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BIOPHYSICS

Estimate of the Degree of Synchronization in the Intermittent Phase Synchronization Regime from a Time Series (Model Systems and Neurophysiological Data)

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A method for determining the degree of synchronization of intermittent phase synchronization regime from a time series has been proposed on the basis of estimating the zero conditional Lyapunov exponent. The efficiency of the method has been tested on model systems near the boundary of the appearance of the synchronous regime. The method has been used to determine the degree of synchronization between various regions of the brain of rats of the WAG/Rij line having a genetic predisposition to epilepsy.

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One of the most widespread types of chaotic synchronization in real physiological systems is the chaotic phase synchronization regime [1, 2]. It is a generalization of classical synchronization of periodic oscillations to the case of nonautonomous and coupled chaotic systems and means the establishment of phase locking between their states at the absence of correlation between their amplitudes [3, 4].

At the boundary of chaotic phase synchronization, the behavior is intermittent; i.e., intermittent phase synchronization occurs [5, 6]. In this case, the phase locking condition is satisfied only in certain time intervals corresponding to laminar (synchronous) sections of the behavior interrupted by short-term intervals of an increase in the phase difference, which are called turbulent sections. Such a behavior is also characteristic of periodic oscillators subjected to external noise [7]. It is observed at the development of epileptic activity of a human being and laboratory animals [8]. In other words, the intermittent phase synchronization regime is a fairly widespread behavior characteristic of both model and real biological systems.

When the control parameters of interacting systems are mismatched quite weakly, the intermittent phase synchronization regime is usually classified as eyelet intermittency characterized by very long sections of the laminar behavior [5, 9]. The degree of synchronization of the intermittent phase synchronization regime can be determined by calculating the statistical characteristics of the durations of laminar phases (distributions of durations of laminar sections at fixed control parameters and the dependences of the average duration of laminar phases on the supercritical parameter) [5, 9] or the dependence of the zero conditional Lyapunov exponent on the control parameter [10]. The zero conditional Lyapunov exponent in the eyelet intermittency regime is negative and its absolute value can be considered as a characteristic of the degree of synchronization of the intermittent phase synchronization regime [10, 11]. Since the laminar phases of the behavior are responsible for the negativeness of the zero conditional Lyapunov exponent (see, e.g., [10, 12]), it can be assumed that the degree of synchronization of the intermittent phase synchronization regime can be estimated by calculating the zero conditional Lyapunov exponent from the parts of a time series that correspond only to laminar phases of the behavior of systems.

The zero conditional Lyapunov exponent for systems with the explicitly specified evolution operator can be calculated by classical methods and algorithms (Benettin algorithm, Gram–Schmidt orthogonalization procedure) [13]. At the same time, when analyzing time series, it is necessary to use modified methods and approaches. In particular, in [11, 14], methods allowing the determination of the zero conditional Lyapunov exponent of nonautonomous and coupled systems in the chaotic phase synchronization regime, in particular, in the presence of noise, from a time series are proposed. The aim of this work is the generalization of the previously developed methods and approaches to the estimation of the degree of intermittent phase synchronization and their application to model and real physiological systems.

First, we consider the main stages of the method for estimating the degree of synchronization of the intermittent phase synchronization regime from the time series. It is worth emphasizing that the time dependence of the phase difference $x(t) = \Delta\varphi(t) - \theta$ between two different signals (e.g., obtained from different regions of the brain of a laboratory animal or a human being) shifted by a certain constant θ (determined experimentally) should be analyzed. As was mentioned above, this dependence should contain only laminar (synchronous) sections of the behavior of interacting systems. The next stages are the construction of the signal distribution $x(t)$ and its approximation by the expression

$$\rho(x) = A\exp\left[-\frac{2}{D}\left(\varepsilon x - \frac{\Omega x^3}{3}\right)\right], \quad (1)$$

where ε and Ω are the control parameters and A is the normalization coefficient, and the search for the approximation parameters A, D, ε, and Ω (for more details, see [11]). Similar to the case of phase synchronization, the last stage is the estimation of the zero conditional Lyapunov exponent, which determines the degree of synchronization of the intermittent phase synchronization regime, by the formula

$$\Lambda = \int_{x_1}^{x_2} \rho(x)\ln|1 + 2\Omega x| dx. \quad (2)$$

The parameters x_1 and x_2 should be determined empirically from the form of $\rho(x)$.

We apply the method of estimating the degree of intermittent phase synchronization from the time series to particular systems. We begin with the consideration of model systems, namely, a classical nonautonomous Van der Pol oscillator under external noise:

$$\ddot{x} - (\lambda - x^2)\dot{x} + x = B\sin(\omega t) + \xi. \quad (3)$$

Here, $\lambda = 0.1$ is the control parameter, B and $\omega = 0.98$ are the amplitude and frequency of the external action, and ξ is the stochastic Gaussian process with zero average and unit standard deviation and two coupled unidirectional chaotic Rössler oscillators:

$$\begin{aligned} \dot{x}_1 &= -\omega_1 y_1 - z_1, \quad \dot{x}_2 = -\omega_2 y_2 - z_2 + \varepsilon(x_1 - x_2), \\ \dot{y}_1 &= \omega_1 x_1 + a y_1, \quad \dot{y}_2 = \omega_2 x_2 + a y_2, \\ \dot{z}_1 &= p + z_1(x_1 - c), \quad \dot{z}_2 = p + z_2(x_2 - c), \end{aligned} \quad (4)$$

where $a = 0.15$, $p = 0.2$, $c = 10$, $\omega_1 = 0.93$, and $\omega_2 = 0.95$ are the control parameters and ε is the coupling parameter [11].

At the chosen values of control parameters, the phase synchronization regime appears at $B_c = 0.029$ in system (3) and at $\varepsilon_c = 0.042$ in system (4). At $B \in (0.0238; 0.029)$ and $\varepsilon = (0.0345; 0.042)$, systems (3) and (4) exhibit intermittent behavior: type-I intermittency in the presence of noise in system (3) and eyelet intermittency in system (4), whose characteristics are identical to each other [15]. The time dependences of the phase difference $\Delta\varphi(t)$[1] in both cases contain both sections of the synchronous behavior (laminar phases), where the phase difference is limited ($|\Delta\varphi| < 2\pi$), and periods of a stepwise change in the phase difference by 2π, which are called turbulent splashes, as was mentioned above. The application of the above method only to sections of synchronous dynamics of interacting systems at various values of the control parameters B and ε in the ranges indicated above makes it possible to obtain the distribution of phase differences satisfying Eq. (1). This statement is illustrated in Fig. 1, which shows (a) the time dependences of the phase difference $\Delta\varphi(t)$ of a nonautonomous Van der Pol oscillator given by Eq. (3) at $B = 0.027$ in the laminar phases of the behavior and (b) the distribution of the phase differences and its approximation by Eq. (1) with the parameters presented in the figure

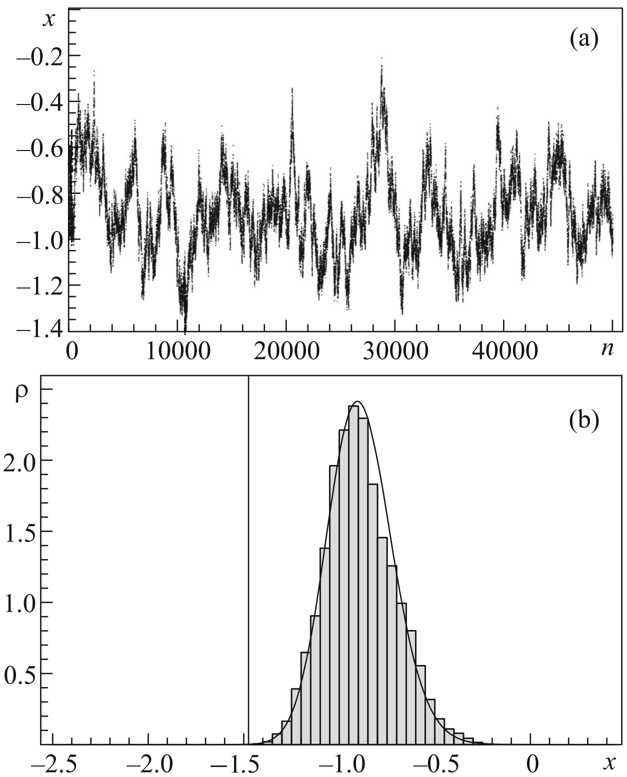

Fig. 1. (a) Time dependence of the phase difference $\Delta\varphi = \varphi(t) - \omega t$ of the nonautonomous Van der Pol oscillator given by Eq. (3) at $B = 0.027$ during the phases of synchronous behavior (n is the discrete time). (b) Distribution of the phase difference and its approximation by Eq. (1) with the parameters $A = 0.000137$, $\Omega = 0.009$, $\varepsilon = 0.00737$, $D = 0.0009$, $x_1 = -2$, $x_2 = 1$, and $\theta = 1.0$.

[1] For the Van der Pol oscillator, $\Delta\varphi(t) = \varphi(t) - \omega t$; for Rössler systems, $\Delta\varphi(t) = \varphi_1(t) - \varphi_2(t)$. The phases of the Van der Pol oscillator, $\varphi(t)$, and of interacting Rössler systems, $\varphi_{1,2}(t)$, can be introduced traditionally, e.g., as angles in the polar coordinate system in the $(x, \dot{x})$ and $(x_{1,2}, y_{1,2})$ planes, respectively.

Calculated conditional zeroth Lyapunov exponents in model systems (3) and (4) with various values of the control parameters: Λ is the value obtained by the proposed method with the accuracy δ calculated by Eq. (5) and Λ_0 is the value obtained by means of the Benettin algorithm and the Gram–Schmidt orthogonalization procedure

System	Parameter	Λ	Λ_0	δ
Nonautonomous Van der Pol oscillator (3)	$B = 0.027$	-0.00904 ± 0.00037	-0.0091 ± 0.00034	0.0066
	$B = 0.028$	-0.0103 ± 0.00035	-0.0106 ± 0.00028	0.0283
	$B = 0.029$	-0.0115 ± 0.00029	-0.0119 ± 0.00024	0.0336
Unidirectional coupled Rössler system (4)	$\varepsilon = 0.035$	-0.0051 ± 0.00106	-0.00547 ± 0.00101	0.0676
	$\varepsilon = 0.037$	-0.0081 ± 0.00109	-0.00847 ± 0.00107	0.0437
	$\varepsilon = 0.040$	-0.0137 ± 0.00104	-0.01302 ± 0.00102	0.0522

caption. The zero conditional Lyapunov exponent thus calculated is $\Lambda = -0.00904$ (see table), which is in good agreement with the calculation of a similar Lyapunov exponent using the Benettin algorithm. A similar situation also occurs at other values of the control parameters of the Van der Pol oscillator given by Eq. (3) and interacting Rössler systems specified by Eq. (4). The table summarizes the Lyapunov exponents in the systems under study at several values of the control parameters as calculated by the proposed method, as well as by the Benettin algorithm and Gram–Schmidt orthogonalization procedure. The same table gives the estimate of the accuracy of the developed method by the formula

$$\delta = \frac{|\lambda - \Lambda_0|}{|\Lambda_0|}, \quad (5)$$

where Λ is the zero conditional Lyapunov exponent obtained by the proposed method and Λ_0 is the zero conditional Lyapunov exponent calculated by means of the Benettin algorithm and Gram–Schmidt orthogonalization procedure. It is seen that the error is small in all cases under consideration; consequently, the developed method is applicable to the determination of the degree of synchronization of the intermittent phase synchronization regime.

We now discuss the application of the method to experimental time series for which the direct calculation of the Lyapunov exponents by classical methods and algorithms is impossible in view of the absence of the evolution operator describing the dynamics of the system (see above). To estimate the Lyapunov exponent, we used real experimental neurophysiological data: electroencephalogram signals from the thalamic reticular and ventrobasal nuclei of the brain of a rat of the WAG/Rij line having a genetic predisposition to epilepsy. All experiments were performed at the laboratory headed by Prof. Gilles van Luijtelaar, Radboud Universiteit Nijmegen (Netherlands). They satisfied ethical norms and included the recording of electroencephalograms of freely moving animals for 24 h.

It is known that epileptic electroencephalograms are intermittent time implementations containing patterns of synchronous activity (spike-wave discharges) alternating with the background activity of the brain (Fig. 2) [16]. Spike-wave discharges have a high degree of synchronization. Therefore, they are laminar phases of the behavior in electroencephalogram signals, whereas background activity sections can be considered as turbulent phases. In other words, to estimate the degree of synchronization of the intermittent phase synchronization regime from electroencephalogram signals, the developed method should be applied only to sections containing spike-wave discharges (gray rectangles in Fig. 2). The separation of spike-wave discharges from electroencephalogram signals was performed automatically by the method [17] based on a continuous wavelet transformation and was controlled by a skilled neurophysiologist. A continuous wavelet transformation with a Morlet mother wavelet

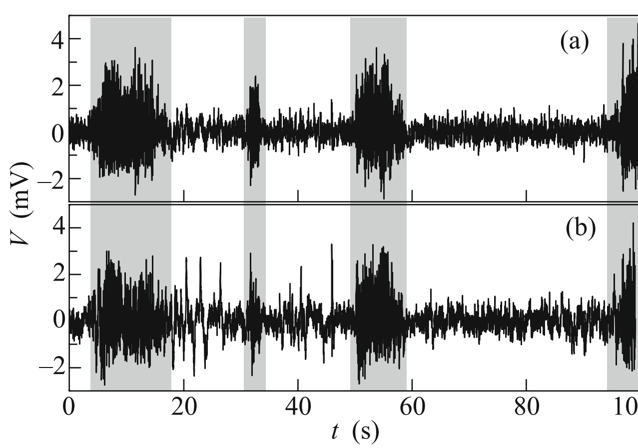

Fig. 2. Electroencephalogram signals from the thalamic (a) reticular and (b) ventrobasal nuclei of the brain of a rat of the WAG/Rij line. Gray rectangles mark the electroencephalogram sections corresponding to spike-wave discharges.

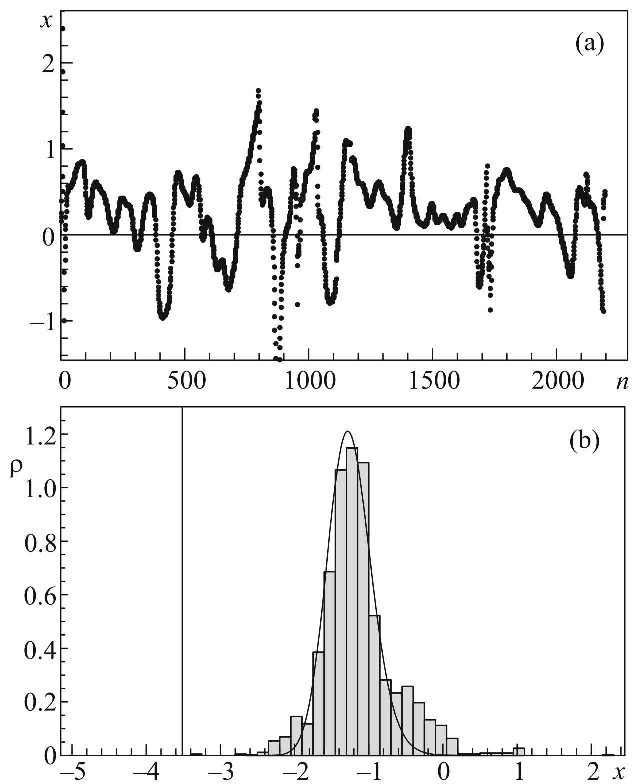

Fig. 3. (a) Time dependence of the phase difference between electroencephalogram signals from the thalamic reticular and ventrobasal nuclei of the brain of a rat of the WAG/Rij line during spike-wave discharges (n is the discrete time). (b) Distribution of the phase difference and its approximation by Eq. (1) with the parameters $A = 0.0016$, $\Omega = 0.09$, $\varepsilon = 0.148$, $D = 0.038$, $x_1 = -3$, $x_2 = 1$, and $\theta = -0.8824$.

was also used to introduce the phase of analyzed signals during spike-wave discharges on both electroencephalogram leads [18, 19].

Figure 3 illustrates the application of the method to electroencephalogram signals under study. This figure shows (a) the time dependence of the phase difference between electroencephalogram signals from the thalamic reticular and ventrobasal nuclei of the brain of a rat of the WAG/Rij line during spike-wave discharges (n is the discrete time) and (b) the distribution of the phase difference and its approximation by Eq. (1). It is seen that analytical formula (1) is a good approximation of the numerically obtained distribution despite the presence of "heavy tails." The Lyapunov exponent calculated from this distribution is $\Lambda = -0.0524357$ s^{-1}, which indicates intermittent phase synchronization between indicated regions of the brain. Such a situation is also characteristic of other regions of the brain of the rat of the WAG/Rij line. However, the calculations show that the degree of synchronization of the intermittent phase synchronization established between different regions of the brain is also different. In particular, the zero conditional Lyapunov exponent calculated from electroencephalogram signals from the frontal lobe of the cerebral cortex and the thalamic ventrobasal nucleus is $\Lambda = -0.429936$ s^{-1}, whereas the same Lyapunov exponent obtained from electroencephalogram signals from the occipital lobe of the cerebral cortex and the thalamic reticular nucleus is $\Lambda = -0.398955$ s^{-1}. This indicates a higher degree of synchronization of the intermittent phase synchronization regime established between these lobes of the brain of a rat as compared to the case presented in Fig. 3.

To summarize, a method for estimating the degree of synchronization of the intermittent phase synchronization regime from a time series has been proposed on the basis of calculating the conditional zeroth Lyapunov exponent. The efficiency of the method has been tested on model systems (nonautonomous Van der Pol oscillator in the presence of noise and two unidirectionally coupled Rössler systems) allowing the calculation of the Lyapunov exponents by classical methods and algorithms. The method has been used to estimate the degree of synchronization of the regime established between various regions of the brain of a rat of the WAG/Rij line. It has been revealed that different regions of the brain are characterized by different degrees of synchronization of the intermittent phase synchronization regime.

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