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Universal Regularity of the Main Spectral Component Synchronization of Interacting Oscillators

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Abstract—We study the dynamics of oscillators at the chaotic phase synchronization boundary. The universal regularity of the difference between the phases of main spectral components varying in dispersion, as estimated by means of the Fourier transformation, and the length of analyzed time interval is found. The revealed regularity describes the behavior of different classes of systems at all values of the control parameter that belong to the investigated region.

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INTRODUCTION

Several types of synchronous behavior of interacting chaotic systems are currently known, and the problem of their interaction is of great interest [1, 2]. Scientists have developed several approaches to analyzing the various types of chaotic synchronization from a common point of view, one of which is the synchronization of spectral components of Fourier spectra. It was shown in [3, 4] that different types of chaotic synchronization (phase synchronization, delayed synchronization, and full synchronization) are particular manifestations of spectral component synchronization. Specifically, phase synchronization in such approach means synchronization of the main components of the Fourier spectra of interacting systems. This is expressed by the independence from initial conditions of the difference in the phases of main spectral components, which are calculated using the Fourier transformation.

Although chaotic synchronization has been studied quite well with regard to spectral components, an investigation of the dynamics in the parameter region that preceeds the establishment of synchronous operation has yet to be performed. It is known from [5, 6] that the phenomenon of needle's eye intermittency is observed in chaotic systems over the short range of the coupling parameter directly adjacent to the area of synchronous dynamics. In this case, we might expect that the distribution of the difference in the phases of main spectral components would be distinct from the δ function that fits the synchronous behavior. The form of such distributions and the way they change with the supercriticality parameter are of great interest. We should therefore consider that the Fourier transformation is formally caculated by an infinite interval, though we have to restrict ourselves to analyzing some finite time realization length both in numerical simulations and in experimental investigations. The question then arises as to how the calculation results relate to the selected length of time realization.

This work is devoted to the investigation of such problems. The first section considers the dynamics of chaotic stream systems in both synchronous and asynchronous operation. A system of two unidirectionally coupled Ressler oscillators is chosen as the investigated system. The second section studies the dynamics of discrete mapping (circle mapping) in the vicinity of the saddle-node bifurcation with noise, which can be considered as a model system describing the processes under establishing of synchronous operation [7]. The results of the first and second section are also compared here.

SYNCHRONIZATION OF THE SPECTRAL COMPONENTS IN TWO UNIDIRECTIONALLY COUPLED RESSLER SYSTEMS

We begin with a system that consists of two unidirectionally coupled Ressler oscillators:

$$\begin{aligned} \dot{x}_d &= -\omega_d y_d - z_d, \\ \dot{y}_d &= \omega_d x_d + a y_d, \\ \dot{z}_d &= p + z_d(x_d - c), \end{aligned} \qquad (1)$$

$$\begin{aligned} \dot{x}_r &= -\omega_r y_r - z_r + æ(x_d - x_r), \\ \dot{y}_r &= \omega_r x_r + a y_r, \\ \dot{z}_r &= p + z_r(x_r - c). \end{aligned} \qquad (2)$$

The d subscript in (1)–(2) fits the master system, while the r subscript fits the slave system. The following parameters of the system were chosen: $a = 0.15$, $p = 0.2$, and $c = 10$. The parameters $\omega_d = 0.93$ and $\omega_r = 0.95$ specify the fundamental frequency of the master and slave systems respectively, and æ is responsible for the relationship between the systems. In this case, the main spectral component of the master system is

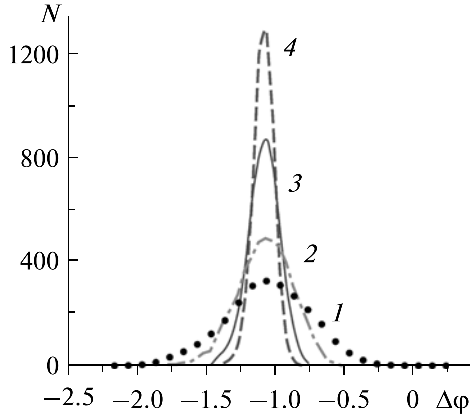

Fig. 1. Distributions of difference in phases of the main specrtal components of Ressler systems at æ = 0.043, introduced as the rotation angle on phase plane (*1*) and calculated using the Fourier transformation for different values of the analyzed time interval: (*2*) $T = 100$, (*3*) $T = 600$, and (*4*) $T = 1600$.

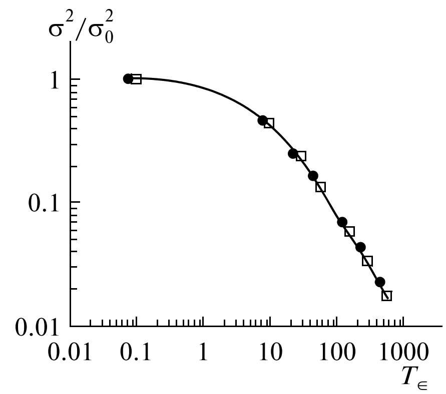

Fig. 2. Relations between the normalized dispersion of the distributed difference between the Ressel system phases and the normalized length of the time interval at æ = 0.04 and æ =0.043.

matched by the value of frequency $f_d = 0.148$ at the fundamental frequency of the slave system in the autonomous operation $f_r = 0.151$. Gauss's law describes the distributions of values of the difference between the master and slave systems at frequency f_d, calculated using the Fourier transformation. Figure 1 presents the distributions obtained from intervals of different length T for the value of coupling parameter æ = 0.043. For our selected system parameters, the operation of phase synchronization is observed when we exceed the critical value of coupling force parameter $æ_{PS} \approx 0.041$. The value æ = 0.043 thus corresponds to synchronous operation. Clearly, the dispersion of such distributions decreases and tends to zero with T tending toward infinity as the length of time intervals T in which the Fourier transformation is implemented increases. The points in Fig. 1 depict the distribution of the difference of phases introduced as a rotation angle on the phase plane [2]. This way of introducing the phase is valid, since the systems posesses a phase-coherent attractor at the given parameters.

The distributions at $æ < æ_{PS}$ are of identical form; their dispersion also decreases as the interval in which the Fourier transformation is calculated increases. We now estimate in what way the dispersion depends on the length of the analyzed interval, and how such relationships for different values of coupling parameter æ over a range of values corresponding to the operation of needle's eye intermittency are related to one another.

An analytical solution to such a problem is impossible for a system of coupled Ressler oscillators. At the same time, it is known that chaotic dynamics can in some cases be considered as random perturbations of regular behavior [8, 9]. It was shown in [5, 6] that the needle's eye intermittency and type I intermittency are of the same kind of behavior. For such a description, we can renormalize the variables as in [7]:

$$q = \frac{\Delta\varphi}{\sqrt{|\epsilon|}}, \quad \tau = t\sqrt{|\epsilon|}, \tag{3}$$

where $\Delta\varphi$ is the difference of phases of interacting oscillators; t is time; $\epsilon = æ - æ_C$ is the supercriticality parameter; and $æ_C = 0.0345$ is the point of saddle-node bifurcation of the imvestigated system. It corresponds to the point at which we establish phase synchronization, when the noise intrinsic to the chaotic dynamics of Ressler systems [7] can be turned off.

We might expect that the relationships between the σ^2 dispersions and the length of the normalized time interval $T_\epsilon = T\sqrt{|\epsilon|}$ for the various values of æ parameter will be close after the introduction of renormalizing (3). Let us normalize for simplicity the dispersion σ^2 to σ_0^2, where σ_0^2 is the dispersion of distribution of phases introduced as the rotation angle on the phase plane. Figure 2 gives the obtained relationships, which obviously coincide as expected. We thus find some universal curve that describes the dispersions of phase difference in accordance with the length of time interval at different values of coupling parameter.

TYPE I INTERMITTENCY WITH NOISE IN CIRCLE MAPPING

As was noted above, it was shown in [5, 6] that the needle's eye and type I intermittencies with noise are the same kind of behavior, and we therefore consider the dynamics of circle mapping with noise to verify the generality of our results:

$$x_{n+1} = x_n + 2\Omega(1 - \cos x_n) - \varepsilon + \xi_n, \quad \mathrm{mod}\, 2\pi, \tag{4}$$

where ε is the control parameter; $\Omega = 0.1$; and $\xi_n$is the δ-correlated Gaussian white noise. The analog of the phase difference of Ressler systems (1)—(2) is the x_n

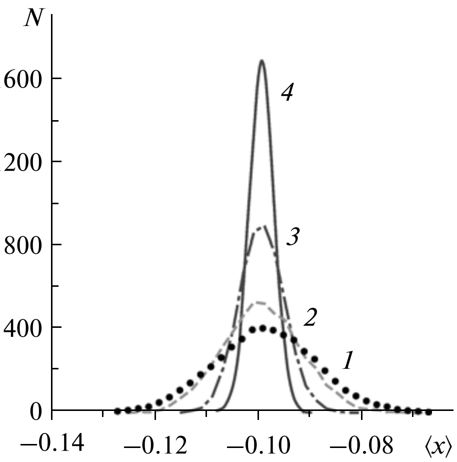

Fig. 3. The distributed x_n value of circle mapping at the control parameter $\varepsilon = 0.1$ for $K = 1$ (*1*), $K = 100$ (*2*), $K = 500$ (*3*), and $K = 200$ (*4*).

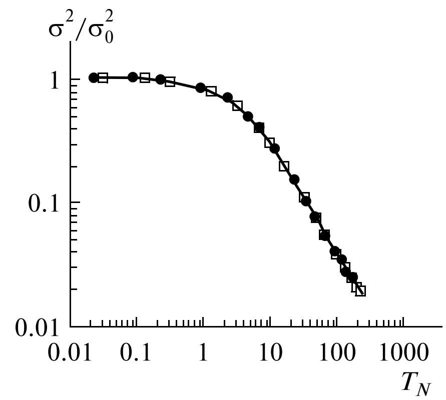

Fig. 4. The relationships between the normalized dispersion of distributed x_n value of circle mapping and the dicrete-time T_N interval at the control parameters $\varepsilon = 0.05$ and $\varepsilon = 0.1$.

variable, while the analog of the Fourier transformation is averaging by some discrete-time K interval:

$$\langle x \rangle = \frac{1}{K}\sum_{i=1}^{K} x_i. \quad (5)$$

The unaveraged x_n value is matched by the difference between the phases of Ressler systems, introduced as the rotation angle on the phase plane.

Figure 3 shows the distributions of x_n value of circle mapping at $\varepsilon = 0.1$ for different lengths of K interval of discrete time in which the averaging is performed. These distributions are identical to the ones presented in Fig. 1 for the Ressler systems. It is evident that they are also in the form of Gaussian distributions, and their dispersion decreases as the discrete-time interval increases. By analogy with [7], we can introduce the renormalizing

$$z = \frac{x_n}{\sqrt{|\varepsilon|}}, \quad \tau = t\sqrt{|\varepsilon|}, \quad D^* = D|\varepsilon|^{-3/2}, \quad (5)$$

where t is the time; D is the noise intensity (in this case, the supercriticality parameter is ε, since the point of saddle-node bifurcation is at zero [7]). After calculating the relationships between the σ^2 dispersion normalized on σ_0^2, which is the dispersion of distribution defined for $K = 1$ and the normalized discrete time interval $T_N = K\sqrt{|\varepsilon|}$, we find that they coincide and are in the same form as for the Ressler systems (Fig. 4).

The comparison between our results for circle mapping with noise and the results of the first section for the Ressler systems is of great importance. This is complicated, however, since the considered systems belong to the class of stream time systems, and circle mapping is a discrete time system. At the same time, it is known that stream systems can be reduced to mappings by using the Poincaré section, and stream system properties are therefore related to mapping properties [10]. During the transition from the stream system to mapping by Poincaré section, the time interval between the crossing of a cross section by the phase trajectory is, on average, inversely proportional to the fundamental frequency of the Fourier spectrum that fits one unit of time in the mapping dynamics. We should also consider that the supercriticality parameters in the stream system and mapping have different typical scales. We now renormalize the time once again to compare the results correctly:

$$s = \alpha_i \tau, \quad (6)$$

where $i = 1.2$, $\alpha_1 = 0.4$, $\alpha_2 = 0.1$, subscript 1 corresponds to the Ressler systems, and subscript 2 corresponds to circle mapping. The α_i coefficients were found empirically. They clearly lie on a certain curve that is universal both for Ressler systems and for circle mapping at any value of the control parameter within the investigated area.

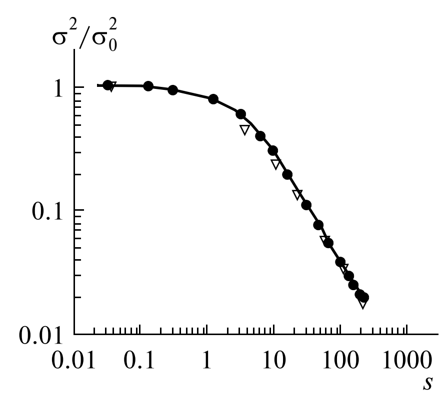

Fig. 5. The relationship between the normalized disperison and the normalized length of time interval. The line with points corresponds to the circle mapping at the control parameter $\varepsilon = 0.1$ and the triangles correspond to the Ressler systems at the coupling parameter æ $= 0.043$.

CONCLUSIONS

We have considered the synchronization of spectral components for chaotic systems in the operations of needle's eye intermittent and phase synchronization. Distributions of the difference of phases calculated by the Fourier transformation and introduced as the rotation angle on the phase plane were obtained for selected model systems of unidirectionally coupled chaotic Ressler oscillators. A universal relationship between the dispersion of such distributions and the length of the analyzed T interval describing the system dynamics at any value of the coupling æ parameter within the investigated area was found. We may expect that such a relationship describes the dynamics of a wide class of chaotic systems near the phase synchronization boundary. The correctness of the obtained results is confirmed by the identical results for circle mapping with noise, which concide with our analytical estimates. Our results can be also considered as additional verification of the unity of the needle's eye and type I intermittments with noise.

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