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SHORT COMMUNICATIONS

Intermittent Behavior at the Boundary of Noise-Induced Synchronization

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Abstract—The intermittent behavior at the boundary of noise-induced synchronization is investigated. It is shown that the on–off type intermittence takes place. The observed effect is illustrated by analyzing model systems with discrete time, as well as stream dynamic systems under the action of a common noise source.

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Intermittent behavior is typical of systems of various origins and is a universal effect. In particular, intermittence is one of classical scenarios in a transition from periodic to chaotic oscillations [1]. In this case, the signal is an intermittent sequence of regular (laminar) phases and chaotic spikes (turbulent phases). Upon an increase in the control parameter, the turbulent spikes become more and more frequent until the motion becomes completely randomized. Depending on the type of stability loss in a periodic regime (determined by multiplicators of the limiting cycle), intermittence of types I–III can be distinguished [2].

The intermittent behavior is also observed in the vicinity of the boundaries of various types of chaotic synchronization; in this case, we can distinguish the intermittent phase locking [3, 4], intermittent generalized synchronization [5], and intermittent synchronization with time lag [6]. The intermittent behavior is subject to certain classification. A transition from generalized synchronization to synchronization with time lag is characterized as the on–off type intermittence [5, 6], while in a transition to phase locking, either type I intermittence and "needle eye" intermittence [3], or "ring" intermittence [4] takes place depending on the detuning of control parameters.

Along with various types of random synchronization, noise-induced synchronization is also known [7]. It has been established that this regime resembles in certain respects the regime of generalized chaotic synchronization [8] both in the diagnostics methods [8] and in the mechanism of its formation, which enables us to treat these two types of chaotic synchronization as a unique type of synchronous behavior of coupled dynamic systems [9].

At the same time, the intermittent behavior at the noise-induced synchronization boundary has not been studied. The similarity of generalized synchronization and noise-induced synchronization suggests the interrelation of the phenomena occurring at the boundaries of their formation. As mentioned above, an intermittent behavior of the on–off type is observed at the boundary of generalized synchronization. We can assume that an analogous type of intermittence must also take place upon a transition to noise-induced synchronization mode.

This study is aimed at the confirmation of this assumption. It will be shown below by numerical simulation of systems with continuous and discrete time that on–off-type intermittence takes place at the noise-induced synchronization boundary. Following [5], we will refer to this regime as intermittent noise-induced synchronization.

Noise-induced synchronization is traditionally treated as follows: a random signal $\xi(t)$ acting on two independent but identical chaotic systems $\mathbf{u}(t)$ and $\mathbf{v}(t)$ (with different initial conditions $\mathbf{u}(t_0)$ and $\mathbf{v}(t_0)$ lying in the attraction basin of the same chaotic attractor) may lead to "synchronization" of these systems; i.e., the systems demonstrate identical behaviors $\mathbf{u}(t) \equiv \mathbf{v}(t)$ after the completion of the transient process [7].

In the diagnostics of noise-induced synchronization, direct comparison of vectors of state $\mathbf{u}(t)$ and $\mathbf{v}(t)$ of the systems subjected to noise action is carried out or the senior Lyapunov exponent Λ of one of the systems under the noise action is calculated. The noise-induced synchronization regime can be stabilized only when all Lyapunov exponents are negative [10].

Intermittent behavior (noise-induced intermittent synchronization) is observed below the synchronous regime threshold. In this case, the noise-induced synchronization regime is observed in systems over the major part of time period; at these instants, the vectors of state of the systems acted upon by noise coincide ($\mathbf{u}(t) \equiv \mathbf{v}(t)$), and the laminar phase of behavior is observed. At the same time, in certain time intervals, the systems behave asynchronously (turbulent phase of their behavior takes place). At such instants, the

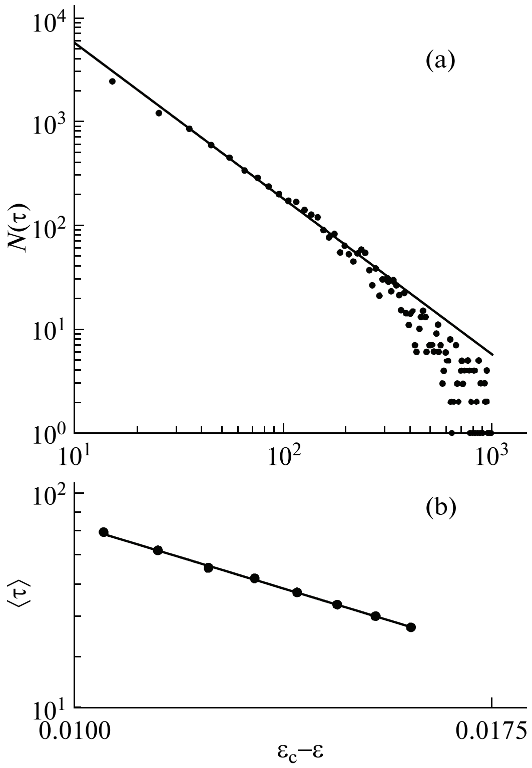

Fig. 1. (a) Distribution of the durations of laminar phases for $\varepsilon = 0.1525$ and (b) dependence of the average duration τ of laminar phases on supercriticality parameter ($\varepsilon_c - \varepsilon$), $\varepsilon_c = 0.1625$, of logistic mappings (3) located in the vicinity of the boundary of noise-induced synchronization and the corresponding approximations by power laws (a) (1) and (b) (2). Data obtained by numerical simulation are shown by points, theoretical dependencies are represented by solid lines.

vectors of state of the systems experiencing the action of noise become different (i.e., $\mathbf{u}(t) \equiv \mathbf{v}(t)$).

In identification of the intermittence type, statistics of laminar phase durations play an important role. Each intermittence type has its own characteristics; it is assumed that two traditionally used characteristics (dependence of the average duration of laminar phases on the supercriticality parameter and the distribution of laminar phase durations) make it possible to determine unambiguously the type of intermittence in the system. In particular, in the on–off-type intermittence, laminar phase duration distribution $N(\tau)$ obeys the power law

$$N(\tau) \sim \tau^{-3/2}, \quad (1)$$

and the dependence of the average duration of laminar phases on supercriticality parameter ($P_c - P$) satisfies the power dependence

$$\langle\tau\rangle \sim (P_c - P)^{-1}, \quad (2)$$

where P is the running value of the control parameter and P_c is its critical value corresponding to the instant of transition to the synchronous regime [11].

Let us consider the stabilization of noise-induced synchronization in model systems with continuous and discrete time.

We begin with simple objects, viz., 1D images demonstrating chaotic dynamics under the action of a common noise source.

By way of example, we consider logistic mappings studied in [9]:

$$\begin{aligned} y_{n+1} &= f(y_n) + \varepsilon(f(\xi_n) - f(y_n)), \\ z_{n+1} &= f(z_n) + \varepsilon(f(\xi_n) - f(z_n)), \end{aligned} \quad (3)$$

where y_n, z_n are the states of systems subjected to noise; $f(x) = \lambda x(1 - x)$ is a nonlinear function, λ is the control parameter, and ξ_n is a random quantity obeying the normal probability density distribution:

$$p(\xi) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(\frac{-(\xi - \xi_0)^2}{2\sigma^2}\right), \quad (4)$$

where $\xi_0 = 0.5$ and $\sigma = 0.12$ are the average value and dispersion, respectively. Term $\varepsilon(f(\xi_n) - f(y_n))$ determines the dissipative nature of a stochastic signal acting on the system under investigation and ε is the parameter determining the intensity of such action. The initial conditions for systems y_n and z_n are chosen different.

It is known that in the absence of external action, the logistic mapping can demonstrate various oscillatory regimes (including chaotic) depending on the choice of control parameter λ [12]. We choose the value of control parameter $\lambda = 3.75$, which corresponds to realization of the chaotic regime in system (3).

For the chosen values of control parameters and upon an increase in parameter ξ, system (3) demonstrates a transition from the asynchronous state to the noise-induced synchronization regime [9]. The synchronous regime sets in for $\varepsilon > \varepsilon_c = 0.1625$. The critical value of parameter ε was determined from the instant of transition of Lyapunov exponent Λ to the range of negative values (see also [9]).

Let us consider the behavior of system (3) in the vicinity of the boundary of noise-induced synchronization. As mentioned above, the intermittent behavior is realized in the system below the threshold of the synchronous regime. To determine the type of intermittence, we analyze the statistical characteristics of durations of laminar phases: the distributions of the durations of laminar phases for the given value of the control parameter (parameter ε in the given case) and the dependence of the average duration of laminar phases on supercriticality parameter ($\varepsilon_c - \varepsilon$).

Figure 1 shows the distribution $N(\tau)$ of laminar phase durations for $\varepsilon = 0.1525$ (a) and the dependence of the average duration of laminar phases on supercriticality parameter $\varepsilon_c - \varepsilon$ (b), as well as corresponding approximations. It can be seen from the figures that the resultant characteristics obey to a high degree of accuracy the power laws in exact correspondence with familiar regularities (1) and (2) for the on–off-type intermittence. The deviation of numerically obtained values from the power dependence in the range of large durations of laminar phases in Fig. 1a is the well-known fact for the on–off intermittence and is due to poor statistics.

Let us now analyze the stabilization of noise-induced synchronization in objects with a higher level of complexity, viz., steam dynamic systems demonstrating chaotic dynamics. By way of example of such systems, we consider the model of uncoupled identical (in control parameters) Lorentz oscillators [7, 13] in the case of action of a common noise source with zero mean value.

The system under investigation has the form

$$\begin{aligned}\dot{x}_1 &= p(y_1 - x_1),\\ \dot{y}_1 &= -x_1 z_1 + r x_1 - y_1 + \varepsilon\xi,\\ \dot{z}_1 &= x_1 y_1 - b z_1,\\ \dot{x}_2 &= p(y_2 - x_2),\\ \dot{y}_2 &= -x_2 z_2 + r x_2 - y_2 + \varepsilon\xi,\\ \dot{z}_2 &= x_2 y_2 - b z_2,\end{aligned} \qquad (5)$$

where $\mathbf{x}_{1,2} = (x_{1,2}, y_{1,2}, z_{1,2})$ are the vectors of state of the systems subjected to the action of common noise source $\xi(t)$, $p = 10$, $b = 8/3$, and $r = 28$ are the control parameters, ε is the parameter characterizing the intensity of external action, and ξ is a random quantity obeying normal probability density distribution (4), where $\xi_0 = 0$ and $\sigma = 1$.

The threshold for the stabilization of the noise-induced synchronization regime was determined by calculating the spectrum of the Lyapunov exponents. When the senior Lyapunov exponent passed through zero, the synchronous regime was diagnosed. For the system considered here, noise-induced synchronization sets in for parameter $\varepsilon_c = 760$.

As in the case of mappings (3), alternation of laminar and turbulent phases of behavior is observed at the boundary of the synchronous regime. To determine the type of intermittence, we analyzed the statistical characteristics of the duration of laminar phases.

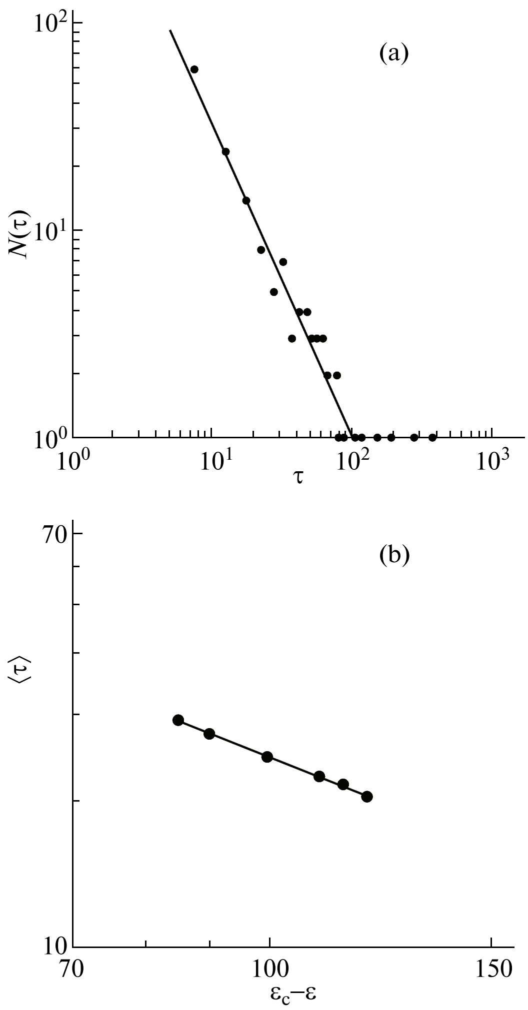

Fig. 2. (a) Distribution of the durations of laminar phases for $\varepsilon = 675$ and (b) dependence of the average duration τ of laminar phases on supercriticality parameter $(\varepsilon_c - \varepsilon)$, $\varepsilon_c = 760$, for Lorentz systems (5) located in the vicinity of the boundary of noise-induced synchronization and corresponding approximations by power laws (a) (1) and (b) (2). Data obtained by numerical simulation are shown by points, theoretical dependencies are represented by solid lines.

Figure 2a shows the distribution of durations of laminar phases for coupling parameter $\varepsilon = 675$. It can be seen that this distribution is successfully approximated by power law (1). The dependence of the average duration of laminar phases on supercriticality parameter $(\varepsilon_c - \varepsilon)$ is shown in Fig. 2b. In this case also, we note that the resultant dependence satisfies familiar regularity (2) to a high degree of accuracy. The deviation of numerically obtained values from the power dependence in the range of large durations of laminar phases in Fig. 2s is due to poor statistics.

Thus, in the case of stream dynamic systems and discrete mappings demonstrating chaotic dynamics, the on–off-type intermittence takes place at the boundary of noise-induced synchronization. This regularity is typical for the transition from the noise-induced synchronization regime and is apparently universal. Our results are of considerable theoretical and applied value for understanding the general regularities in the interaction of complex nonlinear system of various origins. It can be expected that analogous behavior will take place in real systems subjected to fluctuations (in particular, physiological [14] and physical systems [15, 16]).

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