, and the number

Table 1. Parameters of the RCGs with a single modulating groove

No.	f, GHz	V_0, kV	β_0	I_0, A	B_0, T	n_v	d_v, cm	h_v, cm	b_0, cm	L_1, cm	h_1, cm	L_{1v}, cm	Δ_p	r_0, cm	Number of electron layers/electrons per layer	Efficiency, %
1	10	520	0.869	2500	0.357	10	1.26	0.69	1.79	1.06	0.52	2.76	0.8	1.48	2/8	23.5
2	10	228	0.723	1600	1.79	10	1.43	0.754	2.39	2.86	0.759	3.41	0.8	2	2/8	39.6
3	5	291	0.772	823	0.179	7	2.24	1.31	2.67	1.43	1.32	4.42	0.8	1.9	2/8	52.3

Table 2. Parameters of RCGs with a double modulating groove

No.	f, GHz	V_0, kV	β_0	I_0, A	B_0, T	n_v	d_v, cm	h_v, cm	b_0, cm	$L_1 = L_{12} = L_2$, cm	h_1, cm	L_{2v}, cm	Δ_p	r_0, cm	Number of electron layers/electrons per layer	Efficiency, %
4	10	323	0.84	3000	0.35	20	0.61	0.68	3.77	2.2	0.96	0.53	0.8	3.3	3/8	42
5	10	323	0.84	3000	0.35	20	0.605	0.7	3.77	2.23	0.96	0.38	0.8	3.3	2/8	46
6	10	323	0.84	2000	0.35	20	0.6	0.7	3.77	2.3	0.96	0.38	0.8	3.3	2/8	12
7	10	323	0.84	2000	0.35	20	0.612	0.69	3.77	2.23	0.96	0.38	0.8	3.3	2/32	42.8
8	10	323	0.84	1750	0.35	20	0.612	0.69	3.77	2.23	0.96	0.38	0.8	3.3	2/32	13.14

Table 3. Results of RCG optimization

No.	Electronic efficiency of generator	Total efficiency	Efficiency increment
1	23.5	35.84	12.34
2	39.6	64.14	24.54
3	52.3	51.95	–0.35
4	42	55.74	13.74
5	46	63.13	17.13
6	12	68.9	56.9
7	42.8	54.37	11.57
8	13.14	68	54.86

Table 4. Parameters of optimal recuperation systems for generators considered under investigation

No.	L_a, cm	L_r, cm	L_{kt}, cm	U_k, kV	L_m, cm	$B_{0\max}$, T	$B_{0\min}$, T
1	7.3	7.6	17.4	–90	55	0.375	0
2	2.9	1.1	3	–55	6.7	1.7	1.1
3	2	2	9.9	–1.9	15	0.375	0
4	5	8	10	–60	55	0.375	0
5	10	2.3	9.8	–76.97	28.8	0.375	0.195
6	10	1	10	–246	23.4	0.375	0.231
7	9.4	1	10	–38	26	0.375	0.2175
8	10	1	10	–236	23.4	0.375	0.231

of electrons per layer is eight. The attained efficiency is 23.5%.

Variant 2 (3-cm one-wave generator). Working frequency $f = 10$ GHz; the chosen reference frequency ω_0 corresponds to $\lambda_0 = 3$ cm. Voltage across the beam is $V_0 = 228$ kV ($\beta_0 = 0.723$), current $I_0 = 1600$ A, focusing magnetic field $B_0 = 1.79$ T. The regular corrugated segment contains $n_v = 10$ periods, $d_v = 3$, $h_v = 1.58$, $b_0 = 5$ (1.43 cm, 0.754 cm, 2.39 cm). The width and height of the modulating groove and its distance from the beginning of the comb are $L_1 = 6$, $h_1 = 1.59$, and $L_{1v} = 7.15$ (2.86 cm, 0.759 cm, and 3.41 cm), $\Delta_p = 0.8$. The beam radius is $r_0 = 4.2$ (2 cm). The number of electron layers is 2, the number of electrons per layer is 8. The attained efficiency is 39.6%.

Variant 3 (6-cm two-wave generator). Working frequency $f = 5$ GHz; the chosen reference frequency ω_0 corresponds to $\lambda_0 = 6$ cm. Voltage across the beam is $V_0 = 291$ kV ($\beta_0 = 0.772$), current $I_0 = 823$ A, focusing magnetic field $B_0 = 0.179$ T. The regular corrugated segment contains $n_v = 7$ periods, $d_v = 2.35$, $h_v = 1.31$, $b_0 = 2.8$ (2.24 cm, 1.31 cm, 2.67 cm). The width and height of the modulating groove and its distance from the beginning of the comb are $L_1 = 1.5$, $h_1 = 1.38$, and $L_{1v} = 4.63$ (1.43 cm, 1.32 cm, and 4.42 cm), $\Delta_p = 0.8$. Beam radius is $r_0 = 2$ (1.9 cm). The number of electron layers is 2 and the number of electrons per layer is 8. The choice of a relatively strong focusing field has made it possible to elevate the efficiency to 52.3% due to the compensation of dynamic stratification.

Variant 4 (3-cm two-wave generator). Working frequency $f = 10$ GHz; the chosen reference frequency ω_0 corresponds to $\lambda_0 = 3$ cm. Voltage across the beam $V_0 = 323$ kV ($\beta_0 = 0.84$), current $I_0 = 3000$ A, focusing

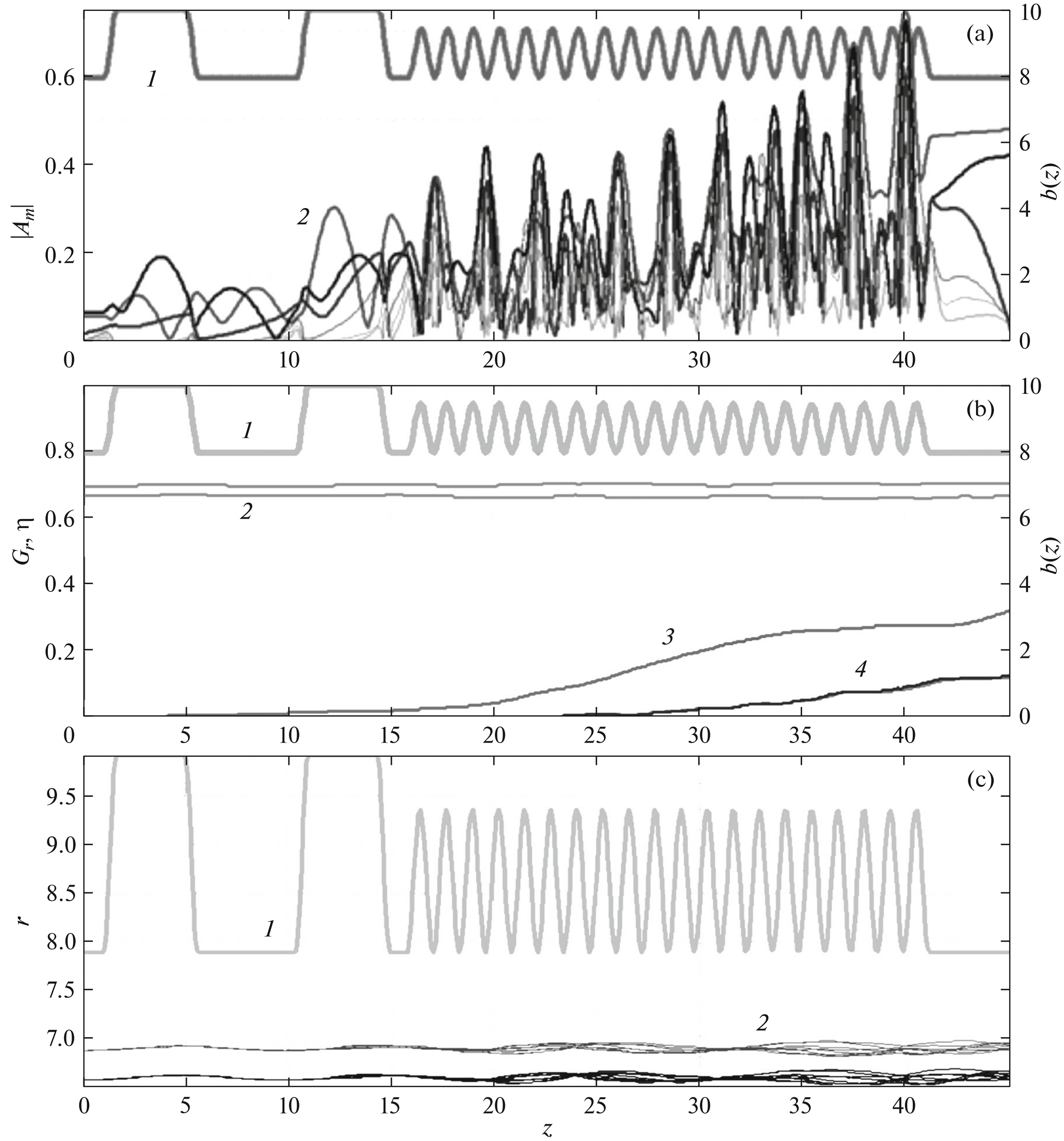

Fig. 2. Characteristics of an optimal variant of RCG (variant 6): (a) (*1*) profile and (*2*) amplitude distribution; (b) (*2*) electron beam boundaries; (*3*) grouping function and (*4*) electronic and wave efficiency; (c) (*2*) electron trajectories.

magnetic field B_0 = 0.35 T. The regular corrugated segment contains n_v = 20 periods, d_v = 1.28, h_v = 1.43, b_0 = 7.9 (0.61 cm, 0.68 cm, 3.77 cm). The width and height of the double modulating groove and its distance from the beginning of the comb are $L_1 = L_{12} = L_2$ = 4.6, h_{12} = 2, L_{2v} = 1.12 (2.2 cm, 0.96 cm, and 0.53 cm), Δ_p = 0.8. Beam radius is r_0 = 6.9 (3.3 cm). The number of electron layers is 3 and the number of electrons per layer is 8. The attained efficiency is 42%.

Variant 5 (3-cm two-wave generator). Working frequency f = 10 GHz; the chosen reference frequency ω_0 corresponds to λ_0 = 3 cm. Voltage across the beam is V_0 = 323 kV (β_0 = 0.84), current I_0 = 3000 A, focusing magnetic field B_0 = 0.35 T. The regular corrugated segment contains n_v = 20 periods, d_v = 1.268, h_v = 1.46, b_0 = 7.9 (0.605 cm, 0.7 cm, 3.77 cm). The width and height of the double modulating groove and its distance from the beginning of the comb are $L_1 = L_{12} = L_2$ = 4.67, $h_{1,2}$ = 2, L_{2v} = 0.79 (2.23 cm, 0.96 cm, 0.38 cm), Δ_p = 0.8. Beam radius is r_0 = 6.9 (3.3 cm). The number of electron layers is 2 and the number of electrons per layer is 8. The attained efficiency is 46%.

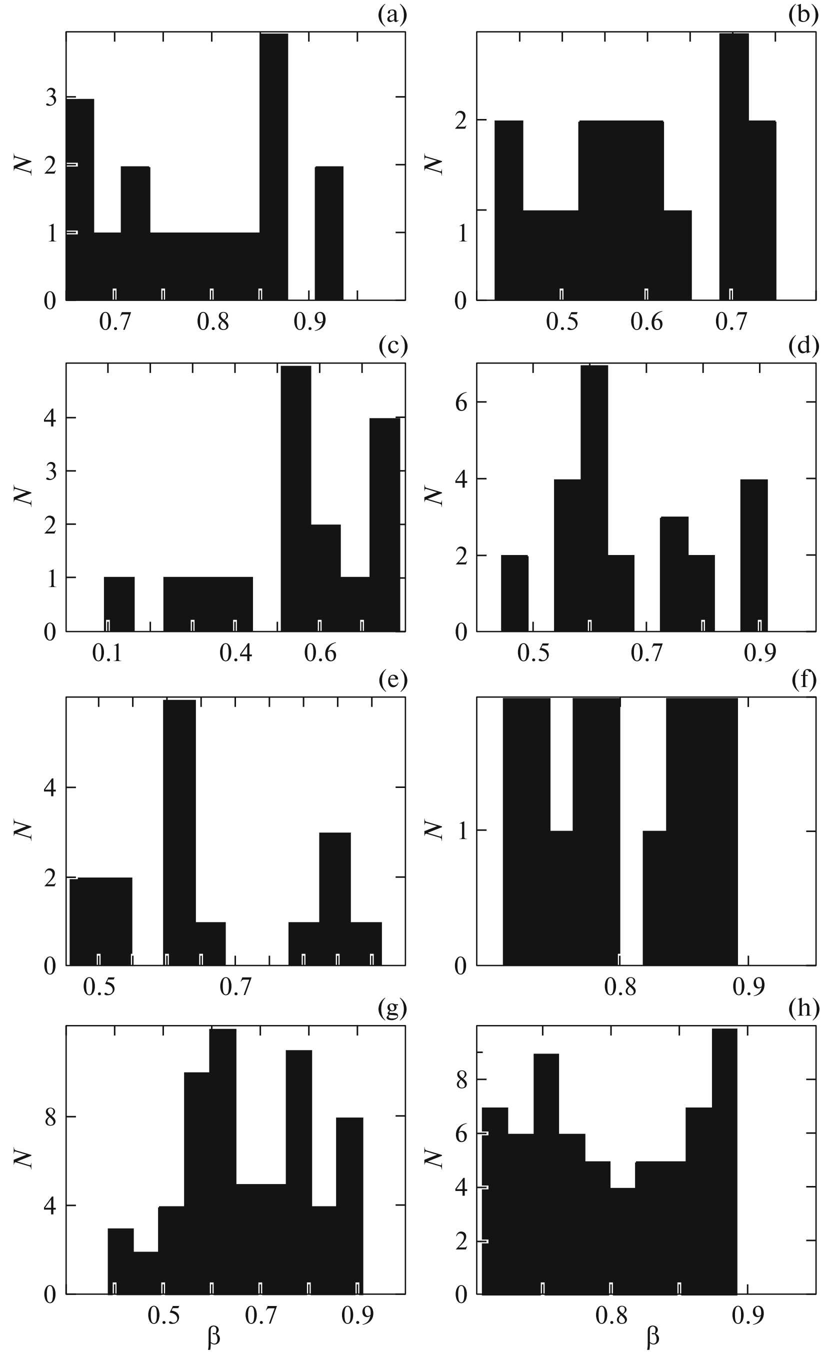

Fig. 3. Electron velocity distribution for generator variants 1–8.

Variant 6 (3-cm two-wave generator). Working frequency $f = 10$ GHz; the chosen reference frequency ω_0 corresponds to $\lambda_0 = 3$ cm. Voltage across the beam is $V_0 = 323$ kV ($\beta_0 = 0.84$), current $I_0 = 2000$ A, focusing magnetic field $B_0 = 0.35$ T. The regular corrugated segment contains $n_v = 20$ periods, $d_v = 1.25$, $h_v = 1.46$, $b_0 = 7.9$ (0.6 cm, 0.7 cm, 3.77 cm). The width and height of the double modulating groove and its distance from the beginning of the comb are $L_1 = L_{12} = L_2 = 4.67$, $h_{1,2} = 2$, $L_{2v} = 0.79$ (2.23 cm, 0.96 cm, 0.38 cm), $\Delta_p = 0.8$. Beam radius is $r_0 = 6.9$ (3.3 cm). The number of electron layers is 2 and the number of electrons per layer is 8. The attained efficiency is 12%.

Variant 7 (3-cm two-wave generator). Working frequency $f = 10$ GHz; the chosen reference frequency ω_0 corresponds to $\lambda_0 = 3$ cm. Voltage across the beam is $V_0 = 323$ kV ($\beta_0 = 0.84$), current $I_0 = 2000$ A, focusing magnetic field $B_0 = 0.35$ T. The regular corrugated

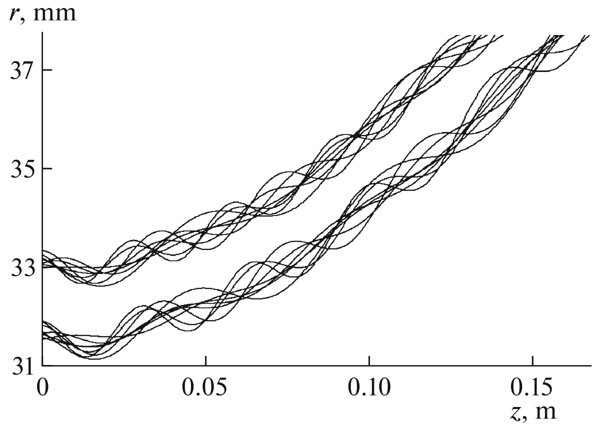

Fig. 4. Deposition of electrons at zero potential at the collector relative to the anode.

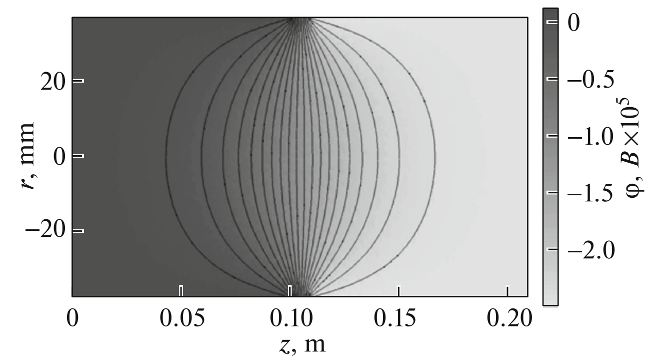

Fig. 5. Electrostatic field potential distribution for variant 6.

segment contains $n_v = 20$ periods, $d_v = 1.282$, $h_v = 1.45$, $b_0 = 7.9$ (0.612 cm, 0.69 cm, 3.77 cm). The width and height of the double modulating groove and its distance from the beginning of the comb are $L_1 = L_{12} = L_2 = 4.67$, $h_{1,2} = 2$, $L_{2v} = 0.79$ (2.23 cm, 0.96 cm, 0.38 cm), $\Delta_p = 0.8$. Beam radius is $r_0 = 6.9$ (3.3 cm). The number of electron layers is 2 and the number of electrons per layer is 32. The attained efficiency is 42.8%.

Variant 8 (3-cm two-wave generator). Working frequency $f = 10$ GHz; the chosen reference frequency ω_0 corresponds to $\lambda_0 = 3$ cm. Voltage across the beam is $V_0 = 323$ kV ($\beta_0 = 0.84$), current $I_0 = 1750$ A, focusing magnetic field $B_0 = 0.35$ T. The regular corrugated segment contains $n_v = 20$ periods, $d_v = 1.282$, $h_v = 1.45$, $b_0 = 7.9$ (0.612 cm, 0.69 cm, 3.77 cm). The width and height of the double modulating groove and its distance from the beginning of the comb are $L_1 = L_{12} = L_2 = 4.67$, $h_{1,2} = 2$, $L_{2v} = 0.79$ (2.23 cm, 0.96 cm, and 0.38 cm), $\Delta_p = 0.8$. Beam radius is $r_0 = 6.9$ (3.3 cm). The number of electron layers is 2 and the number of electrons per layer is 32. The attained efficiency is 13.14%.

The parameters of these variants of generators and their characteristics are given in Tables 1 and 2.

Let us consider the main characteristics of variant 6 in Fig. 2. The main role in the simulation of recuperation is played by the electron velocity distribution. This characteristic for the above generators is shown in Fig. 3. Figure 4 shows the electron trajectories in two radial REB layers moving in a nonuniform magnetic field with zero potential of the collector relative to the anode. It can be seen from Fig. 4 (see also Fig. 2c) that the electron trajectories at the RCG outlet (in the given conditions) have complex shapes and can only be determined by numerical simulation. For this reason, the configuration of the recuperation system should be optimized for each device being simulated individually.

To find effective configurations, we optimized the following parameters: the nonuniform magnetic field profile, geometrical size of the system (L_a, L_r, and L_k), and the voltage across the collector. During optimization, we maintained the conditions of deposition of all electrons on the collector, i.e., the absence of reflected electrons and those that leave the region of the interaction. The results of optimization are given in Table 3. The parameters of the resultant optimal recuperation systems for the generators under investigation are compiled in Table 4.

Variant 3 is characterized by a high efficiency (52.3%); for this reason, electrons with low velocities (0.1c, see Fig. 3c) are present in the device. To prevent these electrons from returning to the region of interaction in the case of recuperation, it is necessary to change the operating regime of the generator, which deteriorates its electronic efficiency. For the variant under investigation, recuperation did not allow us to increase the efficiency or even slightly reduced it.

To analyze recuperation in generators with low efficiency, we changed variant 5 as follows: the current was reduced from 3000 to 2000 A, and the corrugation period was optimized. As a result, the efficiency was reduced from 46 to 12% (variant 6). In the resultant generator, electrons possess high velocities and a relatively uniform distribution, which makes it possible to successfully use recuperation and to increase the efficiency to 56.9%.

In simulating RCGs based on the theory developed in [5–8], it is sufficient to use a small number of coarse particles (mainly, 16 in the variants considered here); however, this can be insufficient for taking into account strongly retarded particles in simulating recuperation. In variants 7 and 8, the number of electrons was increased to 64. This affected the continuity in the velocity distribution of particles, but the efficiency of using recuperation almost remained unchanged. The electrostatic field potential, the profiles of the irregular magnetic field, and the electron trajectories for

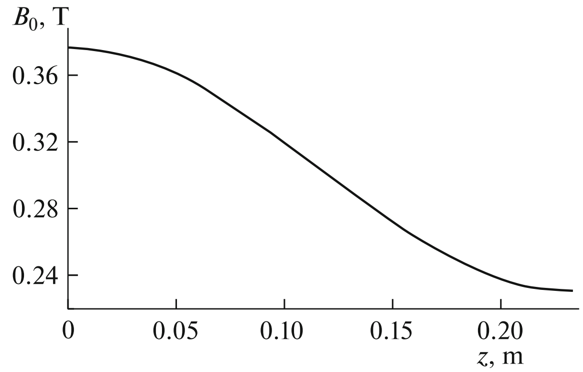

Fig. 6. Nonuniform magnetic field profile for variant 6.

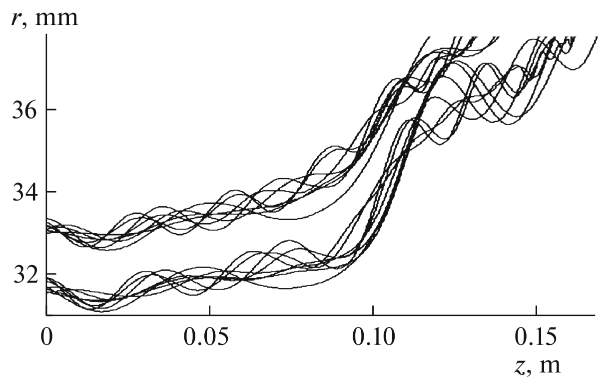

Fig. 7. Electron trajectories for variant 6.

variant 6 are shown in Figs. 5–7, respectively. It can be seen from Fig. 5 that the electrostatic field in the interaction region has the form of an electron lens [11].

CONCLUSIONS

An analysis of electron trajectories and the electron velocity distribution at the RCG exit shows that the recuperation system should be calculated separately for each individual generator.

For a generator with a lower efficiency and, hence, with less retarded electrons that have a less uniform velocity distribution, recuperation ensured a large increase in the efficiency (by more than 50%). In generators with initially high efficiency (exceeding 50%), recuperation may prove to be ineffective.

As a result of optimization, RCG configurations with a total efficiency close to 70% were obtained. It should be noted that, in this work, we disregarded the emission of secondary electrons from the collector because it produces a weaker effect in the system with recuperation, since the shape of the collector remains unchanged (as in the absence of recuperation), and the energy of primary electrons is lower.

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REFERENCES

1. D. I. Trubetskov and A. E. Khramov, *Lectures in Microwave Electronics for Physisists* (Fizmatlit, Moscow, 2003), Vol. 1.
2. D. I. Trubetskov and A. E. Khramov, *Lectures in Microwave Electronics for Physisists* (Fizmatlit, Moscow, 2004), Vol. 2.
3. J. Benford, J. A. Swegle, and E. Schamiloglu, *High Power Microwaves* (Taylor and Francis, New York, 2016).
4. A. A. Rukhadze, S. D. Stolbetsov, and V. P. Tarakanov, Radiotekh. Elektron. (Moscow) **37**, 385 (1992).
5. A. A. Kuraev and D. I. Trubetskov, *Methods of Nonlinear Dynamics and Chaos Theory in Problems of Microwave Electronics,* Vol. 1: *Stationary Processes* (Fizmatlit, Moscow, 2009), pp. 167–274.
6. V. F. Kravchenko, A. A. Kuraev, T. L. Popkova, and A. O. Rak, Fiz. Osnovy Priborostr. **1** (2), 41 (2012).
7. V. F. Kravchenko, A. A. Kuraev, T. L. Popkova, and A. O. Rak, Fiz. Osnovy Priborostr. **2** (1), 78 (2013).
8. A. A. Kuraev, I. V. Lushchitskaya, T. L. Popkova, A. O. Rak, and A. K. Sinitsyn, Izv. Vyssh. Uchebn. Zaved., Prikl. Nonlin. Din. **16** (3), 142 (2008).
9. L. D. Landau and E. M. Lifshitz, *Classical Field Theory,* Vol. 2 of *Course of Theoretical Physics* (Nauka, Moscow, 1988; Pergamon, Oxford, 1975).
10. A. O. Rak, Dokl. Belarus. Gos. Univ. Inform. Radioelektron. **67** (5), 86 (2012).
11. L. A. Artsimovich and S. Yu. Luk'yanov, *Motion of Charged Particles in Electric and Magnetic Fields,* 2nd ed. (Nauka, Moscow, 1972).

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