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Studying the Behavior of a Nonautonomous Van der Pol Oscillator in Different Time Scales with the Presence of Noise near the Synchronization Boundary

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Abstract—A new level of organization of the temporal behavior of two coupled complex systems is revealed. We report for the first time the coexistence of two types of intermittent behavior that occurs simultaneously near the boundary of the synchronization regime of coupled chaotic oscillators. This intricate phenomenon was observed both experimentally in a physiological experiment and numerically. The laws for both the distribution and the mean length of the laminar phases versus the control parameter values are analytically deduced. Very good agreement between the theoretical results and the numerically calculated data is shown.

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INTRODUCTION

Intermittency is typical of many nonlinear systems and is particularly observed in transitioning from periodic to chaotic oscillations [1] and near the boundaries of different regimes of the chaotic synchronization of coupled oscillators [2–5].

Intermittent behavior can be classified. Intermittency of types I–III [1–6], on–off intermittency [7], needle hole intermittency [8], and ring intermittency [9] can be distinguished in particular. Despite some similarities (the presence of two different modes alternating with each other in time series), each type of intermittency has its own peculiarities and characteristics (especially the dependency of laminar phases on control parameters and the distribution of the durations of laminar phases). Mechanisms leading to the emergence of each type of intermittent behavior also differ.

In addition to the types of intermittency listed above, more complicated behaviors of a system displaying two types of intermittency at the same time can take place. Such behavior is referred to as the intermittency of intermittencies [10]. This work is devoted to studying just this type of behavior, which can exist in a system of unidirectionally coupled chaotic oscillators or in an oscillator under external impact in the state preceding synchronization. At the same time, the investigated system can be considered in different time scales introduced by means of continuous wavelet transformation [11, 12], including time scales differing from the main scale. Experimental results [10] confirm that two different types of intermittency (needle hole and ring) do coexist in a certain range of time scales for a system of unidirectionally coupled oscillators (Ressler systems were considered).

It should be noted that the intermittency of intermittencies is a poorly studied type of behavior, and therefore is of great interest from a fundamental point of view, since these studies provide a deeper understanding of the nature and mechanisms of such fundamental phenomena as intermittency and chaotic synchronization.

We may assume that the intermittency of intermittencies also can exist in a nonautonomous periodic oscillator under an external impact in the presence of noise. This work presents numerical simulation results for such a system relative to the theoretical dependences, and good agreement is achieved. Research data allow more detailed comprehension of the mechanisms leading to the emergence of intermittency of intermittencies.

METHOD

Let us consider a nonautonomous Van der Pol generator that is influenced by an arbitrary impact $D\xi(t)$, where $\xi(t)$ is delta-correlated white noise [$\langle\xi(t)\rangle = 0$, $\langle\xi(t)\xi(\tau)\rangle = \delta(t-\tau)$]. The dynamics of the system is described by the equation

$$\ddot{x} - \left(\lambda - x^2\right)\dot{x} + x = A\sin(\omega_e t) + D\xi(t), \qquad (1)$$

where A is the amplitude of the external harmonic impact, and ω_e is its frequency. The values of the control parameters were set at $\lambda = 0.1$ and $\omega_e = 0.98$. At such parameter values and a zero noise level ($D = 0$), the dynamics of a nonautonomous Van der Pol generator becomes synchronous when $A = A_c = 0.0238$, which corresponds to the saddle-main bifurcation of the plane of complex amplitudes [13]. It should be noted that research has been conducted on amplitude values $A > A_c$ and $D = 1$, at which the system displays type I intermittency with noise in the supercritical domain of parameter values.

Our consideration of the behavior of a nonautonomous oscillator under external impact in different time scales [11, 12] is based on the introduction of continuous set of phases of the investigated signals by means of continuous wavelet transformation,

$$W(s, t_0) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} x(t) \psi^*\left(\frac{t - t_0}{s}\right) dt \qquad (2)$$

with mother Morlet wavelet

$$\psi(n) = \frac{1}{\sqrt[4]{\pi}} \exp(j\Omega_0 \eta) \exp\left(\frac{-\eta^2}{2}\right), \qquad (3)$$

where $\Omega_0 = 2\pi$.

Wavelet surface

$$W(s, t_0) = |W(s, t_0)| e^{j\varphi_s(t_0)} \qquad (4)$$

characterizes the behavior of the system in each time scale s at each moment in time t_0. Quantity $|W(s, t_0)|$ characterizes the presence and intensity of corresponding time scale s at the moment in time t_0. In addition, the instantaneous value

$$E(s, t_0) = |W(s, t_0)|^2 \qquad (5)$$

and integral distribution of energy by time scales

$$\langle E(s) \rangle = \int |W(s, t_0)|^2 dt_0 \qquad (6)$$

are introduced.

Continuous phase for each time scale s can be defined via expression $\varphi(s, t) = \arg W(s, t)$ when using wavelet transformation (2). In other words, each time scale s can be characterized using associated phase $\varphi(s, t)$, which is a continuous function of time scale s and time t. Such an assembly of phases completely characterizes the behavior of a nonautonomous oscillator under external impact: the behavior of each time scale can be described using its associated phase $\varphi(s, t)$.

Let us consider time realization $x(t)$ of system (1) and external impact $A\sin(\omega_e t)$. If the interval of time scales $s_l \le s \le s_h$ satisfying phase capture condition

$$|\varphi_x(t) - \varphi_A(t)| < \text{const} \qquad (7)$$

and nonzero energy condition (where part of energy of wavelet spectrum per the given interval of time scales turns out to differ from zero)

$$E_{snhr} = \int_{s_l}^{s_h} \langle E(t) \rangle ds > 0, \qquad (8)$$

can be found for this system, then such a mode is referred to as the synchronization of time scales.

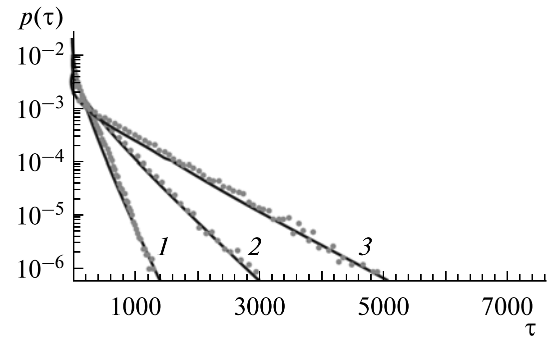

Fig. 1. Distribution of the durations of laminar phases for type I intermittency with noise and ring intermittency for nonautonomous Van der Pol generator under external impact, and analytical dependencies (9) corresponding to these distributions (solid lines). Curve (*1*) $A = 0.02308$, $s = 3.50$, $T_i = 56.0$, $T_r = 148.1$; curve (*2*) $A = 0.02497$, $s = 3.57$, $T_i = 120.0$, $T_r = 6002.9$; curve (*3*) $A = 0.02496$, $s = 3.8$, $T_i = 118.5$, $T_r = 2838$.

A synchronous mode is established at an amplitude of external harmonic impact of $A_s \approx 0.028$ for the selected values of the control parameters, at which the synchronous mode of time scales lies in the range $s \in [s_l; s_h]$, $s_l = 3.98$, $s_h = 8.62$.

It should be noted that research has been conducted in the domain of external impact amplitude values $A < A_s$, i.e., type I intermittency with noise can be detected in the system. Time scales s were selected as asynchronous, allowing us to observe intermittency of the ring type.

Let us compare the quantitative characteristics obtained numerically for the system under consideration with theoretical dependences corresponding to the intermittency of intermittencies, e.g., the dependency of the average duration of laminar behavior on the supercriticality parameter and the distribution of the durations of laminar behavior regions at fixed values of the control parameters. The amplitude of external harmonic impact A and time scale s served as our critical parameters in studying this type of behavior.

Figure 1 shows the distribution of the durations of laminar phases obtained numerically for a nonautonomous Van der Pol generator under external impact in coexistence with type I intermittency with noise (in the supercritical domain) and ring intermittency for three different sets of the values of external impact amplitude A and time scale s in which observations were made. Since the mechanisms leading to ring intermittency and type I intermittency with noise differ, we can distinguish the phase jumps related to the types of intermittency and estimate values T_i and T_r (where T_i is the average duration of a laminar behavior region for type I intermittency with noise, and T_r is the average duration of a laminar behavior region for ring intermittency) in the theoretical correlation for the

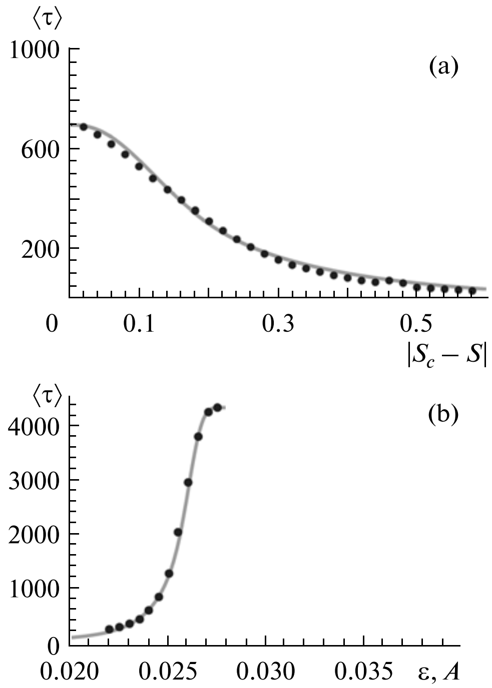

Fig. 2. Dependency of the average duration of laminar behavior regions on the supercriticality parameter. Dots indicate numerical results, while the line shows approximating curve (10). (a) Supercriticality parameter $(s_c - s)$ at an amplitude of external impact is equal to 0.0230; (b) supercriticality parameter is the amplitude of external impact A; time scale was selected as 3.97.

distribution of the duration of laminar phases upon the intermittency of intermittencies obtained in [10]:

$$p(\tau) = \frac{\exp\left(-\frac{\tau}{T_i}\right)}{(T_i + T_r)}\left(1 - \frac{\tau}{T_i}\right)\Gamma\left(0, \frac{\tau}{T_r}\right) + \frac{T_i^2 + T_r^2}{T_i T_r (T_i + T_r)} \times \exp\left(-\frac{\tau}{T_i} - \frac{\tau}{T_r}\right) + \frac{\exp\left(-\frac{\tau}{T_i}\right)}{(T_i + T_r)}\left(1 - \frac{\tau}{T_{ir}}\right)\Gamma\left(0, \frac{\tau}{T_i}\right). \quad (9)$$

It can be seen from Fig. 1 that the obtained numerical distributions for the duration of laminar phases are in very good agreement with theoretical curve (9), indicating there is intermittency of intermittencies in the system.

One more characteristic of intermittent behavior is the dependency of the average duration of laminar behavior ⟨τ⟩ on the supercriticality parameter. Figure 2 shows a comparison of the numerical results with the theoretical curve also known for the intermittency of intermittencies [9] and expressed by the formula

$$\langle\tau\rangle = -\frac{T_i^2 \log\left(\frac{T_i + T_r}{T_i}\right) - 2T_i T_r + T_i^2 \log\left(\frac{T_i + T_r}{T_i}\right)}{T_i + T_r}. \quad (10)$$

It can be seen from Fig. 2 that the numerical results are in good agreement with theoretical curves (10) for when the supercriticality parameter is represented by time scale s (Fig. 2a), and for when the supercriticality parameter is $(A_s - A)$ (Fig. 2b).

CONCLUSIONS

The results presented in this work show that intermittent behavior consisting of two types of intermittency, i.e., the intermittency of intermittencies can be attained for a periodic oscillator under external impact. It was also shown that the intermittency of intermittencies attained in a nonautonomous system with noise obeys the same theoretical laws as chaotic oscillators [10]. We may expect that this type of behavior is typical for a wide range of nonlinear systems displaying the synchronization mode of time scales.

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