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Synchronization of Elements with Different Dimensions of Their Ensembles in a Complex Network

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Abstract—We consider the possibility of studying synchronization regimes in a complex network with the aid of integral characteristics (macroscopic signals) taken from a large number of interacting oscillators. It is shown that calculation of the synchronization index based on the phases of macroscopic signals can lead to the loss of information about asynchronous oscillators, which is proportional to the size of ensembles generating these macroscopic signals.

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One of the most important tasks of modern science in the field of network structures consists in studying the regimes of synchronization between the interacting elements of networks [1, 2]. Investigation of the interaction between elements of complex real systems frequently encounters a lack of information on the vectors of state of some network nodes, and so researchers are dealing with total signals generated by groups of interacting elements [3, 4]. An illustrative example of such situations is offered by the study of pathological and cognitive brain activity by magnetoencephalography and electroencephalography techniques [5–8], where detected signals received from various regions of the brain represent an integral activity of large neuronal ensembles.

At the same time, problems encountered in diagnostics of synchronous regimes in neuronal networks of the brain are of considerable interest for studying pathological activity—in particular, epilepsy—that is characterized by hypersynchronous activity of interacting neurons [4, 5, 9, 10].

This Letter presents the results of investigation of the possibility of determining the degree of synchronization of a complex network of interacting elements with the aid of integral characteristics representing total activities of various parts of the system. In addition, we analyze the possible loss of information about the system in this approach.

The present investigation is based on the Kuramoto model, which was originally proposed in 1975 [11] for mathematical description of the collective dynamics of chemical and biological oscillators [12] and now is one of the most widely used network models. In recent years, various modifications of this model of a network of phase oscillators have been applied to the analysis of clusterization and synchronization processes—in particular, in neuronal networks and social systems [13].

Let us consider a network comprising $N = 200$ coupled oscillators (nodes). Every ith node of the network is characterized by phase φ_i and interacts with all the other $N - 1$ nodes. The dynamics of each oscillator is described by the following equation:

$$\dot{\varphi}_i = \omega_i + \lambda \sum_j^N w_{ij} \sin(\varphi_j - \varphi_i), \tag{1}$$

where ω_i are randomly set real frequencies in the [1, 10] interval, w_{ij} is the weight of link between the ith and jth nodes, and λ is the coupling strength. The initial phases of interacting elements are randomly set and uniformly distributed over the $[-\pi, \pi]$ segment; the weights of links between nodes are also randomly distributed.

In order to quantitatively characterize the degree of network synchronization, let us introduce the synchronization index that measures the degree of phase coherency of the oscillators [14, 15]:

$$\sigma = \frac{1}{TN} \int_{t-T}^{t} \left| \sum_{i=1}^{N} e^{i\varphi_i(t)} \right| dt, \tag{2}$$

where φ_i is the phase of the ith oscillator and T is the length of a time series used for modeling processes in the network. Values of the synchronization index close to zero imply that a very small number of oscillators are in a regime of synchronization, an increase in σ indicates that a growing number of oscillators are synchronized, and $\sigma = 1$ corresponds to complete phase synchronization in the network under consideration.

For analysis of synchronization in terms of integral characteristics, we introduce the following functions

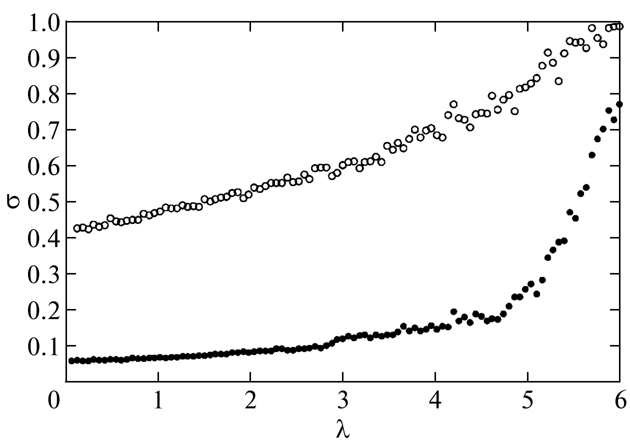

Fig. 1. Plots of synchronization index σ vs. coupling strength λ for different sizes of subsets $M = 1$ (black symbols) and $M = 50$ (open symbols) in a network of $N = 200$ elements.

representing signals averages over a certain subset of network elements:

$$X_k(t) = \frac{1}{M}\sum_{n=k}^{k+M} x_n(t), \quad (3)$$

where M is the number of elements over which the signal is averaged and k is the serial number of the first element in this subset. The entire network is divided into nonoverlapping subsets containing equal numbers M of elements. Each signal $x_n(t)$ is defined as

$$x_n(t) = A\cos(\varphi_i(t)), \quad (4)$$

where φ_i is the phase of the ith oscillator at time moment t; in what follows, the amplitude of every oscillator is assumed to be unity ($A = 1$).

The introduced characteristics (3) can be treated as analogous to the electroencephalographic signal that characterizes the contribution of a group of oscillators (neurons of a local brain region close to the receiving electrode [16]) to the integral signal measured in experiment. In order to study the degree of synchronization of a system using its integral characteristics, we calculate the integral synchronization index by using Eq. (2), where phases of separate elements are replaced by the phases of total signals taken from subsets of M elements.

Figure 1 shows plots of the synchronization index and integral synchronization index versus the strength of coupling between interacting oscillators. As can be seen, the synchronization index exhibits a nonlinear behavior by weakly increasing with the coupling strength up to $\lambda \approx 4.7$ and then growing sharply with further increase in λ.

Let us now consider the behavior of the integral synchronization index. When the entire network of $N = 200$ elements is divided into subsets of $M = 50$ elements, we obtain four macroscopic signals which are used to calculate the integral index. These σ values are significantly greater than those obtained using separate elements, which is evidence of a significant loss of information on the dynamics of coupled oscillators. In addition, the shape of the plot also strongly changes, since the integral synchronization index exhibits almost linear increase with the coupling strength. In the region of $\lambda > 5$, the difference between two depen-

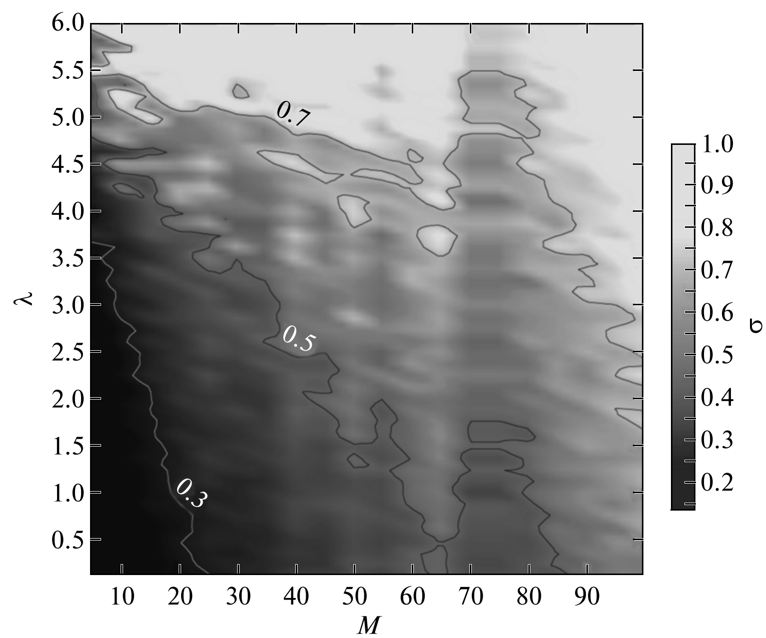

Fig. 2. Two-parametric dependence of integral synchronization index σ on oscillator coupling strength λ and number M of elements in a subset generating a macroscopic signal. Number of network elements: $N = 200$.

dences decreases. This circumstance indicates that estimation of the degree of system synchronization with the aid of integral characteristics leads to loss of information on asynchronous oscillators.

In order to study the possibility of determining the degree of synchronization from integral characteristics in more detail, we have carried out a two-parametric study of the integral synchronization index as dependent on coupling strength λ and size M of a subset generating the macroscopic signal. The results are presented in Fig. 2, where the level lines correspond to $\sigma = 0.3$, 0.5, and 0.7. As can be seen from these data, deviations from the true value (synchronization index calculated using the phases of each element) linearly grow with increasing number of elements in the subset.

In conclusion, we have considered the possibility of studying synchronization regimes in a complex network with the aid of integral characteristics (macroscopic signals) taken from a large number of interacting oscillators. The dependence of integral synchronization index σ on the size of the ensemble of oscillators from which the macroscopic signal is received has been determined. It is shown that, as size M of this ensemble increases, the dependence of σ on the strength of coupling between oscillators tends toward linear. It is established that the increase in M is accompanied by the growing loss of information about asynchronous oscillators, which shows that this method of determination of the degree of synchronization is correct in application to networks of strongly coupled elements.

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