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SHORT COMMUNICATIONS

Intermittency of Intermittencies at the Phase Synchronization Boundary in the Presence of Noise

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Abstract—The intermittent behavior at the boundary of phase synchronization in the presence of noise is investigated. It is shown that in a certain range of the coupling parameter and noise intensity, the system experiences the intermittency of needle's eye- and ring-type intermittencies. The basic results are demonstrated with two unidirectionally coupled Ressler chaotic oscillators.

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Intermittency is one of the most frequently encountered phenomena in the nature [1]. It is characteristic of a wide class of nonlinear systems, including physical, physiological, and biological systems (see, for example, [2–6]). Several types of intermittencies are distinguished today. These are, first of all, type-I, type-II, and type-III intermittencies [1]; on–off intermittency [7]; needle's eye intermittency [4, 8]; and ring intermittency [9]. Although their underlying mechanisms and statistical characteristics differ, two types of behavior, as a rule, alternate at fixed values of control parameters in all known cases of a system's time realization. Later, it was found that under certain conditions (say, at the boundary of phase synchronization between nonautonomous and coupled chaotic oscillators or in the case of systems with a periodic dynamics), two types of intermittent behavior may coexist in a certain range of time scales. Such a mode was called the intermittency of intermittencies [10]. It represents a radically new level of complexity in the nonlinear system dynamics and naturally attracts much attention of the researchers.

In [10], a general theory of the coexistence of two types of intermittencies in nonlinear systems was constructed. For the case when the needle's eye intermittency (type-1 intermittency in the presence of noise in the supercritical range of the control parameter) and the ring intermittency alternate, analytical expressions were derived for the laminar phase distribution with the control parameter fixed and the mean duration of laminar phases depending on the supercriticality parameter. It was also shown that numerical simulation data for chaotic oscillators and systems demonstrating a periodic dynamics in the presence of noise near the phase synchronization boundary are in good agreement with theoretical predictions in a certain range of time scales.

The aim of this work was to see whether the intermittency of intermittencies may occur in chaotic systems near the phase synchronization boundary in the presence of noise. It will be shown below that when noise exceeds some level (depending on the system under test, control parameter values, and noise signal characteristics), the needle's eye intermittency and ring intermittency coexist in a certain range of the control parameter.

Consider the intermittency of intermittencies near the phase synchronization boundary in the presence of noise using two unidirectionally chaotic Ressler oscillators,

$$\begin{aligned} \dot{x}_1 &= -\omega_1 y_1 - z_1, \quad \dot{x}_2 = -\omega_2 y_2 - z_2 + \varepsilon(x_1 - x_2), \\ \dot{y}_1 &= \omega_1 x_1 + a y_1, \quad \dot{y}_2 = \omega_2 x_2 + a y_2 + D\xi, \\ \dot{z}_1 &= p + z_1(x_1 - c), \quad \dot{z}_2 = p + z_2(x_2 - c). \end{aligned} \qquad (1)$$

Here, $\mathbf{x}_{1,2}(t) = (x_{1,2}, y_{1,2}, z_{1,2})^T$ are the state vectors of the master and slave systems, respectively; $a = 0.15$, $p = 0.2$, $c = 10$, $\omega_1 = 0.93$, and $\omega_2 = 0.95$ are control parameters; ξ is a random Gaussian process with a zero mean and unit variance, and D is the noise intensity. Stochastic differential equations (1) were integrated using the Runge–Kutta fourth-order method (adapted to stochastic differential equations [11]) with time step $\Delta t = 0.001$. The phase synchronization was confirmed by analyzing the phase difference between interacting systems and checking the fulfillment of the phase lock condition

$$|\Delta\phi| = |\phi_1(t) - \phi_2(t)| < \text{const.} \qquad (2)$$

Phases $\phi_{1,2}(t)$ of chaotic signals were considered as rotation angles in the planes $(x_{1,2}, y_{1,2})$ [12]. First, we will analyze the influence of noise on the position of the phase synchronization boundary in system (1). Calculations show that when the noise intensity

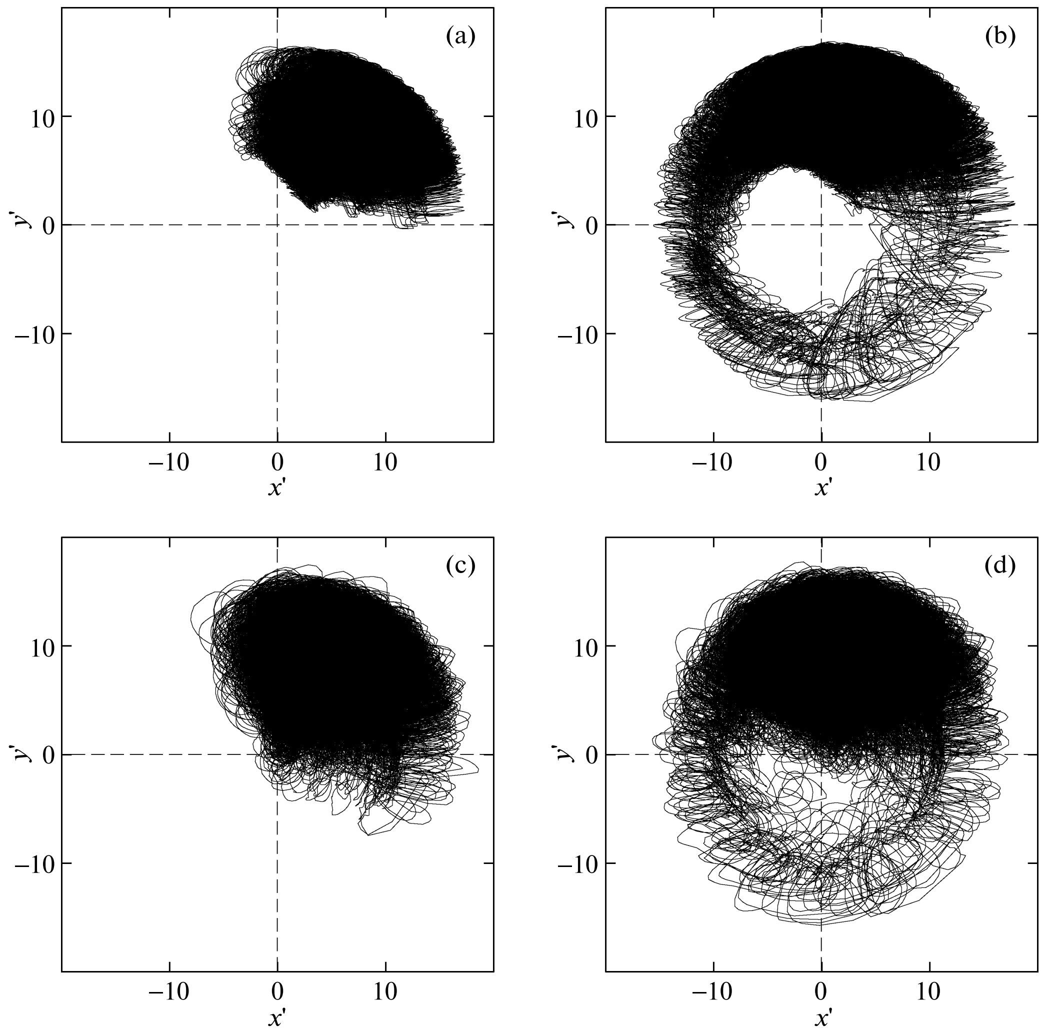

Fig. 1. Phase trajectories of the slave Ressler oscillator on the rotating plane (x', y'): (a) $\varepsilon = 0.045$ and $D = 1.5$ (phase synchronization mode), (b) $\varepsilon = 0.037$ and $D = 1.5$ (needle's eye intermittency), (c) $\varepsilon = 0.045$ and $D = 10$ (ring intermittency), and (d) $\varepsilon = 0.037$ and $D = 10$ (intermittency of the needle's eye intermittency and ring intermittency).

exceeds some critical level, the synchronous mode breaks, because the chaotic attractor of the system loses phase coherency. Clearly, in domains where the phase synchronization boundary remains almost unchanged ($D < 9$), noise has a minor effect both on the threshold of the synchronous mode and on the characteristics of an intermittency near its boundary. However, noise may change the intermittency characteristics in the domain where the attractor of the slave system loses coherency ($D > 9$).

To find the characteristics of an intermittency arising in the system, let us analyze the behavior of the slave system on a rotating plane (see [9]),

$$x' = x_2\cos\phi_1 + y_2\sin\phi_1,$$
$$y' = -x_2\sin\phi_1 + y_2\cos\phi_1. \quad (3)$$

Here, $\phi_1 = \phi_1(t)$ is the phase of the master system and x_2 and y_2 are the coordinates of the slave system. Figure 1 plots the phase trajectories of the system under study on the (x', y') plane for different values of noise intensity D and coupling parameter ε. In Fig. 1a, the case is shown when the system exhibits a synchronous dynamics ($D = 1.5$, $\varepsilon = 0.045$). In our case, the attractor of the slave system is phase-coherent and the phase trajectory of the slave system on the rotating plane represents a "noisy" point not covering the origin. When the noise intensity is low ($D = 1.5$, $\varepsilon = 0.037$), the needle's eye intermittency is observed below the synchronous mode boundary (as in the case when noise is absent, Fig. 1b). The attractor of the slave system in this case remains phase-coherent, and the phase trajectory on the rotating plane represents a noisy limiting cycle. With an increase in the noise intensity, the coherent properties of the slave system attractor change: it becomes basically phase-incoherent, and the phase trajectory embraces the origin on the rotating plane (Figs. 1c, 1d). The origin can be embraced in two ways. If the coupling parameter exceeds the syn-

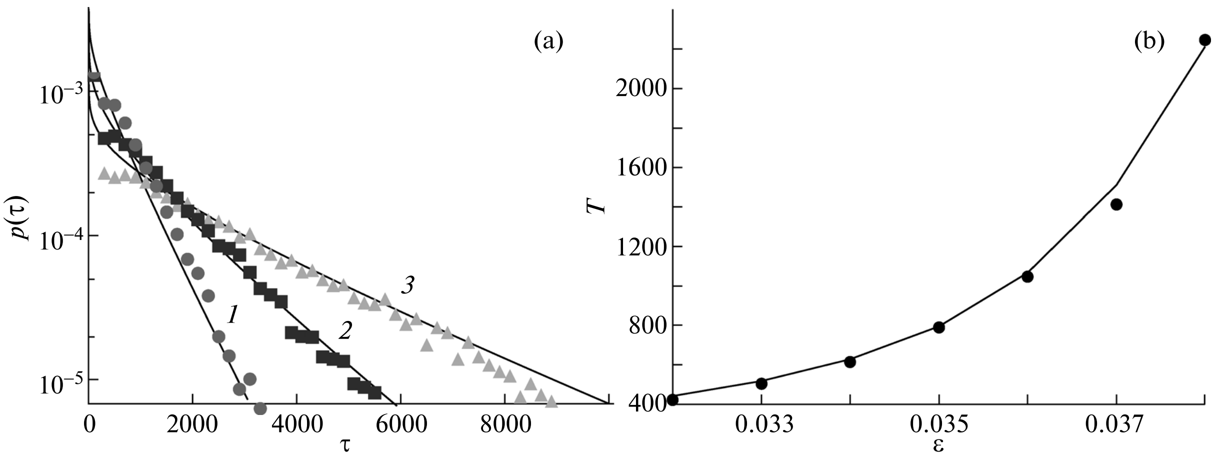

Fig. 2. (a) Normalized laminar phase duration distribution in two unidirectionally coupled Ressler oscillators under the conditions when the needle's eye intermittency alternates with the ring intermittency (intermittency of intermittencies) in the presence of noise ($D = 10$) at different values of the coupling parameter (circles) and their theoretical approximations by expression (4) (continuous lines): (*1*) $\varepsilon = 0.034$, $T_1 = 780$, and $T_2 = 4500$; (*2*) $\varepsilon = 0.036$, $T_1 = 2700$, and $T_2 = 4500$; and (*3*) $\varepsilon = 0.034$, $T_1 = 780$, and $T_2 = 4500$ and (b) laminar phase mean duration vs. the coupling parameter for the same system under the conditions when the needle's eye intermittency alternates with the ring intermittency (circles) and its theoretical approximation by expression (5) (continuous line).

chronous mode threshold without noise ($D = 10$, $\varepsilon = 0.045$) but the phase synchronization at a given noise intensity is still absent, the phase trajectory represents a noisy point embracing the origin (Fig. 1c). In this case, the system exhibits the ring intermittency. If the control parameter is below the phase synchronization threshold in the absence of noise ($D = 10$, $\varepsilon = 0.037$), the phase trajectory on the rotating plane represents a noisy limiting cycle embracing the origin (Fig. 1d). In the given case, the needle's eye intermittency and the ring intermittency alternate.

To be sure that the intermittency of the intermittencies does take place in the system, let us analyze the intermittency statistical characteristics: the laminar phase duration distribution at fixed values of the control parameters and the dependence of the laminar phase mean duration on the supercriticality parameter. It was shown [10] that when the needle's eye intermittency and ring intermittency coexist, the laminar phase duration distribution must obey the relationship

$$p(\tau) = \frac{\exp(-\tau/T_1)}{(T_1+T_2)}\left(1-\frac{\tau}{T_1}\right)\Gamma\left(0, \frac{\tau}{T_2}\right) + \frac{T_1^2+T_2^2}{T_1T_2(T_1+T_2)}\exp\left(-\frac{\tau}{T_1}-\frac{\tau}{T_2}\right) + \frac{\exp(-\tau/T_2)}{(T_1+T_2)}\left(1-\frac{\tau}{T_2}\right)\Gamma\left(0, \frac{\tau}{T_1}\right), \quad (4)$$

where $\Gamma(a, z)$ is an incomplete gamma-function. In this situation, the laminar phase mean duration will be described by the relationship

$$T = -\frac{T_1^2\log\left(\frac{T_1+T_2}{T_1}\right) - 2T_1T_2 + T_2^2\log\left(\frac{T_1+T_2}{T_2}\right)}{T_1+T_2}, \quad (5)$$

$T_{1,2}$ are the laminar phase durations obtained numerically under the conditions when only one type of intermittency (needle's eye intermittency or ring intermittency) is observed [10].

Figure 2a plots the laminar phase duration distributions obtained numerically for system (1) exhibiting the intermittency of the intermittencies at different values of the control parameters and their theoretical approximations. It is seen in Fig. 2a that numerical simulation data are in good agreement with theoretical predictions in all the cases. This counts in favor of the supposition that the intermittency of intermittencies can be observed at the phase synchronization boundary.

The dependence of the laminar phase duration distribution on the coupling parameter is also a factor supporting the presence of the intermittency of the intermittencies. Figure 2b plots these dependences for system (1), which were obtained numerically (circles) and analytically by relationship (5) (continuous line). It is easy to see that the numerically obtained data are consistent with the predictions. Similar results were obtained for the unidirectionally coupled hydrodynamic models of noisy Pierce diodes.

Thus, noise in unidirectionally coupled chaotic systems gives rise to new effects at the phase synchronization boundary. Specifically, when the noise intensity is high, the intermittency of the needle's eye intermittency and the ring intermittency is observed at the synchronous mode threshold. This effect is of a rather general nature. Such behavior might also be expected in real systems.

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REFERENCES

1. P. Berge, Y. Pomeau, and C. Vidal, *L'ordre Dans le Chaos* (Hermann, Paris, 1984).
2. C. M. Kim, G. Yim, J. Ryu, and Y. Park, Phys. Rev. Lett. **80**, 5317 (1998).
3. J. L. Perez Velazquez, et. al., Eur. J. Neurosci. **11**, 2571 (1999).
4. S. Boccaletti, E. Allaria, R. Meucci, and F. T. Arecchi, Phys. Rev. Lett. **89**, 194101 (2002).
5. J. L. Cabrera and J. Milnor, Phys. Rev. Lett. **89**, 158702 (2002).
6. E. Yu. Sitnikova, A. E. Hramov, V. V. Grubov, A. A. Ovchinnkov, and A. A. Koronovskii, Brain Res. **1436**, 147 (2012).
7. J. F. Heagy, N. Platt, and S. M. Hammel, Phys. Rev. E **49**, 1140 (1994).
8. A. S. Pikovsky, G. V. Osipov, M. G. Rosenblum, M. Zaks, and J. Kurths, Phys. Rev. Lett. **79**, 47 (1997).
9. A. E. Hramov, A. A. Koronovskii, M. K. Kurovskaya, and S. Boccaletti, Phys. Rev. Lett. **97**, 114101 (2006).
10. A. E. Hramov, A. A. Koronovskii, O. I. Moskalenko, M. O. Zhuravlev, V. I. Ponomarenko, and M. D. Prokhorov, Chaos **23**, 033129 (2013).
11. N. N. Nikitin, S. V. Pervachev, and V. D. Razevig, Avtom. Telemekh. **4**, 133 (1975).
12. M. G. Rosenblum, A. S. Pikovsky, and J. Kurths, Phys. Rev. Lett. **78**, 4193 (1997).

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