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Boundary of Generalized Synchronization in Two Unidirectionally Coupled Tunnel Diode Generators

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Abstract—The boundary of generalized synchronization regimes in a system of two unidirectionally coupled oscillators based on tunnel diodes has been studied. Peculiarities in the behavior of the boundary are found in the region of relatively large detunings between eigenfrequencies of the two subsystems. The character of this behavior and physical mechanisms leading to the onset of generalized synchronization regimes in the system are explained based on the analysis of evolution of the spectrum of oscillations in the response subsystem.

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Synchronization of chaotic oscillations is among the most important nonlinear phenomena and is extensively studied [1] as having both basic and practical significance, in particular, in biology, physiology, hidden data transmission using chaotic signals, control of microwave electronic systems, etc. [2–4]. Several types of synchronous behavior have been observed in coupled dynamical systems, among which the generalized synchronization (GS) [5] and its relation with phase synchronization (PS) [6] are the most interesting and important phenomena. In the case of unidirectionally coupled chaotic oscillators, the GS regime onset implies that the states of the drive $[\mathbf{x}_d(t)]$ and response $[\mathbf{x}_r(t)]$ oscillators after termination of a transient process obey a unique functional relationship $\mathbf{F}[\cdot]$ such that $\mathbf{x}_r(t) = \mathbf{F}[\mathbf{x}_d(t)]$. Several methods have been developed for the diagnostics of GS regimes in systems of unidirectionally coupled chaotic oscillators, including the methods of nearest neighbors [5, 7], conditional Lyapunov exponents [8], and auxiliary system [9]. The PS regime establishment implies locking of the phases of chaotic signals, while their amplitudes remain mutually unrelated and appear chaotic [6].

Zheng and Hu [10] studied the relation between GS and PS in a system of coupled chaotic Rössler oscillators and showed that these regimes behave differently, depending on the misfit of control parameters. In the case of relatively small misfit, the GS is more strongly manifested; whereas at sufficiently large detuning between the interacting subsystems, the threshold value of a coupling parameter for the onset of PS is significantly greater than that for the GS. The critical coupling parameter for the onset of GS is virtually independent of the misfit [11, 12] and the onset/breakage of PS is manifested by the appearance/disappearance of phase coherence in the chaotic attractor of one of the interacting subsystems [13, 14].

Recently, factors determining this behavior of the boundaries of synchronous regimes have been explained [15, 16] and physical mechanisms leading to the onset of synchronization have been revealed. In particular, it was established that, in the region of relatively weak detuning of the eigenfrequencies of coupled chaotic Rössler oscillators, the GS arises due to synchronization of the main spectral component and its subharmonics (whereas PS follows a scenario of eigenfrequency locking). In the region of relatively large detuning of the eigenfrequencies, the GS onset is accompanied by synchronization of two spectral components that correspond to the eigenfrequency of the drive subsystem and the main frequency of the response subsystem.

This Letter presents the results of an investigation of the character of boundary between GS and PS regimes in a system of two unidirectionally coupled oscillators based on tunnel diodes [13]. In a dimensionless form, this system can be described by the following set of differential equations:

$$\begin{aligned}
\dot{x}_d &= \omega_d^2(hx_d + y_d - z_d),\\
\dot{y}_d &= -x_d,\\
\dot{z}_d &= (x_d - f(z_d))/\mu,\\
\dot{x}_r &= \omega_r^2(h(x_r - \varepsilon(y_d - y_r)) + y_r - z_r),\\
\dot{y}_r &= -x_r + \varepsilon(y_d - y_r),\\
\dot{z}_r &= (x_r - f(z_r))/\mu,
\end{aligned} \tag{1}$$

where $f(\xi) = -\xi + 0.002\sinh(5\xi - 7.5) + 2.9$ is the dimensionless characteristic of a nonlinear element, $h = 0.2$ and $\mu = 0.1$ are fixed control parameters, ε is the coupling parameter, $\omega_r = 1.02$ is the control

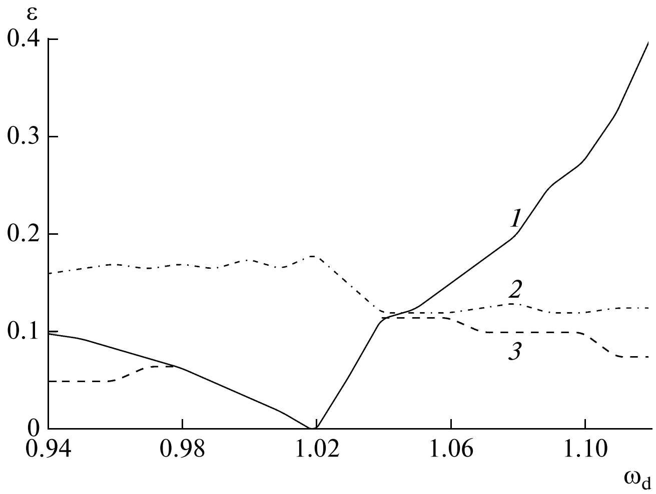

Fig. 1. Boundaries of the appearance of (*1*) PS, (*2*) GS, and (*3*) phase-coherent attractor plotted on the plane of control parameters (ω_d, ε) for system (1) of two unidirectionally coupled oscillators based on tunnel diodes.

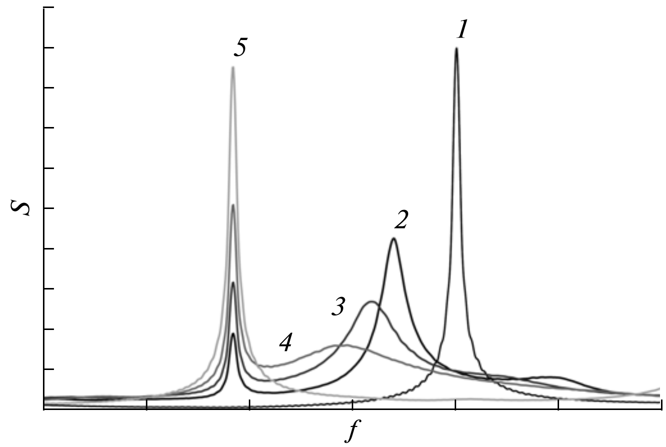

Fig. 2. Fourier spectra of oscillations in the response subsystem at a constant control parameter $\omega_d = 0.96$ of the drive system and various values of coupling parameter (ε): (*1*) 0; (*2*) 0.04; (*3*) 0.05; (*4*) 0.06; (*5*) 0.1.

parameter of the response subsystem that determines its main frequency (fixed), and ω_d is the analogous control parameter of the drive subsystem that was varied from 0.94 to 1.12, thus setting the frequency detuning of the interacting oscillators. For the indicated values of h, μ, and ω_r and the entire interval of ω_d values, the chaotic attractors of both subsystems in the absence of coupling are "phase-coherent" [13].

Figure 1 shows the boundaries of the regimes of GS, PS, and phase coherence for a system of coupled chaotic oscillators described by Eqs. (1), as plotted on the plane of control parameters (ω_d, ε). Here, curves *1* and *2* correspond to the onset of PS and GS, respectively, and curve *3* shows the boundary of phase coherence onset/breakage for the chaotic attractor of the response subsystem. The threshold of GS onset was determined by calculating conditional Lyapunov exponents for Eqs. (1), and then refined using the auxiliary system method. The moment of phase coherence onset/breakage for the chaotic attractor of the response subsystem was determined by calculating the measure of coherence [14] as a function of the coupling parameter. The onset of PS was determined by the condition of phase locking. The instantaneous phase of a chaotic signal; was conventionally defined as the angle of rotation on the (x, y) plane.

As can be seen from Fig. 1, the boundaries of synchronization regimes are essentially asymmetric relative to the $\omega_d = \omega_r$ line, which is related to a strong influence of dissipation in the response subsystem that takes place with an increase in the coupling parameter. On the other hand, the threshold for the onset of GS at small detuning of the interacting subsystems is significantly higher than that for a large misfit. However, this peculiarity is only observed for $\omega_d > \omega_r$, whereas the region of $\omega_d < \omega_r$ is characterized by weak dependence of the synchronization threshold on the drive subsystem parameter. At the same time, on both left and right from $\omega_d = \omega_r$, the breakage of PS at large values of the frequency detuning proceeds via the loss of phase coherence of the chaotic attractor (Fig. 1, curve *3*). However, the threshold value of the coupling parameter that corresponds to the PS onset for large values of the frequency detuning at mega $\omega_d < \omega_r$ is significantly lower than that for the same detuning at $\omega_d = \omega_r$ and is close to ε values corresponding to the onset of PS in the region of relatively small detunings at $\omega_d > \omega_r$.

This behavior of the GS and PS boundaries on the plane of control parameters $(\omega_d, \varepsilon)_{can}$ be explained as follows. Evidently, in the region of relatively small misfit of the eigenfrequencies ($\omega_d \in [0.98, 1.04]$, where the PS breakage takes place without the loss of phase coherence of the chaotic attractor of the response subsystem), the PS in the system under consideration (as well as in the case of interacting Rössler oscillators) arises due to the locking of main frequency components of the drive and response subsystems. At the same time, the GS regime is established via synchronization of the main spectral component and its subharmonics for the response subsystem [15, 16]. In the region of $\omega_d > 1.04$, the behavior is analogous to that observed for the unidirectionally coupled Rössler oscillators with relatively large misfit of eigenfrequencies, whereby the chaotic attractor of response subsystem (1) below the PS boundary becomes "phase-incoherent," which leads to breakage of the PS regime. In addition, Fig. 1 shows that the boundary of GS regimes in the region of relatively large misfit is close to the boundary of appearance/disappearance of phase coherence in the chaotic attractor. Evidently, the GS in this case arises due to the synchronization of two clearly pronounced spectral components (corresponding to frequencies of the drive and response system), the intensities of which near the GS boundary are close.

A much more interesting situation takes place in the region of relatively large misfit of the eigenfrequencies, where $\omega_d < 0.98$. Here, the PS onset/breakage is still determined by the appearance/disappearance of phase coherence in the chaotic attractor of the response subsystem, while the relation between GS and PS regimes is analogous to that in the case of small detunings. Moreover, the GS threshold in this case is about twice that for the coupling parameter corresponding to the GS onset at $\omega_d > 1.04$ and is virtually independent of ω_d in the region below 0.98. Apparently, the mechanism of GS establishment in this case must be different.

Let us consider in more detail the relation between GS and PS, on one hand, and the appearance/disappearance of phase coherence in the chaotic attractor of the response subsystem in this region, on the other hand. For the selected values of control parameters in system (1) of two unidirectionally coupled oscillators based on tunnel diodes and the control parameter of the drive subsystem fixed at $\omega_d = 0.96$, an increase in the coupling parameter leads to the following changes. At $\varepsilon = \varepsilon_{CM} = 0.048$, the chaotic attractor of the response subsystem loses phase coherence; at $\varepsilon = \varepsilon_{PS} = 0.075$ the system enters a PS regime; and at $\varepsilon = \varepsilon_{GS} = 0.175$ the system exhibits GS.

Figure 2 shows the Fourier spectra of oscillations in the response subsystem at various values of the coupling parameter. As can be seen, an increase in ε leads to a decrease in intensity of the spectral component at the main frequency of the response system and an increase in the component at the main frequency of the drive system. The main frequency of the response system shifts toward lower energies of the coupling parameter and eventually (since $\omega_d < \omega_r$) this leads to locking of the main frequencies of interacting subsystems and, hence, the PS onset. This takes place at a certain value of the coupling parameter, for which the intensity of the spectral component of the response subsystem at the frequency of the drive subsystem will become significant. Below the PS threshold, the spectrum of the response subsystem contains two components and its attractor is phase-incoherent. However, the PS is attained via a scenario that is characteristic of the case of relatively small detunings of the eigenfrequencies. Indeed, already at $\varepsilon = 0.1$, the Fourier spectra of oscillations in the response subsystem exhibits a single pronounced spectral component at the frequency of the drive subsystem. Evidently, the GS regime in this case will also arise by a scenario characteristic of the case of relatively small detunings of the eigenfrequencies, which proceeds via synchronization of the main spectral component of the drive subsystem and its subharmonics.

Thus, we have considered the appearance of GS in a system of two unidirectionally coupled oscillators based on tunnel diodes. It is established that a GS regime in the case of a relative strong misfit of parameters can arise by a scenario that is characteristic of relatively small detuning of the eigenfrequencies of subsystems. The behavior of the boundary of GS regimes and its relation to the PS regime in the system under consideration is explained based on the analysis of evolution of the spectrum of oscillations in the response subsystem.

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