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ELECTRONIC PROPERTIES OF SOLID

The Effect of Temperature on the Nonlinear Dynamics of Charge in a Semiconductor Superlattice in the Presence of a Magnetic Field

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Abstract—The space–time dynamics of electron domains in a semiconductor superlattice is studied in a tilted magnetic field with regard to the effect of temperature. It is shown that an increase in temperature substantially changes the space–time dynamics of the system. This leads to a decrease in the frequency and amplitude of oscillations of a current flowing through the semiconductor superlattice. The quenching of oscillations is observed, which is attributed to the change in the drift velocity as a function of electric-field strength under the variation of temperature.

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Semiconductor superlattices represent complex nanostructures that contain several thin (about 10-nm-thick) alternating layers of different semiconductor materials. Proposed by Esaki and Tsu in 1969 [1] for the experimental investigation of various quantum-mechanical phenomena related to resonant tunneling and Bloch oscillations, superlattices provide a unique test bed both for studying and understanding processes in solid state physics [2, 3] and for investigating nonlinear dynamic phenomena [4–7]. Superfast Bloch oscillations, as well as associated nonlinear processes [4, 8], make a superlattice a promising element for the generation, amplification, and detection of high-frequency (terahertz) signals.

Recently, a lot of attention has been devoted to the study of the collective dynamics of electrons in semiconductor superlattices in a tilted magnetic field [6, 9]. Under these conditions, the electrons can perform chaotic oscillations that arise due to a resonance between cyclotron and Bloch oscillations. The electron trajectories, which are strongly localized in space in the absence of resonances, become unbounded after the onset of resonance [6, 9]. Thus, a small variation in the field configuration can significantly change the drift velocity of electrons and, hence, the electronic conductivity of a superlattice [6, 7, 9, 10]. Semiconductor superlattices can be exploited as high-frequency devices by controlling the collective dynamics of electrons. Current oscillations are generated by large groups (or domains) of electrons, rather than by individual electrons. The propagation of domains along a superlattice gives rise to current oscillations with frequencies of 10–100 GHz [11, 12].

Usually, theoretical studies devoted to the generation of domains in a semiconductor superlattice in longitudinal electric and tilted magnetic fields focus on the case when the temperature T is close to zero, ($T \approx 0$). However, a variation of temperature may lead to a significant change in the behavior of the system. In particular, it has been shown recently that an increase in the temperature of a superlattice in a tilted magnetic field gives rise to local maxima in the function of the electron drift velocity versus the longitudinal electric field strength. These maxima correspond to different resonances between the Bloch and cyclotron oscillations; as temperature increases, these maxima become more ($T \approx 0$) pronounced [13, 14]. In this paper, we consider the effect of temperature on the collective space–time dynamics of charge in a superlattice. Also, the effect of diffusive processes, which are usually neglected in simulation, but whose contribution obviously increases with temperature, will be investigated.

To describe the collective dynamics of charge in a semiconductor superlattice, we solve self-consistently the Poisson and charge continuity equations numerically. We discretize the superlattice into N layers of width Δx in order to approximate a continuum [14]. We assume that the electron concentration n_m is constant within each mth layer.

The evolution of charge density in layer m is described by the continuity equation

$$e\Delta x\frac{dn_m}{dt} = J_{m-1} - J_m, \quad m = 1, \ldots, N, \tag{1}$$

where $e > 0$ is the electron charge and J_{m-1} and J_m are the current densities through the left and right boundaries of the mth layer, respectively. Within the drift approximation with regard to diffusion, we can define the current density J_m as

$$J_m = en_m v_d(\bar{F}_m) + D(\bar{F}_m)\frac{n_{m+1} - n_m}{\Delta x}, \tag{2}$$

where $v_d(\bar{F}_m)$ is the electron drift velocity for the mean value $\bar{F}_m$ of the electric field strength in layer m [6], and the diffusion coefficient is given by [4]

$$D(\bar{F}_m) = \frac{v_d(\bar{F}_m)d\exp(-e\bar{F}_m d/kT)}{1 - \exp(-e\bar{F}_m d/kT)}, \tag{3}$$

where k is the Boltzmann constant, and $d = 8.3$ nm is the period of the superlattice.

The following discrete representation of the Poisson equation is valid for each layer m:

$$F_{m+1} = \frac{e\Delta x}{\varepsilon_0\varepsilon_r}(n_m - n_D) + F_m, \quad m = 1, \ldots, N, \tag{4}$$

where $n_D = 3 \times 10^{22}$ m^{-3} is the equilibrium electron concentration determined by the doping level and ε_0 and $\varepsilon_r = 12.5$ are the electric constant and the relative permittivity of the material, respectively.

Assuming that the contacts on the emitter and collector of the superlattice are ohmic and that the current density J_0 through the emitter is determined by the conductance $\sigma = 3788\ \Omega^{-1}$ of the contact, $J_0 = \sigma F_0$, the electric field strength F_0 can be determined from the Kirchhoff equation

$$V = U + \frac{\Delta x}{2}\sum_{m=1}^{N}(F_m + F_{m+1}), \tag{5}$$

where V is the voltage applied to the superlattice and U is the voltage drop across the contacts. Taking into account the fact that layers with increased charge concentration are formed near the emitter, and layers with reduced charge concentration are formed near the collector, we define the voltage drop U by the relation [6]

$$\begin{aligned} U = {} & F_0(\Delta x_l - \Delta x_s) + F_0(\Delta x_l - \Delta x_q) \\ & + F_1\Delta x_s + F_{N+1}\Delta x_q + F_{N+1}\Delta x_q \\ & - \frac{en_0(\Delta x_q)^2}{2\varepsilon_0\varepsilon_r} + \sigma F_0 S R_c. \end{aligned} \tag{6}$$

Here $\Delta x_l = 50$ nm is the length of the contacts, Δx_s and Δx_q are the lengths of the regions with increased and reduced electron concentrations near the contacts, $n_0 = 3 \times 10^{23}$ m^{-3} is the electron concentration in the contact layer, $S = 5 \times 10^{-10}$ m^2 is the contact area, and

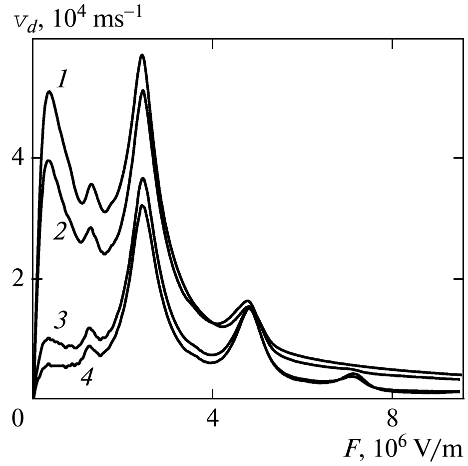

Fig. 1. Drift velocity of an electron as a function of electric field strength for various temperatures and a magnetic field of $B = 15$ T with a tilt angle of $\theta = 40°$. Curve *1* corresponds to the temperature of $T = 0$ K, curve *2*, to $T = 50$ K, curve *3*, to $T = 200$ K, and curve *4*, to $T = 300$ K.

$R_c = 17\ \Omega$ is the circuit resistance. Knowing the current density in each layer, we can calculate the current through the superlattice:

$$I(t) = \frac{S}{N+1}\sum_{m=0}^{N} J_m, \tag{7}$$

which corresponds to the current that can be measured experimentally.

An important role in the model described above is played by the dependence of the drift velocity v_d on the electric field strength $\mathbf{F}$. It is this function that contains information on the spatial structure (period d) and energy characteristics of a semiconductor nanostructure, on the external magnetic field $\mathbf{B}$, and temperature T. Although parameters such as the miniband width Δ (in our case, $\Delta = 19.1$ meV), magnetic displacement vector $\mathbf{B}$, and temperature T do not explicitly appear in the model equations (1)–(7) describing the dynamics of charge domains, they have a significant effect on drift velocity behavior as a function of the electric field strength, $v_d(F)$, and, consequently, on the collective dynamics in the semiconductor superlattice. When modeling the behavior of a semiconductor superlattice, we used the functions of the drift velocity versus the electric field strength, $v_d(F)$ (Fig. 1), measured at various temperatures in a magnetic field of $B = 15$ T with a tilt angle of $\theta = 40°$; the method of calculation of these functions is described in detail in [13].

All numerical values of the control parameters were chosen to correspond to the experimental samples [6]. The spatial step size during modeling was chosen so that to guarantee the convergence of the numerical scheme: the layer width was $\Delta x = 0.24$ nm and the number of layers was $N = 480$. It was shown in [6, 15]

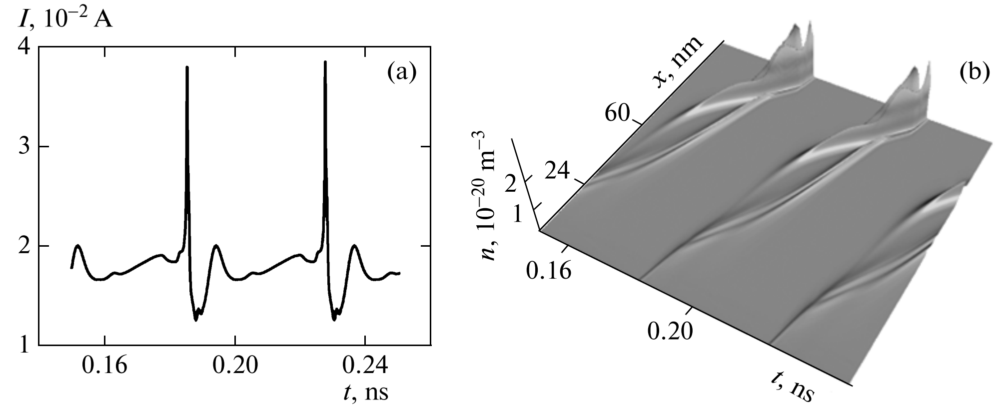

Fig. 2. (a) Current $I(t)$ through a superlattice for $T = 0$ and $V = 0.61$ V. (b) Space–time diagram characterizing the evolution of charge domains as a function of time and coordinate.

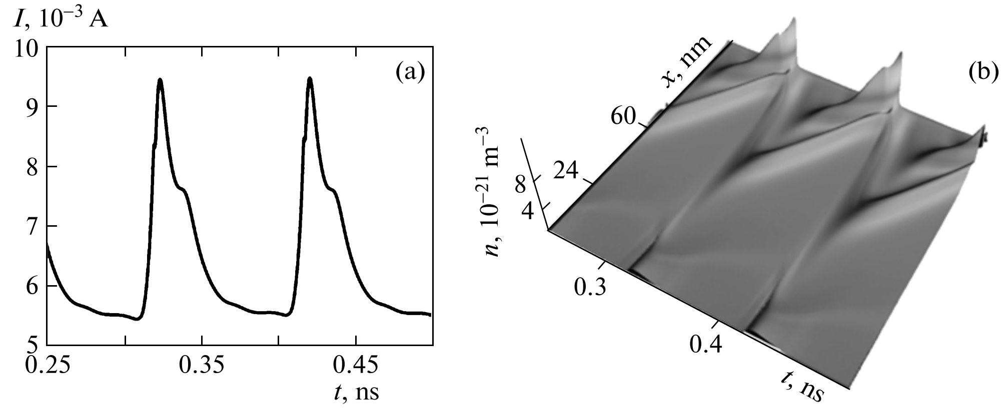

Fig. 3. Same as in Fig. 2 for $T = 300$ K.

that the model (1)–(7) with such discretization parameters describes a number of experimental results, in particular, the current–voltage characteristics of superlattices, to a good degree of accuracy.

Figure 2 shows the results of numerical simulation for the behavior of a superlattice at zero temperature when $V = 0.61$ V corresponds to a region of current oscillations. The current oscillations shown in Fig. 2a correspond to the charge concentration dynamics illustrated in Fig. 2b. The space–time diagram (Fig. 2b) demonstrates the complex dynamics of charge in the superlattice, where the emergence, vanishing, and interaction of domains with high concentration of charge carriers are visible. As descried in [15], the emergence and evolution of these domains is attributed to the maxima of $v_d(F)$ associated with the outset of Bloch oscillations in the miniband dynamics of electrons (the leftmost maxima in Fig. 1), and with the Bloch–cyclotron resonances. The presence of several peaks in the graph of $v_d(F)$ (see Fig. 1) generates several charge domains with the domain associated with the emergence of Bloch oscillations appearing first. Figure 1 shows that the increase in temperature suppresses the peaks of the drift velocity. The peak of the drift velocity associated with the emergence of Bloch oscillations rapidly decreases as temperature increases. The peaks associated with the Bloch–cyclotron resonances vary much more slowly. This variation in the form of the drift velocity inevitably manifests itself in the collective dynamics of charge.

Generally, we find that an increase in temperature leads to the decrease in the frequency and the amplitude of current. This is clearly revealed in Fig. 3a, which represents developed oscillations of current at room temperature $T = 300$ K under the same voltage ($V = 0.61$ V) applied to the semiconductor superlattice as that Fig. 2. Figure 3b shows that the charge concentration in the corresponding domains is suppressed while their period increases. One can also see that, due to diffusion, the increase in temperature leads to the smearing of charge domains, which become more delocalized at high temperature (cf. Figs. 2b and 3b).

Not only the dynamic regimes, but also the processes that occur when setting up oscillations, are of indubitable interest. We have found that, at low temperatures, an increase in temperature shifts the threshold V_c of oscillation to lower values. In a certain range of temperatures (from about 175 to 225 K), we can observe an interesting phenomenon (which is missing in the case of $B = 0$) that consists of a voltage interval $V \in [V_1, V_2]$, $V_1 > V_c$, in which oscillation is quenched (Fig. 4). In other words, at these temperatures, for $V_c < V < V_1$, one can observe an oscillation regime accompanied by the formation of charge domains (Fig. 4a); however, upon a further increase in the voltage ($V_1 < V < V_2$), the oscillations are damped (Fig. 4b), but re-appear when $V > V_2$ (Fig. 4c). The dynamical regimes observed in the semiconductor superlattice are different ($V_c < V < V_1$ compared to $V_1 < V < V_2$) to the left and right of the region of absence of oscillation. This fact is displayed by the surface of charge carrier concentration $n(x, t)$ and the shape of current oscillations $I(t)$ in the superlattice.

By increasing T, we find that the oscillation region $[V_c, V_1]$ decreases and vanishes when $T_c \approx 250$ K (for the given parameters of the semiconductor superlattice). Accordingly, when $T > T_c$, the oscillation threshold coincides with V_2, which, in turn, is comparable with the oscillation threshold at zero temperature.

The emergence of oscillation quenching in the case of a tilted magnetic field **B** can be attributed to variation of the form of $v_d(F)$ as temperature increases. In [15], it was shown that the formation and interaction of domains are largely determined by the form of $v_d(F)$; in fact, the generation of oscillations is attributed to the nonmonotonic character of this function. In turn, the form of $v_d(F)$ is largely determined by the magnetic field [15] and temperature [13, 14]. For $B = 15$ T with a tilt angle of $\theta = 40°$, the function $v_d(F)$ has several maxima (the Esaki–Tsu maxima associated with Bloch–cyclotron resonances) [13, 14]. For low temperatures, the dominant role is played by the Esaki–Tsu maxima and the maximum corresponding to the first-order Bloch–cyclotron resonance. When $T \approx 300$ K, the Esaki–Tsu maximum almost vanishes, while the height of the maximum corresponding to the first-order resonance drops by a factor of two. In addition, higher order resonances appear. For voltages $V \in [V_c, V_1]$, the main role is played by the Esaki–Tsu maximum and the first-order resonance maximum. When $V > V_2$, oscillations are also generated by the maxima corresponding to higher order Bloch–cyclotron resonances. As temperature increases, the heights of the Esaki–Tsu and first-order resonance peaks drop, and, accordingly, the oscillation band corresponding to these peaks decreases until the oscillation band $[V_c, V_1]$ disappears completely.

We have shown that an increase in temperature has a significant effect on the space–time dynamics of charge in a semiconductor superlattice in a tilted magnetic field. In particular, an increase in temperature

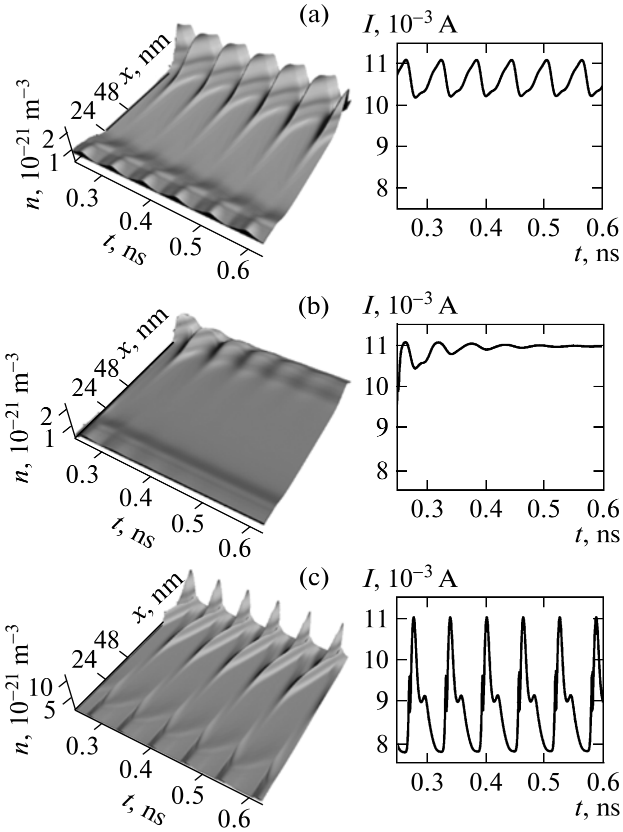

Fig. 4. Space–time diagrams characterizing the evolution of charge domains as a function of time and coordinate, and current $I(t)$ through the superlattice at temperature of $T = 200$ K and voltage of (a) $V = 0.54$ V, (b) $V = 0.55$ V, and (c) $V = 0.56$ V.

leads to a decrease in the peak values of the electron drift velocity, which, combined with diffusive processes, leads to a decrease in the mobility and concentration of charge carriers in the lattice. This results in a decrease in the frequency and amplitude of current oscillations. In addition, a variation in the form of the $v_d(F)$ curve with increasing temperature may lead to the nontrivial phenomenon of oscillation quenching, observed for a certain range of temperatures.

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