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Intermittency near the Phase Boundary of Chaotic Synchronization in Spatially Extended Systems

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Abstract—The intermittent behavior of spatially extended systems is investigated using the example of unidirectionally coupled Pierce diodes. It is shown that the same type of intermittency as in finite-scaled systems is characteristic of this system near the boundary of the chaotic phase synchronization regime, i.e., needle-eye type intermittency, which is in fact also equivalent to type I intermittency with noise in the supercritical region.

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INTRODUCTION

The fundamental phenomenon of intermittency is of great interest to researchers since it is observed in variety of systems (e.g., physical, biological, chemical, and social). There are several types of intermittency: type I–III intermittency [1], on-off intermittency [2], needle-eye intermittency [3], and ring intermittency [4]. Each type is characterized by two regimes that alternate with one another being presented in a temporal series (at fixed values of the controlling parameters). At the same time, each type of intermittency has its own features and characteristics.

One of the most interesting problems in studying intermittency in a system of coupled oscillators is transitioning from a set of asynchronous dynamics to one that is synchronous through intermittency. Two types of the intermittent behavior of chaotic systems that is observed upon the termination of the phase synchronization regime when the eigenfrequencies of interacting oscillators differ slightly, are currently known: type I intermittency and needle-eye intermittency [3]. If we reduce the coupling parameter, the regimes of needle-eye and type I intermittency follow after termination of the synchronous regime. Using the example of a system with few degrees of freedom, the authors of [5, 6] showed that needle-eye intermittency is equivalent to type I intermittency with noise in the supercritical region of the controlling parameter. It is of interest to see if this regularity is observed in more complex, spatially extended systems. This work is devoted to answering this question.

DETERMINING TYPES OF INTERMITTENCY IN UNIDIRECTIONALLY COUPLED PIERCE DIODES

We selected two unidirectionally coupled Pierce diodes as our model system. A Pierce diode [7] consists of two infinite planar parallel grids, penetrated by an infinitely wide electron beam. The space between the grids is filled with a neutralizing background of immobile ions with density equal to the unperturbed charge density in the electron flow. Using the hydrodynamic approximation, the system under study is described by a set of equations of motion and continuity, and by the Poisson equation

$$\begin{aligned} \frac{\partial v_{1,2}}{\partial t} &= -v_{1,2}\frac{\partial v_{1,2}}{\partial x} - \frac{\partial \varphi_{1,2}}{\partial x}, \\ \frac{\partial \rho_{1,2}}{\partial t} &= -\frac{\partial(\rho_{1,2}v_{1,2})}{\partial x}, \\ \frac{\partial^2 \varphi_{1,2}}{\partial x^2} &= -\alpha_{1,2}^2(\rho_{1,2}-1), \end{aligned} \tag{1}$$

with boundary conditions

$$v_{1,2}(0,t) = 1, \quad \rho_{1,2}(0,t) = 1, \quad \varphi_{1,2}(0,t) = 0, \tag{2}$$

where φ is the dimensionless potential of the space-charge field; v is the dimensionless flow density; x is the dimensionless coordinate; t is the dimensionless time; and α is the Pierce parameter, which is the controlling parameter for each system: $\alpha_1 = 2.858\pi$ and $\alpha_2 = 2.862\pi$]. Indices 1 and 2 denote the leading and driven systems, respectively.

Unidirectional coupling between the systems is accomplished using the variation in the dimensionless potential at the right boundary of the driven system, while the potential at the right boundary of the leading system remains invariable:

$$\begin{cases} \varphi_1(1,t) = 0, \\ \varphi_2(1,t) = \varepsilon(\rho_1(1,t) - \rho_2(1,t)). \end{cases} \tag{3}$$

Here, ε is the coupling parameter and $\rho_{1,2}(1,t)$ are the oscillations of dimensionless density of the space charge detected at the output of each system.

In this system, with thus specified parameters, the boundary of the phase chaotic synchronization corresponds to coupling parameter $\varepsilon_{PS} \approx 0.012$.

To compare regimes of type I intermittency with noise and needle-eye intermittencies, the authors of [5, 6] analyzed the duration distributions of laminar phases and the dependence of the average duration of laminar phases on the supercriticality parameter. To construct such distributions, we must isolate the laminar and turbulent phases from their temporal implementation. To accomplish this, the authors used the method proposed in [8]: the phase difference between interacting systems $\Delta\varphi(t)$ is considered to be the magnitude of study, and phase $\varphi(t)$ for each system is introduced as the angle of revolution on the phase plane:

$$\tan\varphi = \frac{y}{x}. \tag{4}$$

These phases are reduced to the range 2π wide. The regions of synchronous (laminar segments) and asynchronous (turbulent segments) dynamics then differ qualitatively, allowing us to separate the entire analyzed temporal implementation into laminar and turbulent phases easily.

Using this approach to analyze our model system, however, a number of difficulties arise due to the more complex dynamics caused by the spatial state of the analyzed system's extention. Consequently, this method cannot be used for such systems with an infinite space. We therefore used a modified method in which we analyze the sliding average of the phase difference instead of the phase difference between interacting systems in this study, in order to separate the laminar and turbulent phases:

$$\Delta\varphi_{\text{av}} = \frac{1}{N}\sum_{i=1}^{N}\Delta\varphi_i, \tag{5}$$

where $\Delta\varphi_i$ is the phase difference in the corresponding instant of counting and N is the number of phase difference values by which averaging is performed. This modified approach makes it possible to separate all laminar and turbulent phases even in temporal implementations of systems with complex dynamics, e.g., spatially extended systems.

Using this method, we analyzed the dynamics of our selected model system: unidirectionally coupled Pierce diodes. We found the duration distributions of laminar phases for various values of the coupling parameter. It is known [9] that such distributions in the needle-eye intermittency regime (and thus type I intermittency with noise in the supercritical region) obey the exponential law

$$N(\tau) = T^{-1}\exp\left(\frac{-\tau}{T}\right), \tag{6}$$

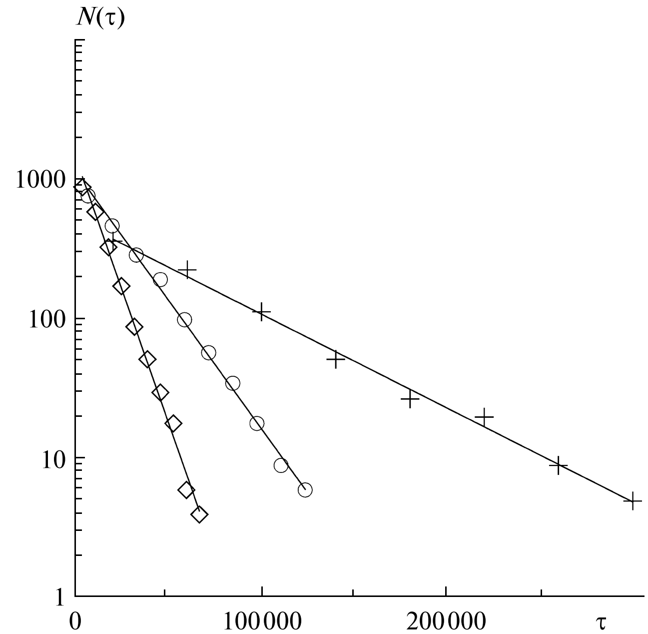

Fig. 1. Time duration distributions of laminar phases of unidirectionally coupled Pierce diodes. Points corresponding to coupling parameter $\varepsilon = 0.0075$ are shown by diamonds; those corresponding to $\varepsilon = 0.008$, by circles; and those corresponding to $\varepsilon = 0.0085$, by crosses.

where T is the average duration of laminar phases, determined for needle-eye intermittency using the expression [3, 10]

$$-\ln\left(\frac{1}{T}\right) = c_0 - c_1\left|\varepsilon_{PS} - \varepsilon\right|^{-1/2}, \tag{7}$$

(c_0 and c_1 are constants), and for type I intermittency with noise in the supercritical region of the controlling parameter with the expression [11]

$$T = \frac{1}{k\sqrt{\varepsilon - \varepsilon_c}}\exp\left(\frac{4(\varepsilon - \varepsilon_c)^{3/2}}{3D}\right), \tag{8}$$

where k is a constant, D is noise intensity, and ε_c is the critical controlling parameter.

It is of interest to see whether these regularities of the intermittent behavior of finite-dimensional systems are observed in spatially extended systems. To answer this question, let us compare the numerical results for coupled Pierce diodes and theoretical dependences (6)–(8). Figure 1 shows the duration distributions of laminar phases and the corresponding theoretical dependences. It can seen that the calculated distributions agree well with the exponential law predicted theoretically.

If the regimes of needle-eye intermittency and type I intermittency with noise are a combined type of behavior, the dependence of the average duration of laminar phases on the supercriticality parameter should satisfy both Eq. (7) and Eq. (8). Figures 2 and 3 show their numerically calculated dependences and the corresponding theoretical curves. It can be seen that the intermittent behavior of two Pierce diodes can be interpreted as both nee-

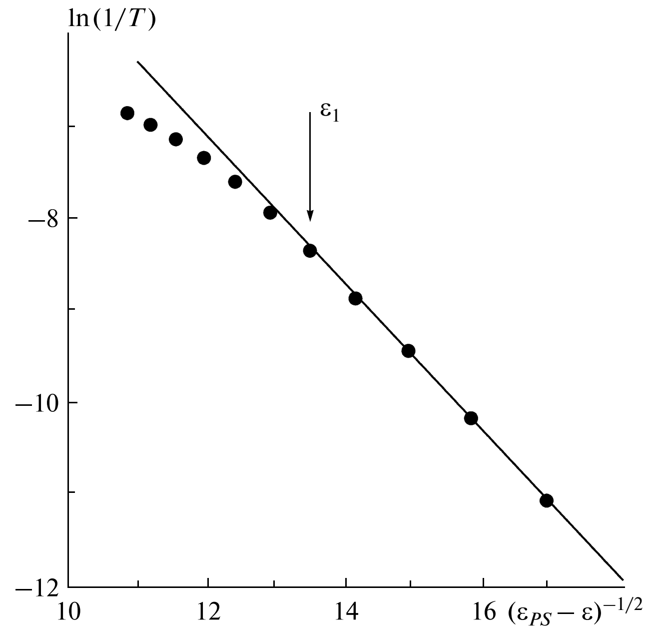

Fig. 2. Dependence of the average duration of laminar phases on the supercriticality parameter. Dots show the numerical results and the solid line shows the theoretical dependence for the needle-eye intermittency regime. The arrow shows value $\varepsilon_1 = 0.0065$ for the emergence of the needle-eye intermittency regime.

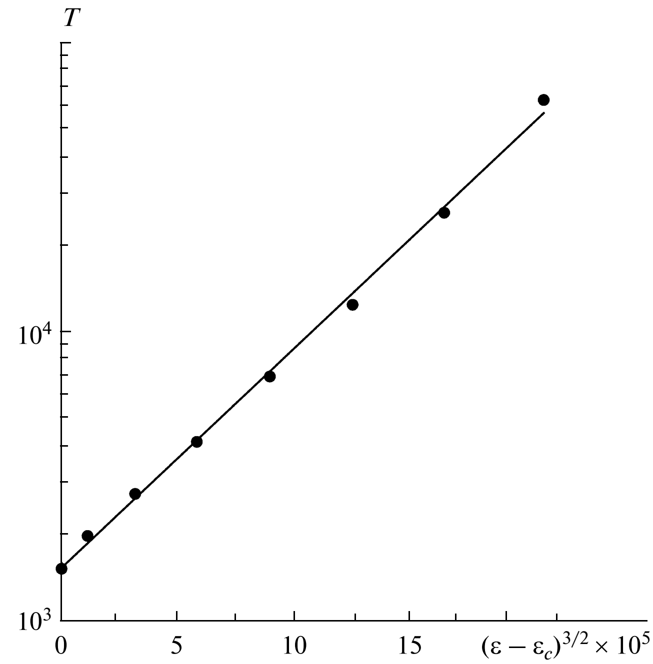

Fig. 3. Dependence of the average duration of laminar phases on the supercriticality parameter. The dots show the numerical results, and the solid line shows the theoretical dependence for the type I intermittency regime with noise.

dle-eye intermittency and type I intermittency with noise, and excellent correspondence between the numerical data and theoretical dependences is observed in both cases. This confirms two things: First, the same type of intermittent behavior as in systems with few degrees of freedom occurs in spatially extended systems near the boundary of the phase chaotic synchronization. Second, our results could be additional proof that needle-eye intermittency and type I intermittency with noise are the same type of dynamics of nonlinear systems.

CONCLUSIONS

The dynamics of spatially extended systems in the region of intermittent behavior was investigated using the example of unidirectionally coupled Pierce diodes. It was shown that the same type of intermittency as in certain finite-dimensional systems, i.e., needle-eye intermittency and type I intermittency with noise, is observed in our investigated system. Our results allow us to conclude that the same regularities of intermittent behavior are characteristic of both spatially extended systems and finite-dimensional systems. We also found additional confirmation that the needle-eye intermittency regime is identical to the type I intermittency regime with noise.

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REFERENCES

1. Dubois, M., Rubio, M., and Berge, P., *Phys. Rev. Lett.*, 1983, vol. 51, p. 1446.
2. Platt, N., et al., *Phys. Rev. Lett.*, 1993, vol. 70, p. 279.
3. Pikovsky, A.S., et al., *Phys. Rev. Lett.*, 1997, vol. 79, p. 47.
4. Hramov, A.E., et al., *Phys. Rev. Lett.,* 2006, vol. 97, p. 114101.
5. Hramov, A.E., et al., *Phys. Lett. A*, 2011, vol. 375, p. 1646.
6. Koronovskii, A.A., Kurovskaya, M.K., Moskalenko, O.I., and Khramov, A.E., *Izv. Vyssh. Uchebn. Zaved. Prikl. Nelin. Dinam.*, 2010, no. 1, p. 24.
7. Trubetskov, D.I. and Khramov, A.E., in *Lektsii po SVCh-elektronike dlya fizikov* (Lectures on Microwave Electronics for Physicists), Moscow: Fizmatlit, 2003, vols. 1, 2.
8. Zhuravlev, M.O., Kurovskaya, M.K., and Moskalenko, O.I., *Tech. Phys. Lett.,* 2010, vol. 36, no. 5, p. 457.
9. Kurovskaya, M.K., *Tech. Phys. Lett.*, 2008, vol. 34, no. 12, p. 1063.
10. Lee, K.J., Kwak, Y., and Lim, T.K., *Phys. Rev. Lett.*, 1998, vol. 81, p. 321.
11. Pomeau, Y. and Manneville, P., *Comm. Math. Phys.*, 1980, vol. 74, p. 189.

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