ISSN 1062-8738, Bulletin of the Russian Academy of Sciences. Physics, 2013, Vol. 77, No. 12, pp. 1444–1447. © Allerton Press, Inc., 2013.
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Stability of the Steady State in a Strongly Coupled Semiconductor Superlattice Described Using a Semiclassical Approach

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Abstract—The stability of the steady state of a semiconductor superlattice described using a semiclassical approach is analyzed. It is shown that generation arises due to the appearance of perturbations characterized by a positive increment of growth. The relationship between the frequency of the current oscillations and the frequency of the perturbation oscillation is determined.

DOI: 10.3103/S1062873813130017

INTRODUCTION

At present, exploiting the terahertz frequency range (0.3–10 THz) is one of most important problems of modern science. One promising line in the development of terahertz-range devices involves using semiconductor nanostructures with periodic modulation of the conductivity band, i.e., superlattices.

Similar structures were proposed by L. Esaki and R. Tsu [1, 2] in 1970 as objects for studying quantum-mechanical phenomena. However, the possibility of using Bloch oscillations in a semiconductor superlattice made it a promising instrument for creating devices that function in the mode of generating and amplifying terahertz-range frequencies [3, 4]. At the same time, the use of the Bloch oscillations in practice turns out to be problematic, since the applied potential difference leads to instability and the formation of domains (regions with increased concentrations of charge carriers) drifting along the superlattice [5, 6]. The inhomogeneities in the distribution the electric field, also associated with the formation of domains, are disastrous for coherent Bloch oscillations [7, 8]. However, the rate of domain repetition in semiconductor superlattices can reach several tens of gigahertz, and this phenomenon can be also used to create microwave devices [9]. Studies in this field are directed toward raising the velocity of domain motion and reducing the voltage necessary for the development of instability. Major problems are determining the critical voltage and estimating the rate of charge domain repetition from the parameters of the semiconductor's structure. Similar estimates can be obtained via numerical simulations [10], but finding them by means of analytical theory is of special interest.

In this work, the stability of a strongly coupled semiconductor superlattice was analyzed by considering the behavior of small perturbations in the reference state.

INVESTIGATED SYSTEM

To describe the dynamics of a strongly coupled semiconductor sublattice,[1] we traditionally use the semiclassical approach in [11] that enables us to describe the dynamics the electron position and wave vector under the action of external electric and magnetic fields F and B, respectively.

Using the semiclassical approach and disregarding diffusion, the collective dynamics of charge carriers in a semiconductor superlattice can be described using a self-consistent system of differential equations that include continuity equation (1) describing the change in electron concentration over time, and Poisson equation (2) describing the distribution of the electric field along the superlattice. In dimensionless quantities, these are written in the form

$$\frac{\partial n}{\partial t} = -\beta \frac{\partial}{\partial x}\left(n\upsilon_d(F)\right), \quad (1)$$

$$\frac{\partial F}{\partial x} = \nu(n-1). \quad (2)$$

System of equations (1)–(2) is solved with respect to dimensionless quantities $n(x, t)$ and $F(x, t)$, where $n(x, t)$ is the volumetric density of charge carriers, $F(x, t)$ is the distribution of the electric field, x and t are

[1] Superlattices are strongly coupled if their barrier width is much less than the characteristic inverse electron wave number inside the barrier [12]. Similar structures function in the mode of electron transport through minibands and can be described using a semiclassical approach.

a dimensionless coordinate and time, and $\beta = 0.03074$ and $\nu = 15.769$ play the role of governing parameters. The transition to dimensional quantities occurs through the relations

$$x = x'/L', \quad t = t'/\tau', \quad n = n'/n'_D,$$
$$F = F'/F'_c, \quad F_c = \hbar/(ed'\tau'), \quad \vartheta'_0 = \delta\Delta' d'/(2\hbar), \tag{3}$$
$$\beta = \vartheta'_0\tau'/L', \quad \nu = eL'n'_D/(F'_c\varepsilon_r\varepsilon_0),$$

where $d' = 8.3$ nm is the superlattice's period, $L' = 115.2$ nm is its length, $n'_D = 3 \times 10^{22}$ m^{-3} is the equilibrium electron concentration determined by the doping level, $e > 0$ is the electron charge, and ε_0 and $\varepsilon_r = 12.5$ are the electric constant and relative dielectric permittivity of the material, respectively. $F'_c = 3.1725 \times 10^5$ V/m is the normalization value of the electric field. Parameters $\delta = \left[\tau'_e/\left(\tau'_e + \tau'_i\right)\right]^{1/2}$ and $\tau' = \delta\tau'_i$ characterize the scattering of electrons in the superlattice and depend on the times of elastic τ'_e and inelastic τ'_i scattering. Here, $\tau' = 250$ fs, $\delta = 1/8.5$. The dimensional parameter values correspond to the semiconductor superlattices used in other experimental works [11].

The dependence of the electron drift velocity $\upsilon_d(F)$ on the electric field strength in Eq. (1) plays an important role in the model described above. This dependence contains information on the spatial structure (period d') and the energy characteristics of the semiconductor nanostructure, external magnetic field[2] B' and temperature T'. Although such parameters as the width of the miniband Δ' (in our case, $\Delta' = 19.1$ meV), the vector of magnetic induction B', and temperature T' are not explicitly included in Eqs. (1) and (2) describing the dynamics of the charge domains, they considerably affect the dependence of the drift velocity on electric field strength $\upsilon_d(F)$ and, as a consequence, the dynamic modes in the semiconductor superlattice.

In this work, we consider a case in which temperature T' is close to zero, and there is no tilted external magnetic field.[3] The dependence can be obtained analytically [1]:

$$\upsilon_d(F) = \frac{F}{1 + F^2}. \tag{4}$$

Assuming that the contacts on the emitter and collector are ohmic and the density of the current flowing through the emitter $J(0, t)$ is determined by the conductivity of the contact, according to the Ohm law we have the boundary condition

$$J(0,t) = sF(0,t), \tag{5}$$

where $s = \sigma' F'_c/(en'_D\vartheta'_0) = 17.6511$ corresponds to the dimensionless conductivity of the emitter, and $\sigma' = 3788$ S^{-1}. Dimensionless value $U_{SL} = U'_{SL}/(F'_c L')$ of the voltage applied to the superlattice can be found from the condition

$$U_{SL} = \int_0^1 F dx, \tag{6}$$

where integration is performed over the length of the system.

ANALYZING STABILITY

To analyze stability, let us consider a perturbed state of the system: $F(x,t) = F_0(x,t) + \tilde{F}(x,t)$, $n(x,t) = n_0(x,t) + \tilde{n}(x,t)$, where $n_0(x,t)$ and $F_0(x,t)$ are steady states, $F_0(x,t) \gg \tilde{F}(x,t)$, $n_0(x,t) \gg \tilde{n}(x,t)$, and $\tilde{F}(x,t), \tilde{n}(x,t)$. The evolution of the spatially distributed perturbations is described by linearized equations of mathematical model (1) and (2),

$$\frac{\partial \tilde{n}}{\partial t} = -\beta\frac{\partial}{\partial x}((n_0 + \tilde{n})(\upsilon_d(F_0) + \upsilon'_d(F_0)F')),$$
$$\frac{\partial \tilde{F}}{\partial x} = \nu\tilde{n}. \tag{7}$$

Introducing $\tilde{n}(x,t) = \breve{n}(x)e^{\sigma t}$ and $\tilde{F}(x,t) = \breve{F}(x)e^{\sigma t}$, where $\sigma = \lambda - i\omega$, we obtain from the first equation of system (7)

$$\upsilon_d(F_0)\frac{d^2\breve{F}}{dx^2} + \left(\frac{\sigma}{\beta} + \upsilon'_d(F_0)\frac{dF_0}{dx} + \upsilon n_0\upsilon'_d(F_0)\right)\frac{d\breve{F}}{dx}$$
$$+ \left(\upsilon\frac{dn_0}{dx}\upsilon'_d(F_0) + \upsilon n_0\upsilon'_d(F_0) + \upsilon n_0\upsilon''_d(F_0)\right)\breve{F} = 0. \tag{8}$$

Here, dependence $\upsilon_d(F_0)$ is given by relation (4), and the expression describing the distribution of the electric field for voltage U_{SL}, $F_0(x)$, can be found from Eqs. (1) and (2) under the condition $\partial n/\partial t = 0$. The solution to Eq. (8) is thus a set of spatially distributed perturbations of electric field $\tilde{F}(x)$, characterized by increment of growth/decay λ and frequency ω. Note that the reduced voltage on the superlattice remains constant; consequently, only perturbations of the electric field can exist in system for which the following condition holds:

$$\breve{U}_{SL} = \int_0^1 \breve{F}(x)dx = 0. \tag{9}$$

As was noted above, applying voltage to the superlattice leads to instability and the formation of drifting

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[2] A tilted external magnetic field can dramatically change the character of electron motion; under certain conditions, electrons in the superlattice are subject to chaotic oscillations that arise as a result of resonance between the electron cyclotron and Bloch oscillations; this in turn considerably changes the characteristics of generation [11, 13].

[3] If necessary, the effect of the temperature and magnetic field on the drift velocity can be included numerically using the method described in [14].

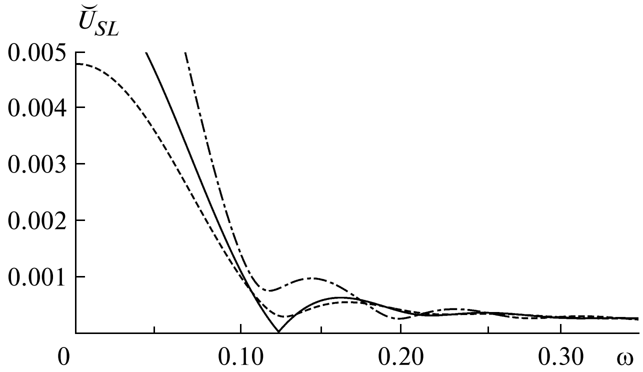

Fig. 1. Dependence $\breve{U}_{SL}(\omega)$ for when the voltage on the superlattice is less than the critical value (dashed line, $U_{SL} = 1.14$), corresponds to the critical value (solid line, $U^*_{SL} = 1.25$), and lies above the critical value (dash-dotted line, $U_{SL} = 1.55$).

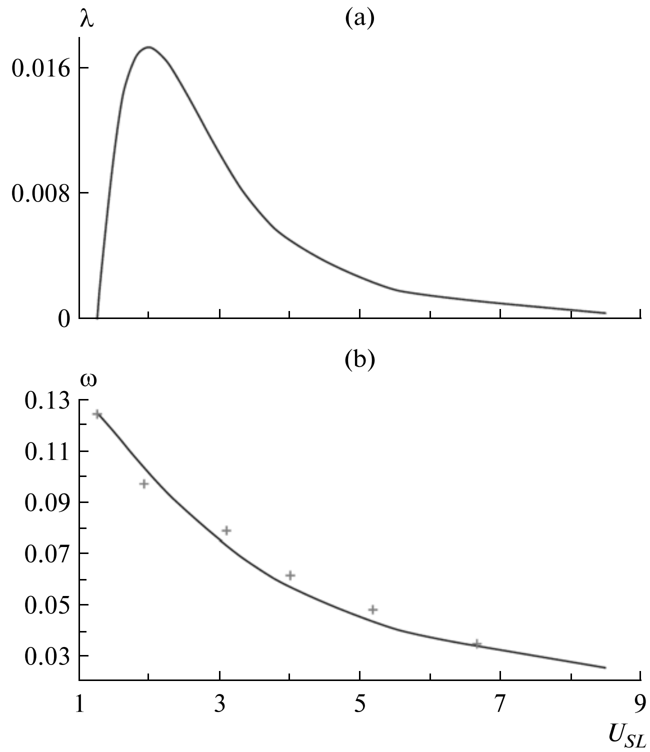

Fig. 2. Dependences of (a) the increment of growth and (b) the oscillation frequency of a perturbation on the applied voltage. The dots in (b) show the frequency of oscillations for a current flowing through a superlattice, calculated numerically.

charge domains. At the moment when the steady state becomes unstable, a perturbation characterized by positive increment of growth λ appears in the system. Consequently, condition $\lambda = 0$ for this perturbation is fulfilled directly upon bifurcation. In Eq. (8), $\mathrm{Re}(\sigma) = 0$, and its solution at a given U_{SL} is a set of perturbations characterized by different frequencies $\omega = \mathrm{Im}(\sigma)$. Introduced boundary condition (9) allows us to choose the perturbations from this set that occur in the superlattice at a given value of governing parameter U_{SL}. Figure 1 shows the values of integral (9) for perturbations with a zero increment of growth and different frequencies ω. The solid line represents a case in which the voltage on the superlattice corresponds to critical $U^*_{SL} = 1.25$. The dashed and dashed-and-dotted lines describe cases in which the voltage is lower ($U_{SL} = 1.14$) and higher than ($U_{SL} = 1.55$) the threshold value, respectively. It can be seen that at the onset of instability in the system, there is a perturbation for which condition (9) holds. The frequency of perturbation oscillation $\omega = 0.0125$.

The results from numerical simulations of Eqs. (1) and (2) show that current oscillations are generated in the superlattice at voltage $U^*_{SL} \sim 1.25$, which agrees with the results obtained by means of analytical theory. It should be noted that the frequency of the oscillations of current flowing through the superlattice is equal to that of the considered perturbation.

The above analysis enables us to find the value of the increment of growth and the frequency of oscillation for the perturbation at any given value of the applied voltage. Similar dependences found for the above perturbation are shown in Fig. 2. It can be seen in Fig. 2a that the perturbation's increment of growth reaches its maximum as the voltage on the superlattice rises, and then falls to zero at $U^{**}_{SL} \sim 9.00$. The results from numerical simulations in this case indicate the termination of generation in the system. Figure 2b shows the dependences of the perturbation's frequency of oscillation (solid line) and the frequency of the current's generated oscillations (dots) on the applied voltage. It can be seen that the frequency of the generated oscillations remains close to the perturbation's frequency of oscillation over a wide range of U_{SL} values. The proposed approach thus allows us to find the voltage necessary for the development of generation by considering the dynamics of perturbations in the reference state, and to estimate the frequency of current oscillations in a semiconductor superlattice.

CONCLUSIONS

The stability of the steady state of a strongly coupled semiconductor superlattice described using a semiclassical approach was considered. By introducing a small perturbation of the reference state, it was shown that the development of instability is associated with the appearance of a perturbation characterized by a positive increment of growth. The voltage at which such perturbations (and thus generation) appear in the system was found. It was established that the frequency of current oscillations was equal to the perturbation's frequency of oscillation. The approach proposed in this work was applied to study the dependence of the frequency of oscillation and the increment of growth of the obtained perturbation when the applied voltage exceeds the critical value. It was found that the increment of growth reaches its maximum when the voltage rises and then falls to zero. Numerical integration of the system's dynamics at a certain value of the governing parameter showed that generation in the system then terminates. The results from numerical integration also indicate that the frequency of oscillations in a current flowing through the superlattice remains close to the perturbation's frequency of oscillation over the zone of generation.

ACKNOWLEDGMENTS

This work was supported by the RF Ministry of Education and Sciences, contracts nos. 14.b37.21.0059 and 14.b37.21.1207; by the Russian Foundation for Basic Research, project no. 12-02-33071; and by the RF Presidential Program for the Support of Young Candidates of Science, Doctors of Science, and Leading Scientific Schools, grants nos. MK-672.2012.2, MD-345.2013.2, and NSh-1430.2012.2.

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Translated by L. Mosina