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Regularities of Spectral Component Behavior near the Phase Synchronization Boundary in Spatially Extended Systems

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Abstract—In this work we study the dynamics of spatially coupled systems using the example of unidirectionally coupled Pierce diodes near the boundary of phase chaotic synchronization. We show that the dynamics in the investigated region of the coupling parameter obeys a universal regularity valid for different space points of the system.

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INTRODUCTION

The synchronization of chaotic oscillations has recently attracted much attention from researchers. The most common types of chaotic synchronization are traditionally phase synchronization [1], generalized synchronization [2], lag-synchronization [3], and complete chaotic synchronization [4]. The question of whether it is possible to consider all types of synchronization simultaneously is of interest. One possible approach is to consider coupled systems from the perspective of time scale synchronization [5] and spectral components [6], which are closely related to each other. Each of the above types of synchronization is a particular manifestation of time scale synchronization or the synchronization of spectral components. When considering the dynamics of coupled systems from the point of time scale synchronization, the focus is usually on the behavior of the phase difference, introduced by using continuous wavelet transformation for each time frame, while the synchronization of individual frequency components of the Fourier spectrum of the interacting systems is studied when the spectral components are synchronized. In this work, we use the second approach, which is associated with analyzing spectral components of the frequency spectra of interacting systems, and use it to study the dynamics of associated spatially distributed systems that can exhibit chaotic dynamics.

Synchronous behavior in terms of spectral components has been thoroughly studied [6], but the dynamics of systems in the range of parameters prior to the establishing of a synchronous mode has yet to be investigated fully. It is known that intermittent behavior is observed in this field in chaotic systems [7, 8]. In addition, the system dynamics near the boundary of establishing synchronous operation in terms of the synchronization of spectral components were studied earlier using the example of systems with several degrees of freedom, i.e., coupled Ressler oscillators and circle maps [9] (which are our reference model objects); and for systems with an infinite number of degrees of freedom, i.e., spatially extended systems [10], whose dynamics is in some cases fundamentally different from the dynamics of classical finite-dimensional models that exhibit chaotic behavior. In [10], a signal received from one point of interaction was analyzed. In spatially extended systems, however, signals received from different locations can differ from one another, and it is therefore an issue of whether the results obtained in [10] are valid for signals received from other locations of the interaction space of the studied systems. This work is devoted to studying this issue.

STUDYING THE DYNAMICS OF COUPLED PIERCE DIODES NEAR THE BOUNDARY OF PHASE SYNCHRONIZATION

Synchronization of spectral components in the range of parameters prior to the establishing of a synchronous mode for systems with infinite-dimensional phase space was investigated using the example of unidirectionally coupled Pierce diodes. A Pierce diode [11] is two infinite flat parallel grids, permeated by an infinitely wide electron beam. The space between the grids is filled with a neutralizing background of fixed ions with a density equal to the unperturbed charge density in the electron beam.

At certain values of control parameters, we can use the hydrodynamic approximation to describe a system of coupled Pierce diodes in which the electron beam is

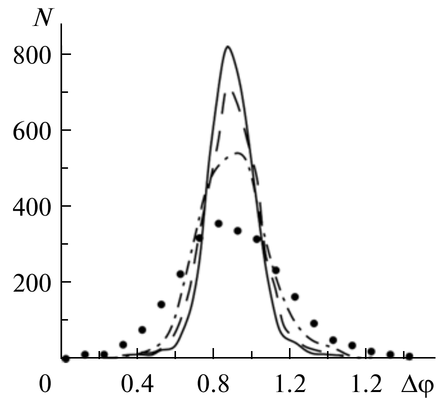

Fig. 1. Distributions of the phase difference of two unidirectionally Pierce diodes at $\varepsilon = 0.01$: $T = 3900$ (solid line); $T = 3000$ (dotted line); $T = 1950$ (dashed-and-dotted line); $T = 750$ (points).

considered as a continuum: a charged liquid, whose velocity at each point in space is a unique function of time [11]. The system under study is described by the equations of motion, continuity, and the Poisson equation

$$\begin{aligned}\frac{\partial \nu_{1,2}}{\partial t} &= -\nu_{1,2}\frac{\partial \nu_{1,2}}{\partial x} - \frac{\partial \varphi_{1,2}}{\partial x},\\ \frac{\partial \rho_{1,2}}{\partial t} &= -\frac{\partial(\rho_{1,2}\nu_{1,2})}{\partial x},\\ \frac{\partial^2 \varphi_{1,2}}{\partial x^2} &= -\alpha_{1,2}^2(\rho_{1,2} - 1)\end{aligned} \quad (1)$$

with boundary conditions

$$\nu_{1,2}(0,t) = 1, \quad \rho_{1,2}(0,t) = 1, \quad \varphi_{1,2}(0,t) = 0, \quad (2)$$

where φ is the dimensionless potential space-charge field, ρ is the dimensionless charge density, ν is the dimensionless beam density, x is a dimensionless coordinate, t is dimensionless time, and α is the Pierce parameter that is the control parameter for each system: $\alpha_1 = 2.858\pi$, $\alpha_2 = 2.862\pi$. Indices 1 and 2 denote master and slave systems, respectively.

Unidirectional communication between systems is affected by changing the value of the dimensionless potential at the right edge of the slave system, while the potential at the right edge of the slave system remains unchanged:

$$\begin{cases}\varphi_1(1,t) = 0,\\ \varphi_2(1,t) = \varepsilon(\rho_2(1,t) - \rho_1(1,t)).\end{cases} \quad (3)$$

Here ε is the coupling parameter, and $\rho_{1,2}(1,t)$ represents the fluctuations recorded at the output of each system for the dimensionless density of the spatial charge.

We study the dynamics of systems at values of the coupling parameter that are close to the boundary of phase synchronization $\varepsilon_{PH} \approx 0.0098$. In terms of the synchronization of spectral components, the establishing of the phase synchronization mode corresponds to synchronization of the main spectral components of the Fourier spectrum for the interacting systems. The frequency corresponding to the main spectral component can be determined by the Fourier transformation

$$S_{1,2}(f) = \int_0^T \rho_{1,2}(t) e^{-2\pi i f t} dt. \quad (4)$$

It should be noted that the integral in expression (4) is formally calculated using an infinite time interval. During a numerical simulation, however, the length of the analyzed time interval T is always limited.

Using (4), we can introduce the phase of a spectral component as the argument of a complex number. For the main spectral component, $\varphi = \arg S(f_m)$, where $f_m = 0.2579$ is the frequency of the master system's main spectral component. A phase introduced in this way is constant in time and depends only on the initial conditions and the length of time series T, which is used to calculate the Fourier transformation. In the synchronous mode, in the absence of chaotic dynamics, the phase difference of interacting systems is the same for any initial conditions; i.e., the distribution of the phase differences has the form of a δ function. Due to the chaotic dynamics and the limited length of T, however, such distributions must take the form of Gauss distributions tending to the δ function with increasing T. It was shown in [10] exactly how the dispersion of such distributions depends on the T value, and how these relations correlate at different values of coupling parameter ε. It was shown that the dependence of the dispersion of the phase difference distribution of the main spectral components of signals from unidirectionally coupled Pierce diodes, received from a fixed point in interaction space $x_1 = 0.2$, on the T value at different coupling parameters of ε is universal. The form of this dependence virtually coincides with the analogous dependences calculated for systems of other classes, e.g., Roessler systems and circle maps [9].

As was noted above, the dynamics of coupled Pierce diodes near the phase synchronization boundary [10] were studied for a single value of spatial coordinate $x_1 = 0.2$. To study the question of whether the results are valid over the interaction space $(0 < x < 1)$ of systems with an infinite number of degrees of freedom, we performed a similar calculation for the other value of longitudinal coordinate $x_2 = 0.75$. Figure 1 shows the distributions of the phase difference between the interacting systems obtained at this point; we can see that they have the form of Gauss distributions whose dispersion diminishes as the length of the investigated time interval T grows. It is interesting how the dispersion

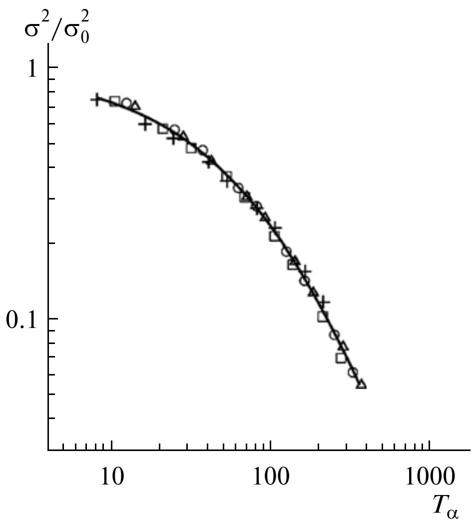

Fig. 2. Dependence of the dispersion of the phase difference distributions of unidirectionally coupled Pierce diodes on the normalized length of the investigated time series T_α for the value of the coupling parameter: $\varepsilon = 0.008$ (crosses), $\varepsilon = 0.01$ (squares), $\varepsilon = 0.012$ (circles), $\varepsilon = 0.014$ (triangles).

of these distributions depends on the length of the investigated time interval T, and how these relations correlate for different values of the coupling parameter ε.

For a correct comparison of the results at different values of ε, by analogy with [12] we introduce the following renormalization

$$T_\alpha = T\sqrt{\alpha}, \tag{4}$$

where $\alpha = \varepsilon - \varepsilon_c$ is the supercriticality parameter, and ε_c is the critical value $\varepsilon_c \approx 0.005$. As in [9], let us normalize dispersion to the value of the dispersion of distributions at $T = 0$. The obtained dependences are shown in Fig. 2. It can be seen that for different values of parameter ε, the curves almost coincide on some universal dependence, and the form of this relation is similar to the curve obtained in [10] for unidirectionally coupled Pierce diodes at another value of the longitudinal coordinate of x.

CONCLUSIONS

We studied the dynamics of coupled spatially extended systems near the boundary of phase chaotic synchronization using the example of coupled Pierce diodes. It was shown that at the investigated point in space, systems exhibit behavior similar to that observed previously for other value of the longitudinal coordinate x of interacting systems. On the basis of these results, we may expect that the resulting patterns are characteristic for signals received from all points of the interaction space of systems with an infinite number of degrees of freedom near the boundary of phase chaotic synchronization.

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