Estimation of degree of synchronization in epileptic brain

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ABSTRACT

The method for calculation of zero conditional Lyapunov exponent from time series has been proposed. Such method is shown to define the degree of synchronization of the regime realized in the system. It has been applied to real experimental neurophysiological time series represented by electroencephalograms of WAG/Rij rats having genetic predisposition to absence-epilepsy. The degree of synchronization in epileptic brain has been found.

Keywords: Synchronization, zero Lyapunov exponent, neurophysiological data, WAG/Rij rats, degree of synchronization

1. INTRODUCTION

Lyapunov exponents are the powerful tool for the analysis of the complex system dynamics.[1–4] They can be used for identification of the transition between different regimes, e.g., from periodic and quasi-periodic oscillations to the chaotic ones,[5,6] from chaotic dynamics to the hyperchaotic ones,[7] for revelation of the presence of hyperbolic attractor,[3,8] as well as for the different types of the synchronous regime detection (see, e.g.[9–13]).

To calculate the spectrum of Lyapunov exponents different methods and procedures have been proposed.[14–17] At the same time, the most part of them can be used only in such case when the evolution operator of the system under study is known explicitly (see, e.g.[4,16–18]). At the same time, sometimes we deal with the experimental time series that demands the modification of the standard methods and algorithms for the experimental time series analysis. Several methods allowing to calculate one or two highest Lyapunov exponents from time series are known at present.[2,17,19–23] They are usually used for identification of the presence of chaos in autonomous systems whereas they are almost inapplicable for the analysis of synchronization in coupled or non-autonomous systems due to impossibility of the calculation of the lower Lyapunov exponents from time series.

Among the spectrum of Lyapunov exponent of non-autonomous or coupled dynamical systems the so-called zero conditional Lyapunov exponent plays a crucial role for the analysis of synchronization. In particular, the transition of such Lyapunov exponent in the field of the negative values may be considered as a precursor of the phase synchronization regime onset in periodical systems subjected to the noise influence and chaotic oscillators[24,25] whereas the absolute value of such Lyapunov exponent can characterize the degree of the phase synchronization regime realized in the system under study. Therefore the estimation of such Lyapunov exponent from time series is an important task of nonlinear dynamics.

Recently we have proposed the method for estimation of the zero conditional Lyapunov exponent from time series and shown its validity on the model systems of nonlinear dynamics.[26] In present paper we make the modification of such method and apply it to real experimental neurophysiological time series.

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Dynamics and Fluctuations in Biomedical Photonics XIII, edited by Valery V. Tuchin, Kirill V. Larin, Martin J. Leahy, Ruikang K. Wang, Proc. of SPIE Vol. 9707, 970710
 · doi: 10.1117/12.2207391

2. DATA AND METHODS

2.1 Method description

The main idea of the method for estimation of zero conditional Lyapunov exponent from time series proposed in Ref.[26] consists in the approximation of the probability density of the shifted phase difference $x = \Delta\varphi - \psi$ (where $\psi = \text{const}$) of interacting systems being in the phase synchronization regime by the distribution

$$\rho(x) = A\exp[-\frac{2}{D}(\varepsilon x - \frac{\Omega x^3}{3})], \tag{1}$$

obtained for the circlemap being the simple discrete model of synchronization phenomenon

$$x_{n+1} = x_n + 2\Omega(1 - \cos x_n) - \varepsilon + \xi_n, \quad \mod \quad 2\pi \tag{2}$$

where x_n plays the role of the phase difference, ξ_n is the noise term ($\langle\xi_n\rangle = 0$, $\langle\xi_n\xi_m\rangle = D\delta(n-m)$), Ω and ε are the control parameters, D is the noise variance, A is a normalization factor.[25] The value of conditional Lyapunov exponent in such case can be calculated by the formula

$$\Lambda_0(\varepsilon) = \int_{x_1}^{x_2} \rho(x)\ln|1 + 2\Omega x|dx, \tag{3}$$

at that the values of x_1 and x_2 should be found empirically from the form of $\rho(x)$, and Ω-parameter should be defined by approximation of the distribution.

2.2 EEG data and wavelet transform

As the real experimental neurophysiological time series we have used electroencephalograms (EEG) of WAG/Rij rats having genetic predisposition to absence-epilepsy. We have analyzed EEG signals registered from four different fields of epileptic brain (occipital cortex, frontal cortex, reticular (RTN) and ventroposteromedial (VPM) nuclei of the thalamus). All experimental work with EEG recordings has been done by experienced neurophysiologists from the Moscow Institute of Higher Nervous Activity and Neurophysiology.

In Fig. 1 fragments of EEG signals registered from different fields of epileptic brain of WAG/Rij rat have been shown. It is clearly seen the presence of spike-wave discharges being the patterns of the synchronous activity in epileptic brain.[27]

To define the degree of synchronization between different fields of epileptic brain we should reveal, first of all, the patterns of the synchronous regime from EEG signals. For these purposes the continuous wavelet transform should be used.[28] Continuous wavelet transform is given by

$$W(s,t_0) = \frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty} x(t)\psi^*\left(\frac{t-t_0}{s}\right) dt, \tag{4}$$

where $x(t)$ is the signal under study, $\psi(\eta) = (1/\sqrt[4]{\pi})\exp(j\Omega_0\eta)\exp\left(-\eta^2/2\right)$ is the Morlet complex mother wavelet function, $\Omega_0 = 2\pi$ is parameter of the wavelet, time scale s corresponds to the width of the wavelet function ψ, t_0 is the shift of wavelet along the time axis, the symbol "*" denotes the complex conjugation.[29,30] The time scale s is usually used instead of the frequency f of Fourier transform. In the case of the use of the complex Morlet wavelet function with parameter $\Omega = 2\pi$ they are inversely proportional to each other, i.e. $s = 1/f$.

The use of the complex wavelet basis allows characterizing the system dynamics on the different time scales s, at that the absolute value $|W(s,t_0)|$ indicates the presence and intensity of the time scale s mode in the time series $x(t)$, whereas its argument $\phi(s,t) = \arg W(s,t)$ is a phase naturally introduced for every time scale.

To reveal the presence of spike-wave discharges in epileptic EEG it is necessary to calculate the instantaneous energy distributions $E(s,t) = |W(s,t)|^2$ averaged over the range of time scales $s \in (s_1, s_2)$, i.e.

$$\langle E(t)\rangle = \int_{s_1}^{s_2} E(s,t)ds, \tag{5}$$

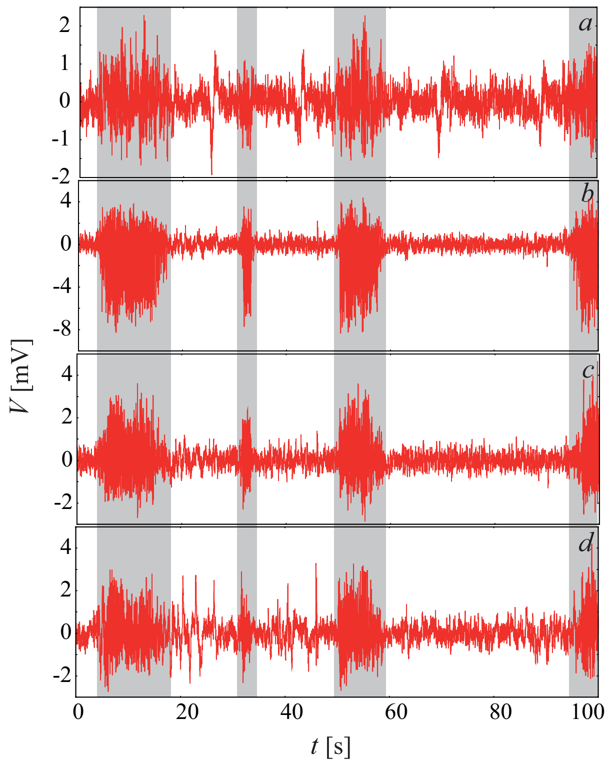

Figure 1. Fragments of EEG signals registered from different fields of epileptic brain: (a) occipital cortex, (b) frontal cortex, (c) reticular nucleus of thalamus (RTN), (d) ventroposteromedial nucleus of thalamus (VPM). Spike-wave discharges in EEG signals are schematically marked by grey rectangles

at that the boundaries of time scales should correspond to the frequency range 7-15 Hz which is typical for the spike-wave discharges.[28] If the value of $\langle E(t)\rangle$ exceeded a certain threshold $\Delta E = 0.5$, the spike-wave discharge is realized in the system.

To define the degree of synchronization between different fields of epileptic brain we should apply the method described in subsection 2.1 to the time series of the phase difference $\Delta\phi = \phi_1(s,t) - \phi_2(s,t)$ between two different fields *1* and *2* corresponding to the time intervals of the spike-wave discharges realization. At that, the phase difference should be calculated on the time scale $s = s_{max} \in (s_1, s_2)$ corresponding to the maximal value of the instantaneous energy distribution $E(s,t)$ computed by the EEG signal, for example, from the frontal cortex of epileptic brain. Our calculations show that in all considered cases in the time intervals corresponding to the spike-wave discharges the phase locking condition

$$|\Delta\phi| < \text{const} \tag{6}$$

is satisfied that makes possible to apply the method considered in Sec.2.1 to the analyzed signals.

In Fig. 2,a the time dependence of the phase difference calculated between the EEG signals from RTN and VPN fields of epileptic brain of WAG/Rij rat during the spike-wave discharges is shown. Fig. 2,b illustrates the probability density distribution obtained for such phase difference and its approximation by the analytical formula (1). It is clearly seen that despite of the presence of heavy tails in the phase difference distribution the analytical relation is a good approximation of the distribution obtained experimentally. The value of zero conditional Lyapunov exponent calculated in such a way by formula (3) is about -0.23 that testifies a high enough degree of the phase synchronization realized in epileptic brain. It should be noted that degree of synchronization being realized between the other fields of the brain cortex and thalamus is a little lower in comparison with the considered case.

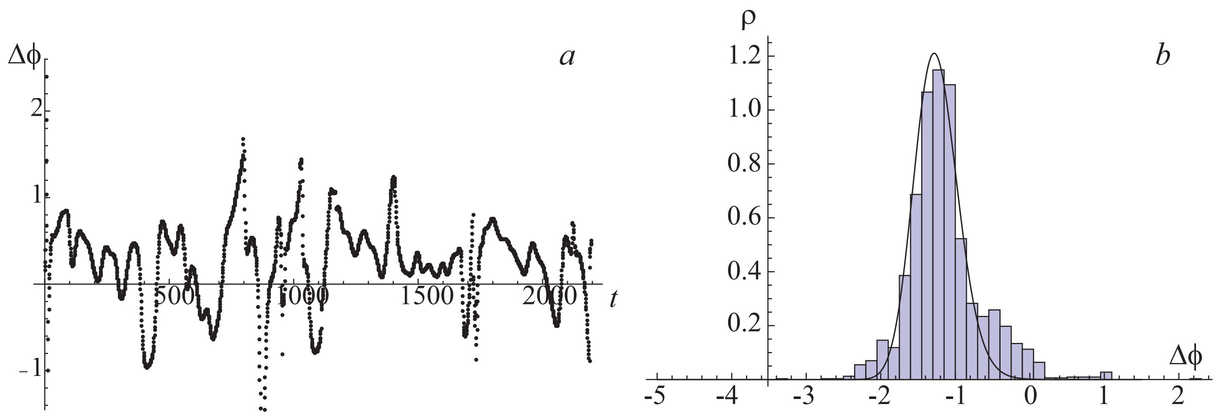

Figure 2. (a) Time dependence of the phase difference $\Delta\phi$ calculated between EEG signals corresponding to RTN and VPN fields of the brain of WAG/Rij rat during the spike-wave discharges. (b) Distribution of the phase differences and its approximation by analytica relation (1). The parameters of approximation are the following: $A = 0.0016$, $\Omega = 0.09$, $\varepsilon = 0.148$, $D = 0.038$, $x_1 = 3$, $x_2 = 1$

3. CONCLUSIONS

In present paper we have proposed the method for estimation of the value of zero conditional Lyapunov exponent from time series. We have shown that such exponent can characterize the degree of the phase synchronization realized in the system. We have applied such method to real experimental neurophysiological time series and defined the degree of synchronization in epileptic brain.

ACKNOWLEDGMENTS

The study of neurophysiological experimental data has been supported by the Russian Science Foundation (project No. 14-12-00224). The development of mathematical method of Lyapunov exponent estimation from time series has been supported by Ministry of Education and Science of Russian Federation (projects 3.23.2014/K and 931).

REFERENCES

[1] Dünki, R. M., "Largest lyapunov-exponent estimation and selective prediction by means of simplex forecast algorithms," *Phys. Rev. E* **62**(5), 6505–6515 (2000).

[2] Porcher, R. and Thomas, G., "Estimating lyapunov exponents in biomedical time series," *Phys. Rev. E* **64**(1), 010902(R) (2001).

[3] Thamilmaran, K., Senthilkumar, D. V., Venkatesan, A., and Lakshmanan, M., "Experimental realization of strange nonchaotic attractors in a quasiperiodically forced electronic circuit," *Phys. Rev. E* **74**, 036205 (2006).

[4] Hramov, A. E., Koronovskii, A. A., Maximenko, V. A., and Moskalenko, O. I., "Computation of the spectrum of spatial lyapunov exponents for the spatially extended beam-plasma systems and electron-wave devices," *Physics of Plasmas* **19**(8), 082302 (2012).

[5] Schuster, H. G., [*Deterministic Chaos*], Physik-Verlag, Weinheim (1984).

[6] Ziehmann, C., Smith, L., and Kurths, J., "The bootstrap and Lyapunov exponents in deterministic chaos," *Physica D* **126**, 49–59 (1999).

[7] Kuznetsov, S. P. and Trubetskov, D. I., "Chaos and hyperchaos in a backward-wave oscillator," *Radiophysics and Quantum Electronics* **47**(5,6), 341–355 (2004).

[8] Kuznetsov, S. P., "Example of a physical system with a hyperbolic attractor of the smale-williams type," *Phys. Rev. Lett.* **95**, 144101 (2005).

[9] Pyragas, K., "Weak and strong synchronization of chaos," *Phys. Rev. E* **54**(5), R4508–R4511 (1996).

[10] Hramov, A. E. and Koronovskii, A. A., "Generalized synchronization: a modified system approach," *Phys. Rev. E* **71**(6), 067201 (2005).

[11] Goldobin, D. S. and Pikovsky, A. S., "Synchronization and desynchronization of self-sustained oscillators by common noise," *Phys. Rev. E* **71**(4), 045201(R) (2005).

[12] Goldobin, D. S. and Pikovsky, A. S., "Synchronization of self-sustained oscillators by common white noise," *Physica A* **351**, 126–132 (2005).

[13] Hramov, A. E., Koronovskii, A. A., and Moskalenko, O. I., "Are generalized synchronization and noise-induced synchronization identical types of synchronous behavior of chaotic oscillators?," *Phys. Lett. A* **354**(5–6), 423–427 (2006).

[14] Oseledec, V. I., "A multiplicative ergodic theorem: Lyapunov characteristic number for dynamical systems," *Trans. Moscow Math. Soc.* **19**, 197–231 (1968).

[15] Benettin, G., Galgani, L., and Strelcyn, J. M., "Kolmogorov entropy and numerical experiments," *Phys. Rev.* **A14**, 2338 (1976).

[16] Benettin, G., Galgani, L., Giorgilli, A., and Strelcyn, J. M., "Lyapunov characteristic exponents for smooth dynamical systems and for Hamiltonian systems: A method for computing all of them. P. I. Theory. P. II. Numerical application," *Meccanica* **15**, 9–30 (1980).

[17] Wolf, A., Swift, J., Swinney, H. L., and Vastano, J., "Determining lyapunov exponents from a time series," *Physica D* **16**, 285 (1985).

[18] Koronovskii, A. A., Hramov, A. E., Maximenko, V. A., Moskalenko, O. I., Alekseev, K. N., Greenaway, M. T., Fromhold, T. M., and Balanov, A. G., "Lyapunov stability of charge transport in miniband semiconductor superlattices," *Phys. Rev. B* **88**, 165304 (2013).

[19] Eckmann, J. P., Kamphorst, S. O., Ruelle, D., and Ciliberto, S., "Lyapunov exponents from a time series," *Phys. Rev. A* **34**(6), 4971–4979 (1986).

[20] Brown, R., Bryant, P., and Abarbanel, H. D. I., "Computing the lyapunov spectrum of a dynamical system from an observed time series," *Phys. Rev. A* **43**(6), 2787–2806 (1991).

[21] Rosenstein, M. T., Collins, J. J., and De Luca, C. J., "A practical method for calculating largest lyapunov exponents from small data sets," *Physica D* **65**(1-2), 117–134 (1993).

[22] Dieci, L. and van Vleck, E. S., "Computation of a few lyapunov exponents for continuous and discrete dynamical systems," *Applied Numerical Mathematics* **17**, 275–296 (1995).

[23] Mammone, N., Principe, J. C., Morabito, F., Shiau, D., and Sackellares, C., "Visualization and modelling of STLmax topographic brain activity maps," *J. Neurosci. Methods* **189**(2), 281–294 (2010).

[24] Osipov, G. V., Hu, B., Zhou, C. S., Ivanchenko, M. V., and Kurths, J., "Three types of transitons to phase synchronization in coupled chaotic oscillators," *Phys. Rev. Lett.* **91**(2), 024101 (2003).

[25] Hramov, A. E., Koronovskii, A. A., and Kurovskaya, M. K., "Zero Lyapunov exponent in the vicinity of the saddle-node bifurcation point in the presence of noise," *Phys. Rev. E* **78**, 036212 (2008).

[26] Moskalenko, O. I., Koronovskii, A. A., and Hramov, A. E., "Lyapunov exponent corresponding to enslaved phase dynamics: Estimation from time series," *Phys. Rev. E* **92**, 012913 (2015).

[27] Hramov, A. E., Koronovskii, A. A., Makarov, V. A., Pavlov, A. N., and Sitnikova, E., [*Wavelets in Neuroscience*], Springer Series in Synergetics, Springer, Heidelberg, New York, Dordrecht, London (2015).

[28] Hramov, A. E., Koronovskii, A. A., Midzyanovskaya, I. S., Sitnikova, E., and Rijn, C. M., "On-off intermittency in time series of spontaneous paroxysmal activity in rats with genetic absence epilepsy," *Chaos* **16**, 043111 (2006).

[29] Torresani, B., [*Continuous wavelet transform*], Paris: Savoire (1995).

[30] Koronovskii, A. A. and Hramov, A. E., [*Continuous wavelet analysis and its applications*], Moscow, Fizmatlit (2003).