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Generalized Synchronization of Coupled Virtual Cathode Generators

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Abstract—A detailed analysis is performed of generalized synchronization in unidirectionally coupled virtual cathode generators, chain simulated by a 1D PIC electron flow. Calculations of the Lyapunov exponent spectrum for PIC systems are used to diagnose synchronous behavior.

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INTRODUCTION

Chaotic synchronization is an important fundamental phenomenon that has a wide range of practical applications [1]. Chaotic synchronization is used in particular for controlling the behavior of complex coupled systems [2, 3] and for hidden data transmission [4]. Generating and receiving complex signals within the GHz and THz ranges of frequency using nonlinear antennas [5] is another promising field of application. These antennas are sets of microwave generators with complex space-time behavior, combined into a network. They can be created using solid-state devices or devices of an electron-wave or beam-plasma nature [6]. It is known that many nonlinear phenomena, including synchronization, can be created in such networks of coupled self-oscillatory systems [7–9]. The designing of nonlinear antennas thus requires the development and use of new methods for analyzing and controlling the complex and nontrivial effects of collective interaction between coupled oscillatory systems [10].

Calculating the Lyapunov exponent spectrum is one approach that allows us to make quantitative estimates of the behavior of both autonomous and nonautonomous oscillatory systems. This way of analyzing the complex behavior of oscillatory systems works well when investigating the behavior of lumped-parameter systems of radiophysical nature, and with spatially distributed beam-plasma and electron-wave systems described in the continuous medium approximation [11]. However, it is not always possible to use hydrodynamics equations describe electron flow systems that interact with an electromagnetic field.

The particle-in-cell (PIC) method developed in the mid-1950s for solving hydrodynamic problems [12] and simulating plasmas [13, 14] is a most efficient tool for modeling beam-plasma systems. Using this approach to calculate the Lyapunov exponents [15] allows us to analyze oscillatory behavior in systems simulated via PIC.

Modifying the procedure for calculating the Lyapunov exponents for autooscillatory systems in PIC method enables us to study nonlinear processes that occur in chains and networks of spatially distributed coupled autooscillatory systems. This approach allows us in particular to make precise estimates of the boundaries of different types of synchronization when varying governing parameters and coupling parameters; to study possible scenarios of a transition to synchronous behavior for a particular system; and to perform quantitative assessments of processes in spatially distributed coupled autooscillatory systems.

DIAGNOSTICS OF SYNCHRONIZATION

We studied the phenomenon of generalized synchronization using a 1D numerical model of a chain of virtual cathode (VC) generators [16–20]. The use of this type of microwave device as units for the construction of nonlinear antennas seems promising, due to the broad spectral band of output oscillations, complex nonstationary processes in the electron flow during VC formation, and oscillatory behavior which can be easily adjusted via variation of the governing parameter [17].

We studied generalized synchronization in a system of coupled vircators by calculating the Lyapunov exponent spectrum. To perform our calculations, we used a modification of the approach proposed in [15], which was designed for analyzing the behavior of an autonomous beam-plasma system with a VC simulated by means of PIC. In considering the behavior of a nonautonomous system with a VC, the input from an external signal is calculated from the velocity and density modulation of the electron flow at the entrance to the drift space [21]. Additional calculations for signal disturbance in the transmission line and flow disturbance in the drift space are thus required. Allowing for the nonautonomous behavior of the electron flow in a VC generator makes it possible to properly construct the Lyapunov exponent spectrum for a system of coupled autooscillatory systems with VCs.

Our results from analyzing the generalized synchronization of spatially distributed systems with VCs obtained by calculating the Lyapunov exponent spectrum were confirmed using other methods for investigating synchronous behavior, particularly modified nearest neighbor and auxiliary system methods [22]. These approaches were developed and successfully used to study generalized synchronization in lumped-parameter systems, and might therefore yield completely inaccurate results when applied directly to spatially distributed systems. Adapting the nearest neighbor method for spatially distributed systems involves calculating the average distance between states U_r^1 and U_r^2 of a slave system, which are images of the closest states U_d^1 and U_d^2 of the master system. The resulting value is then normalized according to the average distance between randomly selected states of the slave system, and the quantitative parameter of the degree of synchronization is calculated:

$$d = \frac{1}{N\delta}\sum_{k=0}^{N-1}\left\|U_r^1 - U_r^2\right\|, \quad (1)$$

where δ is the average distance between randomly selected states of the slave system, and N is the number of averaging iterations.

It should be noted that in this case, the state of the system is a spatially distributed value, rather than a finite-dimensional vector.

The auxiliary system approach is used in a manner similar to the one in [18]: in addition to master and slave systems, an auxiliary system is introduced that is identical to the slave system, apart from the initial conditions at which systems start operating. In this work, the difference between the states of slave and auxiliary systems was considered through the entire space, due to the spatial distribution of interacting systems. This approach allows us not only to determine the presence or absence of generalized synchronization, but to reveal regions of space where asynchronous or synchronous behavior prevails.

RESULTS AND DISCUSSION

Calculating the Lyapunov exponent spectrum to analyze the synchronous behavior of unidirectionally coupled VC generators simulated via 1D PIC allowed us to estimate with precision the boundary of generalized synchronization. The governing parameters of interacting generators were selected such that both systems displayed complex space-time behavior: $\Delta\varphi_d = 0.5$, $\Delta\varphi_r = 0.4$, $\alpha_d = \alpha_r = 0.9$. The slave generator state is in this case characterized by a single positive

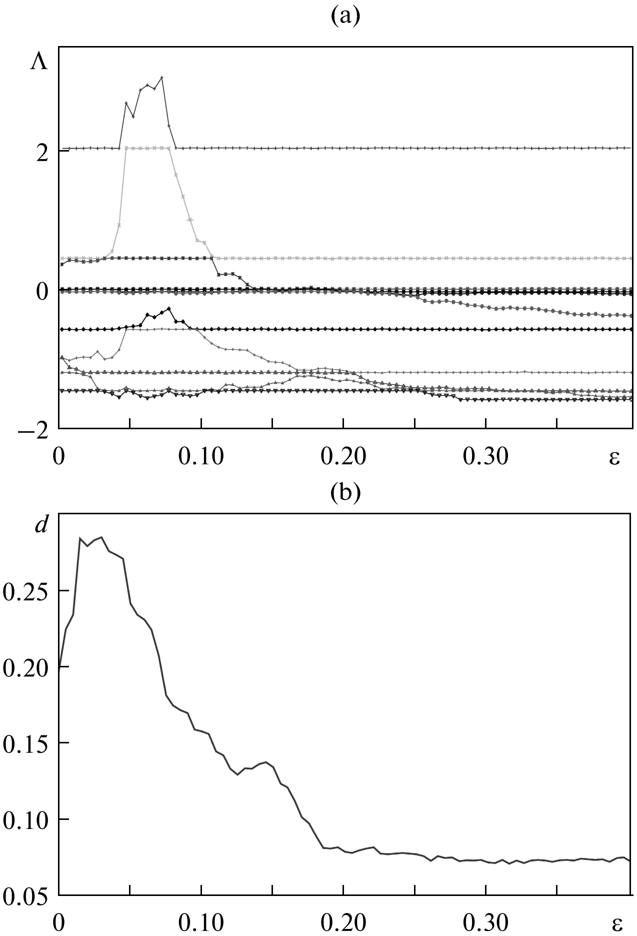

Fig. 1. (a) Dependence of the Lyapunov exponent spectrum and (b) measure of synchronization d on the coupling parameter in the system of unidirectionally coupled VC generators. The parameters of interacting generators are $\Delta\varphi_d = 0.5$, $\Delta\varphi_r = 0.4$, $\alpha_d = \alpha_r = 0.9$.

Lyapunov exponent, $\Lambda_r^1 = 0.36$ (chaotic behavior), and the master system's state is characterized by two positive Lyapunov exponents: $\Lambda_d^1 = 2.03$ and $\Lambda_d^2 = 0.45$ (hyperchaotic behavior). The dependences of the main Lyapunov exponents on the coupling parameter of the systems are presented in Fig. 1. The spectrum includes Lyapunov exponents that correspond to the master system and do not change when the coupling parameter is varied, since coupling is unidirectional. The parameters that do change characterize the slave system behavior. It can be seen that the Lyapunov exponents of the slave system grow at ε = 0–0.07, due to complication of the behavior as a result of external interference. As the coupling parameter grows, the slave system parameters fall sharply: they are close to zero at ε = 0.135–0.195 and pass through zero at $\varepsilon_{GS} = 0.2$.

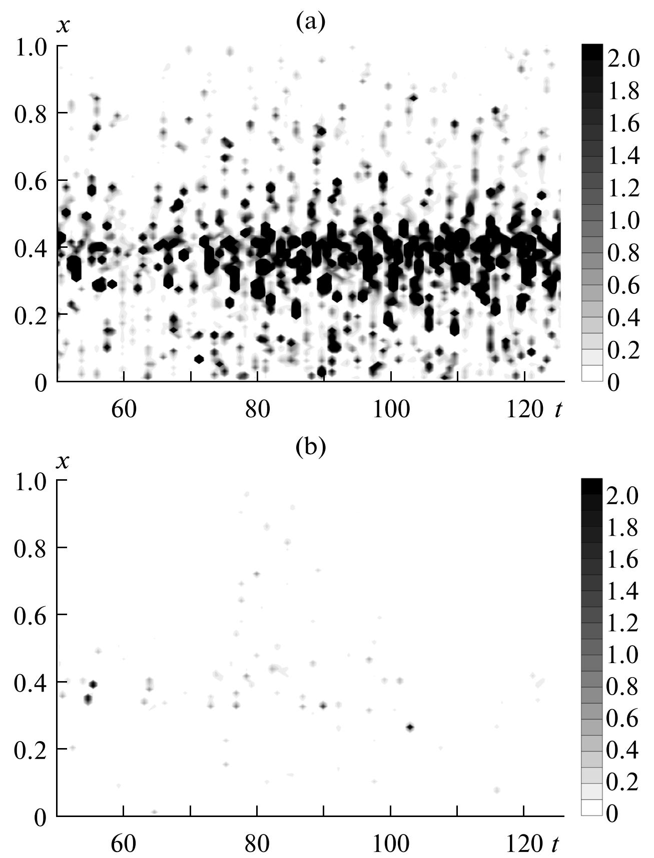

Fig. 2. Difference between states of slave and auxiliary systems, depending on the coordinate and time corresponding to (a) asynchronous behavior $\varepsilon = 0.1$ and (b) synchronization threshold $\varepsilon_{GS} = 0.2$.

This behavior of the Lyapunov exponent spectrum of unidirectionally coupled VC generators indicates that as coupling strengthens, the slave system tends toward synchronization, i.e. toward the destruction of its own behavior and adopting that of the master system.

The results from our study of generalized synchronization were confirmed by other methods for analyzing synchronous dynamics. The dependence of measure of synchronization *d*, obtained using the nearest neighbor method modified for analyzing spatially distributed autooscillatory systems, is presented in Fig. 1b. It can be seen that *d* falls rapidly as coupling between the interacting systems increases, and levels off at $\varepsilon_{GS} = 0.2$.

Generalized synchronization was also analyzed; the results are presented in Fig. 2. When there was no synchronization at $\varepsilon = 0.1$, the states of the slave and auxiliary systems were not identical. The difference between the two was non-zero and reached its maximum in the region of VC oscillation $x = 0.15$–0.55. The difference between the slave and auxiliary systems becomes zero over the drift space after the termination of the transient process at the boundary of synchronization, $\varepsilon_{GS} = 0.2$, testifying to the perfect match between states of the slave and auxiliary systems and the establishing of generalized synchronization.

CONCLUSIONS

A modified method for calculating the Lyapunov exponent spectrum for beam-plasma systems simulated via PIC was proposed. The method allowed us to analyze the joint behavior of spatially-distributed coupled PIC systems. It was shown that this algorithm makes it possible to estimate with precision the boundary of generalized synchronization, as was confirmed by using different approaches to analyzing synchronous behavior: the nearest neighbor and auxiliary system methods adapted for spatially distributed systems.

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