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Appearance of Generalized Synchronization in Mutually Coupled Beam–Plasma Systems

A. A. Koronovskii, O. I. Moskalenko*, V. A. Maksimenko, and A. E. Hramov

Saratov State University, Saratov, Russia

**e-mail: moskalenko@nonlin.sgu.ru*

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Abstract—The phenomenon of generalized synchronization onset between mutually coupled beam–plasma systems (Pierce diodes) with supercritical currents has been discovered. It is established that the appearance of a synchronous regime is related to the change in one Lyapunov exponent from a positive to negative value. The results of the analysis are confirmed by the nearest-neighbor method.

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The synchronization of chaotic oscillations in beam–plasma systems (BPSs) is among the most important direction of research in radio physics and microwave electronics [1, 2]. The phenomenon of chaotic synchronization is now widely used, in particular for hidden data transmission and chaos control in microwave systems [2–4]. Several types of synchronous behavior have been observed in coupled BP systems, including phase, complete, generalized, and time-scale synchronization [5, 6]. The case of generalized synchronization (GS) is among the most interesting and important phenomena [6, 7]. This synchronization regime was originally introduced only for unidirectionally coupled chaotic systems, but then the GS concept was generalized so as to apply to mutually coupled dynamical systems and networks with complicated topologies [8, 9]. However, the investigations of GS in BPSs are still restricted to those with unidirectional coupling [6].

In this context, it was of interest to study the possible appearance of GS in mutually coupled BPSs with supercritical currents. This Letter presents the results of the first study of this synchronization regime between mutually coupled Pierce diodes, which are classical model systems of microwave electronics.

By analogy with the case of unidirectional coupling, the GS of mutually coupled BPSs $\mathbf{u}_{1,2}(x, t)$ is defined as a regime in which a unique functional relationship $\mathbf{F}[\cdot]$ is established between their states, so that[1]

$$\mathbf{F}[\mathbf{u}_1(x, t), \mathbf{u}_2(x, t)] = 0. \quad (1)$$

It should be noted that relation (1) is also valid for systems with unidirectional coupling, since the functional equation $\mathbf{u}_2(x, t) = \mathbf{F}[\mathbf{u}_1(x, t)]$ that is conventionally used to determine the GS of these systems can be considered as a particular case of Eq. (1).

The diagnostics of GS in systems with unidirectional coupling is traditionally based on the methods of nearest neighbors [7, 10], conditional Lyapunov exponents [11], and auxiliary system [12]. Evidently, all of these methods can be generalized so that they are applicable to the case of mutual coupling between systems (see, e.g., a modification of the auxiliary system method [8]). However, the results of investigations performed for systems with small numbers of degrees of freedom, the application of the auxiliary system method in the case of mutually coupled systems can lead to incorrect results [13], while the nearest neighbor method and the calculation of conditional Lyapunov exponents allow for the appearance of a synchronous regime in mutually coupled systems with small numbers of degrees of freedom to be adequately described. In the resent study, these methods are generalized so that they apply to distributed BPSs.

Let us consider a system of mutually coupled Pierce diodes, the dynamics of which in the hydrodynamic approximation is described by the following self-consistent set of the equation of motion, equation of continuity, and Poisson's equation [2, 14]:

$$\frac{\partial v_{1,2}}{\partial t} = -v_{1,2}\frac{\partial v_{1,2}}{\partial x_{1,2}} - \frac{\partial \varphi_{1,2}}{\partial x}, \quad (2)$$

$$\frac{\partial \rho_{1,2}}{\partial t} = \frac{\partial(\rho_{1,2}v_{1,2})}{\partial x}, \quad (3)$$

$$\frac{\partial^2 \varphi_{1,2}}{\partial x^2} = -\alpha_{1,2}^2(\rho_{1,2} - 1), \quad (4)$$

with the corresponding boundary conditions

$$v_{1,2}(0, t) = 1, \quad \rho_{1,2}(0, t) = 1, \quad \varphi_{1,2}(0, t) = 0, \quad (5)$$

[1] The notion of state will be defined below.

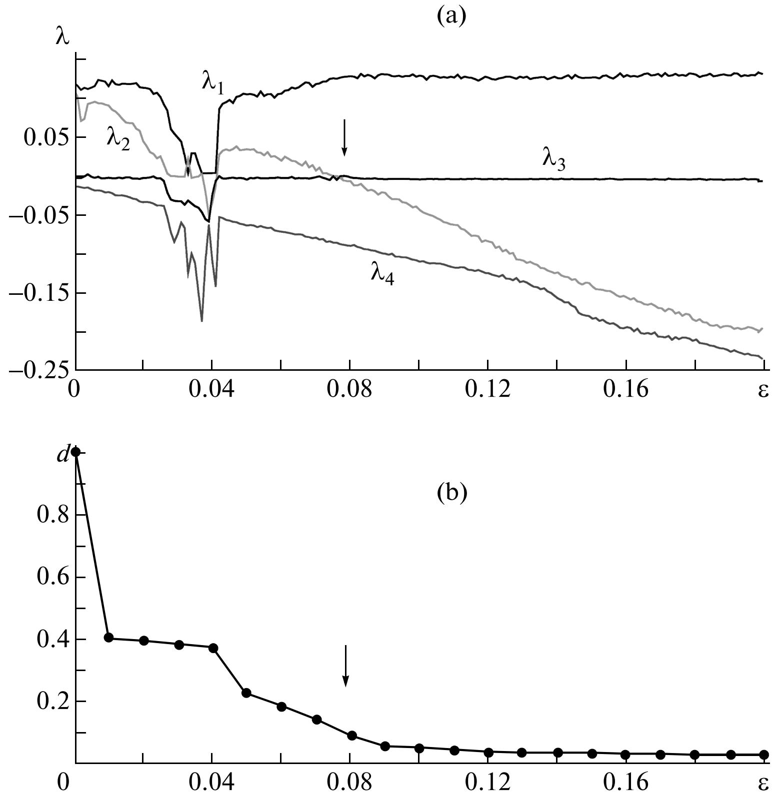

Fig. 1. Plots of (a) four highest Lyapunov exponents λ and (b) quantitative measure of proximity d versus coupling parameter ε. The arrow indicates the critical value $\varepsilon_c = 0.078$ of the coupling parameter that corresponds to the appearance of GS in the system of mutually coupled BPSs.

where φ is the dimensionless potential of the space-charge field; ρ is the dimensionless charge density; ν is the dimensionless flux density; x is the dimensionless coordinate; t is the dimensionless time; and subscripts 1 and 2 refer to the first and second mutually coupled BPSs, respectively. The only control parameter that characterizes the system dynamics is the Pierce parameter α (representing the unperturbed angle of electron flight at the plasma frequency), the values of which for the interacting systems in the numerical calculations are set as follows: $\alpha_1 = 2.858\pi$ and $\alpha_2 = 2.860\pi$.

The mutual coupling between systems 1 and 2 is provided by the variable values of dimensionless potential at the right-hand boundaries of both systems:

$$\varphi_{1,2}(1, t) = \varepsilon(\rho_{1,2}(x = 1, t) - \rho_{2,1}(x = 1, t)). \quad (6)$$

where ε is the coupling coefficient and $\rho_{1,2}(x = 1, t)$ are the oscillations of the dimensionless charge density as detected at the output of each system.

The GS diagnostics in this study is based on calculations of the spectrum of Lyapunov exponents. Figure 1a shows the dependences of four highest Lyapunov exponents λ on the coupling coefficient ε. The spectrum of λ was calculated using a recently proposed method [15]. As can be seen (similar to the case of unidirectional coupling, e.g. [6]), these Lyapunov exponents remain almost unchanged when the coupling parameter ε is varied, so that one of these exponents (λ_1) is always (except periodicity windows) positive and one exponent (λ_3) is always zero. At the same time, two other Lyapunov exponents, i.e., initially positive λ_2 and initially zero λ_4, depend on the coupling parameter and become negative when ε exceeds certain critical values. It can be suggested that, by analogy with the case of unidirectional coupled systems, the passage of the initially positive Lyapunov exponent λ_2 to the region of negative values (in the case under consideration, at $\varepsilon = \varepsilon_c = 0.078$) is related to the appearance of a GS regime in the mutually coupled BPSs.

In order to confirm this assumption, let us use the nearest-neighbor method. By analogy with [10], the proximity of interacting systems is quantitatively characterized by the meads distance d between two states of the second system normalized to the mean distance δ between randomly chosen states of the first system:

$$d = \frac{1}{N\delta}\sum_{k=0}^{N-1}\left\|\mathbf{u}_2^k - \mathbf{u}_2^{kn}\right\|, \quad (7)$$

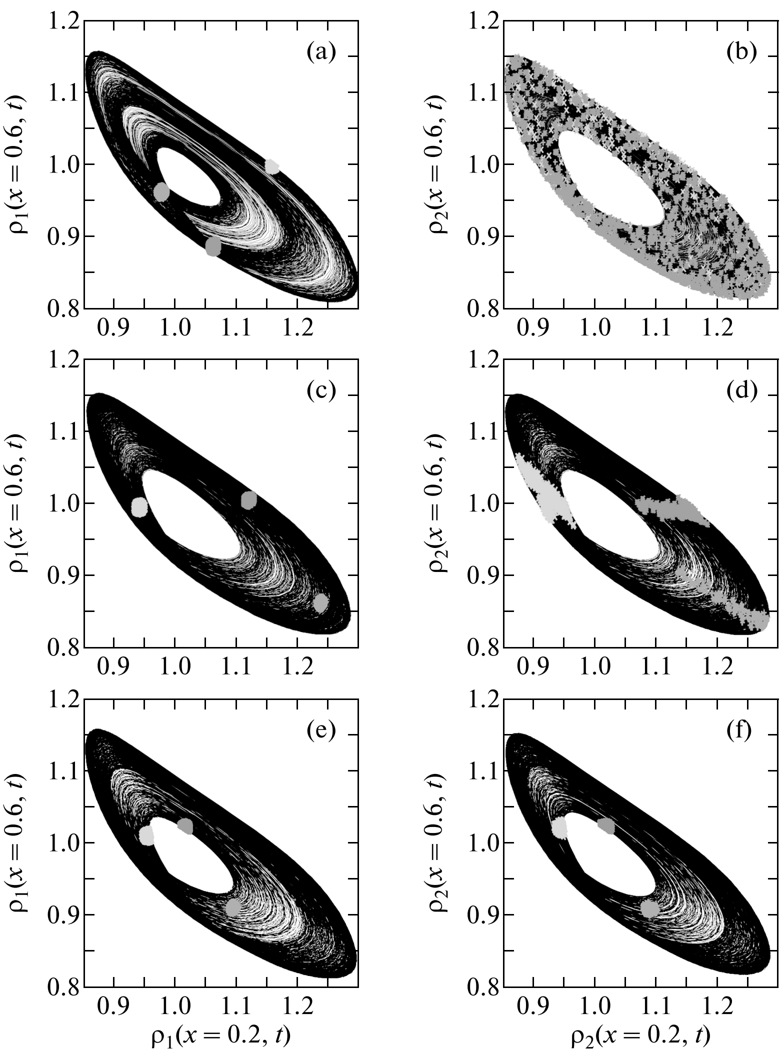

Fig. 2. Reconstructed attractors of mutually coupled Pierce diodes plotted on the plane $[\rho_{1,2}(x = 0.2, t), \rho_{1,2}(x = 0.6, t)]$ for various values of coupling coefficient ε: (a, b) 0.002 (asynchronous state); (c, d) 0.05 (phase synchronization); (e, f) 0.10 (GS regime); (a, c, e) reconstructed attractors of first Pierce diode with randomly chosen points; (b, d, f) phase space of second system with the corresponding nearest-neighbor states.

where N is the number of iterations. The vectors of state of the interacting systems are defined as $\mathbf{u}_{1,2}(x, t) = (\rho_{1,2}v_{1,2}, \varphi_{1,2})^T$.

Figure 1b shows a plot of the quantitative measure of proximity d versus coupling parameter ε. As can be seen, an increase in the coupling parameter is accompanied by the monotonic decrease in d from 1 to 0 and the ε_c value corresponds approximately to the middle of a descending part of the curve within $\varepsilon \in [0.04; 0.12]$, which is evidence for the GS regime onset in the system. It should be noted that, in the given case, the GS regime differs from the complete synchronization (which takes place at $\varepsilon \approx 0.17$).

The existence a GS regime in the mutually coupled BPSs with supercritical current is additionally confirmed by the nearest-neighbor behavior in the phase space of interacting systems. As is known, a Pierce diode with the given values of control parameters can be described by a finite-dimension model of three differential equations obtained using the Galerkin method [16]. This circumstance makes it possible to reconstruct the phase space of interacting Pierce diodes (2)–(4) [17] and analyze the nearest-neighbor behavior using a procedure analogous to that for the systems with small numbers of the degrees of freedom [13].

Figure 2 shows the results of this analysis, which are represented by the reconstructed attractors of interacting Pierce diodes plotted on the plane $[\rho_{1,2}(x = 0.2, t), \rho_{1,2}(x = 0.6, t)]$ for various values of the coupling coefficient ε. Attractors of the first system (Figs. 2a, 2c, 2e) show three pairs of randomly chosen points and their nearest neighbors, and Figs. 2b, 2d, 2f illustrate the corresponding states in the phase of the second system. As can be seen for a small value of the coupling parameter ($\varepsilon = 0.002$), all points in the phase space of the second system are randomly distributed over the entire attractor (Fig. 2b). As ε increases, the points tend to group in a limited region of the attractor (onset of phase synchronization) and the radius of this region decreases with increasing coupling (cf. Figs. 2d and 2f). Finally, at $\varepsilon > \varepsilon_{LE}$, all states of the second BPS that correspond to the indicated nearest neighbors of the first diode are also close and vice versa (Figs. 2e and 2f), which is evidence of the appearance of GC.

Thus, we have considered the appearance of GS between mutually coupled BPSs with supercritical currents as represented by the Pierce diodes in a hydrodynamic approximation). It is established that the onset of a GS regime is related to the passage of the (initially positive) second Lyapunov exponent to the region of negative values.

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REFERENCES

1. A. A. Koronovskii, D. I. Trubetskov, and A. E. Hramov, *Methods of Nonlinear Dynamics and Chaos in Problems of Microwave Electronics*, Vol. 2, *Nonstationary and Chaotic Processes* (Fizmatlit, Moscow, 2009) [in Russian].
2. D. I. Trubetskov and A. E. Hramov, *Lectures on Microwave Electronics for Physicists* (Fizmatlit, Moscow, 2003, Vol. 1; 2004, Vol. 2) [in Russian].
3. N. N. Zalogin and V. V. Kislov, *Broadband Chaotic Signals in Radio Engineering and Information Systems* (Radiotekhnika, Moscow, 2006) [in Russian].
4. A. A. Koronovskii, O. I. Moskalenko, and A. E. Hramov, Usp. Fiz. Nauk **179**, 1281 (2009) [Phys. Usp. **52**, 1213 (2009)].
5. P. V. Popov, R. A. Filatov, A. A. Koronovskii, and A. E. Hramov, Pis'ma Zh. Tekh. Fiz. **31**, 9 (2005) [Tech. Phys. Lett. **31**, 221 (**005**)].
6. R. A. Filatov, A. E. Hramov, and A. A. Koronovskii, Phys. Lett. A **358**, 301 (2006).
7. N. F. Rulkov, M. M. Sushchik, L. S. Tsimring, and H. D. I. Abarbanel, Phys. Rev. E **51**, 980 **1995**).
8. Z. Zheng, X. Wang, and M. C. Cross, Phys. Rev. E **65**, 056–211 (2002).
9. Y. Hung, Y. Huang, M. Ho, and C. Hu, Phys. Rev. E **77**, 016–202 (2008).
10. U. Parlitz, L. Junge, W. Lauterborn, and L. Kocarev, Phys. Rev. E **54**, 2115 (1996).
11. K. Pyragas, Phys. Rev. E **56**, 5183 (1997).
12. H. D. I. Abarbanel, N. F. Rulkov, and M. M. Sushchik, Phys. Rev. E **53**, 4528 (1996).
13. O. I. Moskalenko, A. A. Koronovskii, A. E. Hramov, and S. A. Shurygina, *Proceedings of the 18th IEEE Workshop on Nonlinear Dynamics of Electronic Systems (2010)*, pp. 70–73.
14. V. G. Anfinogentov and D. I. Trubetskov, Radiotekh. Elektron. **37**, 2251 (1992).
15. A. A. Koronovskii, O. I. Moskalenko, N. S. Frolov, and A. E. Hramov Pis'ma Zh. Tekh. Fiz. **36** (14), 19 (2010).
16. A. E. Hramov and I. S. Rempen, Int. J. Electron. **91**, 1 (2004).
17. A. E. Hramov and A. A. Koronovskii, Europhys. Lett. **80**, 10–001 (2007).

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