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NONLINEAR MICROWAVES IN CRYSTALS

Transition to Microwave Generation in Semiconductor Superlattice

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Abstract—We investigate excitation of microwave generation in a semiconductor superlattice under the effect of the applied constant voltage at near-zero temperature in the absence of the external magnetic field. It is shown that the generation is caused by the positive feedback arising from the total constant voltage drop across the superlattice.

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1. INTRODUCTION

Semiconductor superlattices proposed in [1] are an important subject both for the study and understanding of processes in solid state physics [2, 3] and for the investigation of the observed phenomena from the standpoint of nonlinear dynamics [4–7]. Due to the additional potential produced by alternating thin (about 10 nm) layers of various semiconductor materials, superlattices show a number of interesting properties that are not typical of ordinary semiconductors. Interest in semiconductor superlattices is also maintained by a prospect of creating devices that operate both in the generation mode [8] and as amplifiers of microwave signals [9]. It is known that the dc voltage applied to the superlattices can generate, at its particular value, high-frequency oscillation of the current through this structure.

Loss of stability of a steady state was theoretically investigated in [10], but dynamic mechanisms which cause loss of stability have not been quite understood so far. In particular, it is not clear what processes result in formation of traveling charge domains: whether it is due to the fact that the injection current exceeds a particular critical value or the cause is the positive feedback which arises from the voltage drop across the lattice contacts and is able, at certain voltage values, to maintain self-oscillation processes. The study of these mechanisms can be helpful for predicting processes that take place in the semiconductor superlattice at the pregeneration regime, as in the Gunn diode [10–12].

In this work we analyze the steady state of the semiconductor superlattice and processes that lead to the generation. It is shown that loss of stability of the steady-state solution and transition to generation are associated with the feedback effect that maintains a constant potential difference at the boundaries of the system.

2. SYSTEM UNDER INVESTIGATION

To describe electron transport in superlattices, we use the miniband model based on the semiclassical approach. In this approach the collective dynamics of charge carriers in the semiconductor superlattices is described by a self-consistent system of differential equations, which includes the continuity equation (1) describing electron concentration variation with time and the Poisson equation (2) describing electric field distribution along the superlattice:

$$\frac{\partial n}{\partial t} = -\frac{\partial J}{\partial x}, \qquad (1)$$

$$\frac{\partial F}{\partial x} = R(n-1). \qquad (2)$$

Here we introduce the dimensionless variables $n(x,t)$, the bulk density of charge carriers, and $F(x,t)$, the electric field distribution; $J(x,t)$ is the density of the current through the superlattice, x and t are the

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coordinate and the time, and the constant $R = 0.1146$ plays the role of a control parameter. Conversion to dimensionful quantities is through the relations

$$x = \frac{x'}{d'}, \quad t = \omega'_{B0} t', \quad n = \frac{n'}{n'_D},$$
$$J = \frac{J'}{e n'_D \omega'_{B0} d'}, \quad F = \frac{F'}{F'_0}, \tag{3}$$
$$\omega'_{B0} = \frac{e d' F'_0}{\hbar}, \quad R = \frac{e d' n'_D}{F'_0 \varepsilon_0 \varepsilon_r},$$

where $d' = 8.3$ nm is the superlattice period, $n'_D = 3\times10^{22}\ \mathrm{m}^{-3}$ is the equilibrium concentration of electrons determined by the doping level, $e > 0$ is the electron charge, $F'_0 = 3.145\times10^6\ \mathrm{V\,m^{-1}}$ is the normalization value of the electric field, ω'_{B0} is the frequency of the Bloch oscillations of electron in the external electric field F'_0, and ε_0 and $\varepsilon_r = 12.5$ are the electric constant and relative permittivity of the material, respectively. Dimensionful values of the control parameters are chosen in accordance with the parameters of the semiconductor superlattices used in the experiments [6].

Within the drift approximation, the current density $J(x,t)$ is defined as

$$J = n v_d(F) + D(F)\frac{\partial n}{\partial x}, \tag{4}$$

where $v_d(F)$ is the drift velocity of electrons ($v_d = v'_d/\omega'_{B0} d'$) and $D(F)$ is the diffusion coefficient defined as

$$D(F) = v_d(F)\frac{\exp(-\kappa F)}{1 - \exp(-\kappa F)}, \quad \kappa = \frac{e F'_0 d'}{k'_B T'}. \tag{5}$$

At low temperatures T' the current density is calculated with the diffusion term ignored,

$$J = n v_d(F). \tag{6}$$

Dependence of the electron drift velocity $v_d(F)$ on the electric field strength plays an important part in the model. It is this dependence that contains information on the spatial structure (period d') and energy characteristics of the semiconductor nanostructure, external magnetic field[1] B', and temperature T'. Though such parameters as the miniband width Δ' (in our case $\Delta' = 19.1$ meV), the vector of the magnetic induction B', and the temperature T' do not explicitly appear in the model equations (1) and (2) which describe dynamics of charge domains, they substantially affect the character of the drift velocity dependence on the electric field strength, $v_d(F)$, and consequently on the dynamic regimes in the semiconductor superlattice.

We consider the case where the temperature T' is close to zero and there is no external tilted magnetic field[2]. In this case, the dependence $v_d(F)$ can be obtained analytically,

$$v_d(F) = \delta v_0 \frac{F\tau}{1 + (F\tau)^2}, \tag{7}$$

where $v_0 = \Delta'/2eF'_0 d'$, $\delta = \sqrt{\tau'_e/(\tau'_e + \tau'_i)}$, and $\tau = \omega'_{B0}\tau'$ ($\tau' = \delta\tau'_i$) are the parameters characterizing elastic and inelastic electron scattering. We use the values $\tau = 9.9$ ($\tau' = 250$ fs), $v_0 = 0.366$, and $\delta = 1/8.5$. Assuming that the emitter and the collector have ohmic contacts and the density of the current through the emitter, $J(0,t)$, is governed by the conductivity of the contact, we obtain from Ohm's law the boundary condition

$$J(0,t) = sF(0,t), \tag{8}$$

where $s = \delta' F'_0 / e n'_D \omega'_{B0} d' = 7.5315$ is the emitter conductance, and $\delta' = 3788\ \mathrm{cm}^{-1}$. The voltage $V = V'/F'_0 d'$ applied to the superlattice can be found from the condition

$$V = U + U_{SL}, \quad U_{SL} = \int_0^L F\,dx, \tag{9}$$

where integration is carried out over the length of the system $L = L'/d'$ (in our case $L = 13.90$). The quantity $U = U'/F'_0 d'$ determines the voltage drop across the contacts, and $U_{SL} = = U'_{SL}/F'_0 d'$ is the voltage drop across the semiconductor sample.

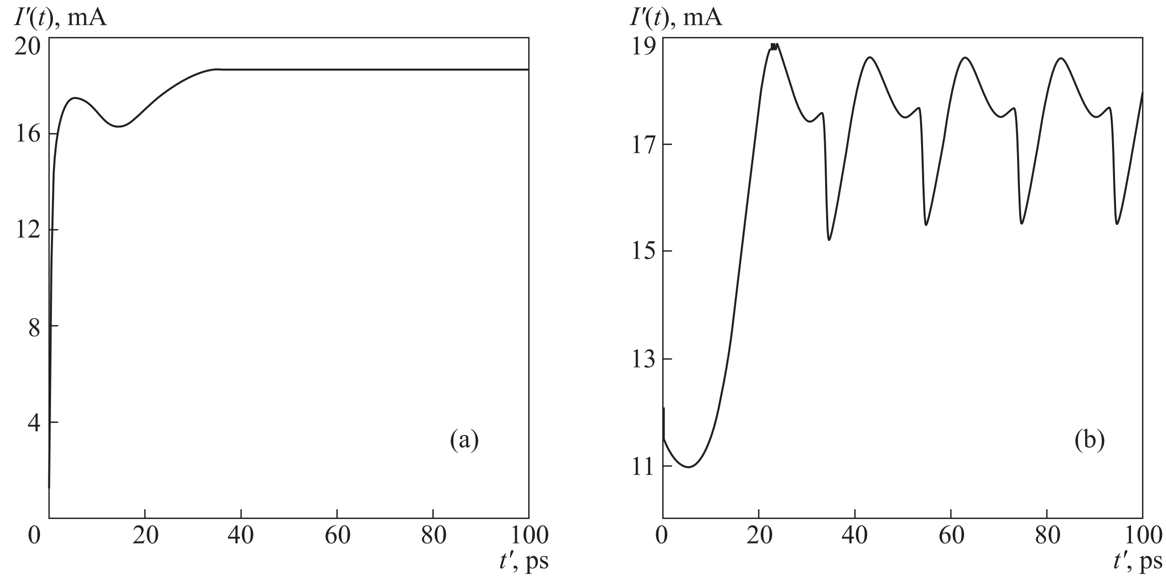
3. NUMERICAL MODEL AND LOSS OF STABILITY

To develop the finite difference model which describes processes occurring in the system, the superlattice is divided into N narrow layers with the width Δx (we use $N = 480$ and $\Delta x = L/N = 2.896\times10^{-2}$). The system of equations (1) and (2) in the finite difference representation has the form

$$\Delta x \frac{\partial n}{\partial t} = J_{m-1} - J_m, \quad m = 1, ..., N, \tag{10}$$
$$F_{m+1} = R(n_m - 1) + F_m, \quad m = 1, ..., N. \tag{11}$$

[1] The external tilted magnetic field can substantially change the character of the motion of electrons; under particular conditions electrons are capable of making chaotic oscillations as a result the resonance between the cyclotron and Bloch oscillations of the electrons, which in turn causes considerable changes in the generation characteristics [6, 13].

[2] If necessary, the effect of the temperature and the magnetic field on the drift velocity can be numerically taken into account by the method described in [14].



Time realizations of the current through the superlattice: (a) boundary condition $J_0 = \mathrm{const} = 0.0236$ and (b) boundary condition $U_{SL} = \mathrm{const} = 2.446$.

The system of (10) and (11) is numerically integrated using the explicit scheme and the step in time $\Delta t = 0.005$. In the generation regime, the current density is deduced from (6) using the drift velocity $v_\mathrm{d} = v_\mathrm{d}(\bar{F})$ calculated for the average electric field strength $\bar{F}$ in the given layer. The total voltage drop across the superlattice is defined by the relation

$$V = U + \frac{\Delta x}{2}\sum_{m=1}^{N}(F_{m+1} + F_m), \qquad (12)$$

where U is the voltage drop across the contacts. Considering formation of regions with a higher concentration of charge carriers near the emitter and their lower concentration near the collector, we describe U as

$$U = 2F_0\Delta x_l - F_0(\Delta x_s + \Delta x_q) + F_1\Delta x_s + F_{N+1}\Delta x_q - \frac{Rn_0(\Delta x_q)^2}{2} + F_0SR. \qquad (13)$$

Here $\Delta x_l = 6.02$ is the length of the contacts, $\Delta x_s = 1.8$ and $\Delta x_q = F_{N+1} - F_0/Rn_0$ are the lengths of the regions with higher and lower concentrations of electrons near the contacts, $n_0 = n'_0/n_D = 3.3$ is the concentration of electrons in the contact layer, and $SR = R_\mathrm{c}A\sigma'/d'$, where $R_\mathrm{c} = 17\,\Omega$ is the contact resistance including the resistance of the measuring line and $A = 5\times10^{-10}\,\mathrm{m}^2$ is the area of the contact [4, 6, 8].

The overall current through the superlattice is

$$J = \frac{A}{N+1}\sum_{m=0}^{N} J_m. \qquad (14)$$

The results of the numerical integration show that when the constant voltage $V_\mathrm{c} \approx 13.79$ is applied to the sample, periodic current oscillation is generated in the semiconductor superlattice. However, mechanisms for the loss of stability of the steady state have not been established so far; it can be assumed that the loss of stability results either from the excess of a particular critical value for $J(0,t)$ (which triggers development of instability in the nonlinear active medium) or from the effect of the positive feedback occurring through the constant drop of the voltage V. These two assumptions were verified by integrating the equation with different boundary conditions imposed. To verify the first mechanism, the current density at the entrance to the superlattice $J_0 = J(0,t)$ was kept constant and then gradually increased to the critical value $J_0 = J_\mathrm{c} \approx 0.0227$ (corresponding to the voltage V_c). To verify the second mechanism (feedback through providing a fixed voltage drop across the superlattice), the boundary condition was given as $U_{SL} = \mathrm{const}$ and, accordingly, U_{SL} was gradually increased. Note also that the voltage U at the contacts was ignored in the numerical integration.

The results of the numerical simulation unambiguously indicate that the loss of stability of the steady state is due to the feedback arising from the constant potential difference in the superlattice. The figure shows time dependences of the current. For convenience, all quantities are given in dimensional units.

The figure (a) corresponds to the boundary condition $J_0 = \mathrm{const} = 0.0236$ ($U_{SL} = 2.446$) with the current J_0 being higher than the critical value J_c. The figure (b) shows a similar dependence under the condition $U_{SL} = \text{const} = 2.446$. It is seen that the magnitude of the current through the superlattice varies with time, i.e., the steady state is unstable and periodic oscillation occurs in the system.

4. CONCLUSIONS

Thus, the efficiency evaluations of the alternative mechanisms for the loss of stability of the steady state have shown that generation in the system is due to the feedback arising from the constant voltage drop across the superlattice.

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