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Method for Separating Laminar and Turbulent Intervals in Intermittent Time Series of Systems near the Phase Synchronization Boundary

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Abstract—A new method is proposed for separating the intervals of synchronous (laminar) and asynchronous (turbulent) behavior in intermittent time series of coupled chaotic systems occurring near the boundary of the phase synchronization regime. Using this method, it is possible to determine the durations of turbulent and laminar intervals, which are necessary for an analysis of the statistical characteristics of a given dynamical system. The proposed approach directly employs the instantaneous phases of chaotic signals and provides exact values of the durations of turbulent and laminar intervals in the system behavior. The validity of the new method is verified by a comparison of the results to analogous data obtained previously, which shows their good agreement.

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The phenomenon of intermittency has been given increasing attention in recent years [1–4]. In these investigations, an important role is played by the statistical characteristics of intermittent behavior such as the distribution of durations (lengths) of laminar and turbulent intervals depending on the control parameters of a given dynamical system, the average laminar interval length as a function of the supercriticality parameter, etc. In order to determine these characteristics, various methods are used that are based on the separation of synchronous and asynchronous behavior. As a rule, these methods employ various transformations of the measured time series, in particular, the continuous wavelet transform [5, 6]. These methods ensure quite accurate separation of the regions of synchronous and asynchronous behavior, but their common disadvantage is a considerable computation time that significantly increases with the length of time series. However, long time series are especially important for a correct analysis of the statistical characteristics of the intermittent behavior.

One important and interesting type of intermittent behavior, which is observed at the boundary of the phase synchronization of two coupled chaotic oscillators, is known as the eyelet intermittency [3, 4, 7]. The present Letter describes a new method for separating the regions of synchronous and asynchronous behavior in the time series of coupled chaotic systems exhibiting the intermittency of this type. An important advantage of the proposed method consists in that it directly employs the instantaneous phase difference and does not involve additional transformations, which significantly decreases the computation time. The new method has been verified by application to a system of two unidirectionally coupled chaotic Rössler oscillators [8].

Let us consider two unidirectionally coupled Rössler oscillators, the dynamics of which is described by the following differential equations:

$$\begin{aligned} \dot{x}_d &= -\omega_d y_d - z_d, \\ \dot{y}_d &= \omega_d x_d + a y_d, \\ \dot{z}_d &= p + z_d(x_d - c), \end{aligned} \tag{1}$$

$$\begin{aligned} \dot{x}_r &= -\omega_r y_r - z_r + \varepsilon(x_d - x_r), \\ \dot{y}_r &= \omega_r x_r + a u_r, \\ \dot{z}_r &= p + z_r(x_r - c). \end{aligned} \tag{2}$$

Oscillators (1) and (2) will be referred to below as the driving and response subsystems, respectively. The coupling has a dissipative character and is determined by the coupling parameter ε. The values of the control parameters are selected as follows: $a = 0.15$, $p = 0.2$, and $c = 10.0$. The values of fundamental frequencies of the response and driving subsystems are fixed at $\omega_d = 0.93$ and $\omega_r = 0.95$, respectively.

The phase synchronization boundary in the system under consideration corresponds to the case where the instantaneous phase difference defined as

$$\Delta\varphi(t) = \varphi_1(t) - \varphi_2(t) \tag{3}$$

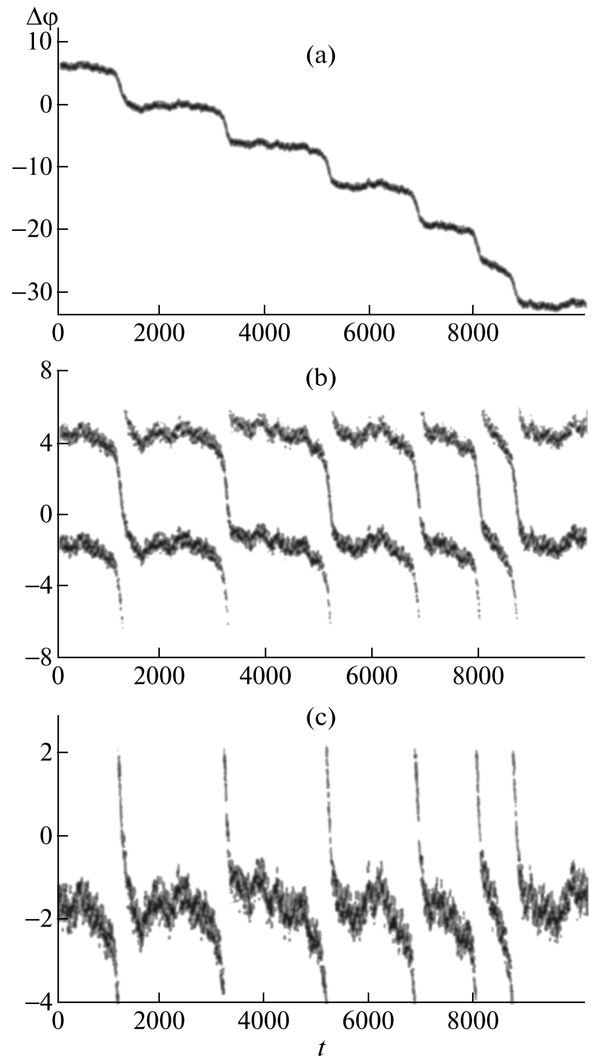

Fig. 1. Time series of the phase difference $\Delta\varphi(t)$ plotted for the instantaneous phases (a) monotonically increase from $-\infty$ to $+\infty$, (b) vary within the interval $[-2\pi, 2\pi]$, and (c) fall within a 2π-wide interval for the system under consideration with a coupling parameter of $\varepsilon = 0.036$.

for the interacting chaotic subsystems is limited rather than increases with the time, that is, obeys the following condition of phase locking:

$$|\Delta\varphi(t)| = |\varphi_1(t) - \varphi_2(t)| < \text{const.} \quad (4)$$

Here, $\varphi_1(t)$ and $\varphi_2(t)$ are the instantaneous phases of chaotic subsystems (1) and (2), respectively. These phases can be introduced in different ways, in particular, using the angle of rotation on the plane of the chaotic attractor projection, the Poincaré section, or the Hilbert transform [9, 10].

If the control parameters are close to (but outside) the domain of phase synchronization, the onset of synchronous regime in the coupled chaotic subsystems is preceded by intermittent behavior, whereby time series of the variables contain the regions of synchronous dynamics (laminar intervals) separated by sudden interrupts (turbulent intervals), during which the phase difference changes by 2π.

The phase of the chaotic signal, which is introduced using one of the aforementioned traditional methods, is assumed to be monotonically increasing from $-\infty$ to $+\infty$. The regions of synchronous and asynchronous behavior are usually separated using an analysis of the time series of the phase difference $\Delta\varphi(t)$. Figure 1a presents the typical time series of the phase difference for two unidirectionally coupled Rössler oscillators (1) and (2). Regions where the phase difference $\Delta\varphi(t)$ changes within 2π obey the phase locking condition and, hence, correspond to the laminar behavior. The other regions correspond to the turbulent behavior, where phase locking is not observed.

Despite the fact that the regions of laminar and turbulent behavior on the $\Delta\varphi(t)$ time series are visually easy to recognize, the numerical separation of these intervals encounters some problems that are related to chaotic fluctuations, which can lead to erroneous detection of the onset of laminar and turbulent behavior. In order to eliminate these difficulties involved in the numerical analysis, let us assume that the instantaneous phases of $\varphi_1(t)$ and $\varphi_2(t)$ vary within the interval $[0, 2\pi]$ rather than monotonically increase from $-\infty$ to $+\infty$. Then, the phase difference $\Delta\varphi(t)$ changes within $[-2\pi, +2\pi]$ and its time series takes the form depicted in Fig. 1b.

A comparison of Figs. 1a and 1b shows that, in the latter presentation, there exist a certain interval $[\Delta\varphi_{\min}, \Delta\varphi_{\max}]$ of phase differences (in the given example, $\Delta\varphi_{\min} \approx -\pi$ and $\Delta\varphi_{\max} \approx 2\pi$) such that the $\Delta\varphi$ going outside this interval corresponds to the onset of turbulence. It is important to note that all $\Delta\varphi(t)$ fluctuations caused by the chaotic character of oscillations in the interacting subsystems are localized in the regions of laminar behavior, whereas the phase difference during the turbulent interval exhibits almost no fluctuations. Using this circumstance, it is possible to unambiguously and very precisely determine the beginning and end of each turbulent interval during numerical analysis of the time series of $\Delta\varphi(t)$.

Evidently, if the $\varphi_1(t)$ and $\varphi_2(t)$ values vary within the interval $[0, 2\pi]$, then the phase difference $\Delta\varphi(t)$ changes within $[-2\pi, +2\pi]$. In this case, the phase difference $\Delta\varphi(t)$ is a many-valued function that is not convenient for the numerical analysis of systems under consideration. In order to remove this disadvantage, we can use the fact that the phase has a period of 2π and convert the phase difference from the 4π-wide interval to an analogous 2π-wide interval. In addition, this should be done so that the values of phase differences corresponding to the laminar intervals in the

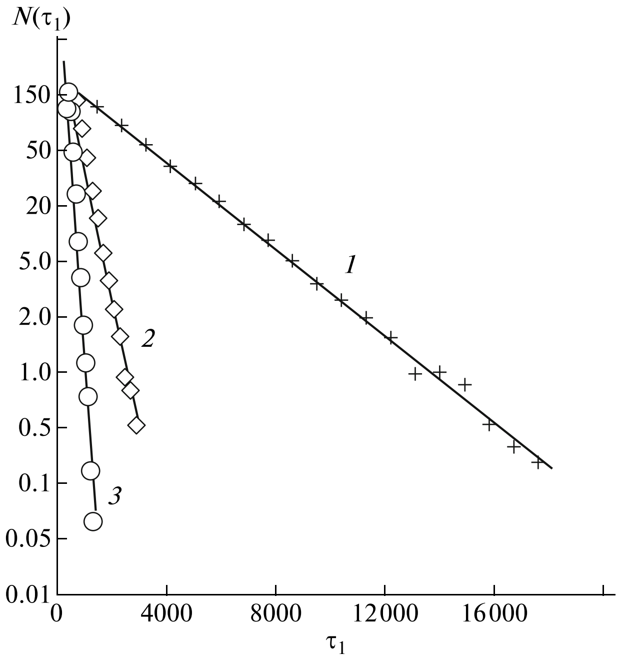

Fig. 2. Distributions $N(\tau_1)$ of the laminar interval duration τ_1 (points) and the corresponding exponential approximations (solid lines) for the system under consideration with various values of the coupling parameter ε: (*1*) $\varepsilon = 0.036$ (+), $N(\tau_1) = 213.6\exp(-4 \times 10^{-4}\tau_1)$; (*2*) $\varepsilon = 0.032$ (◇), $N(\tau_1) = 368.8\exp(-2 \times 10^{-3}\tau_1)$; (*3*) $\varepsilon = 0.028$ (○), $N(\tau_1) = 1211.3\exp(-7 \times 10^{-3}\tau_1)$. Ordinates are plotted on a logarithmic scale.

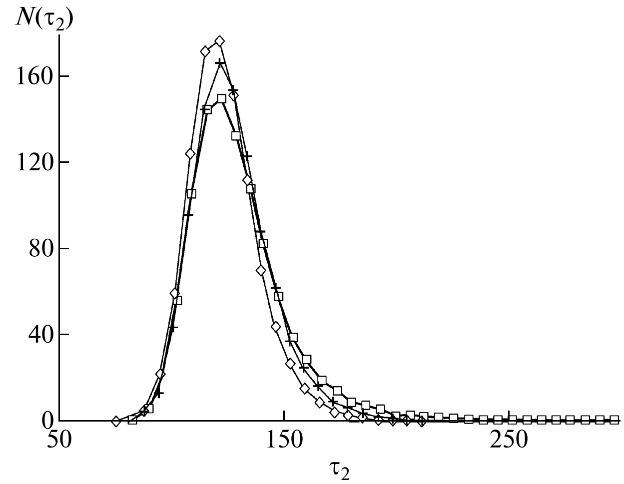

Fig. 3. Distributions $N(\tau_2)$ of the laminar interval duration τ_2 (points) and the corresponding approximations (solid curves) for the system under consideration with various values of the coupling parameter $\varepsilon = 0.036$ (□); 0.032 (+); 0.028 (◇). All $N(\tau_2)$ profiles are close to Gasussian distributions.

system behavior would fall approximately at the middle of their possible range (Fig. 1c). In this case, the beginning and end of each interval with turbulent dynamics can be detected as the point where $\Delta\varphi(t)$ intersects certain preset threshold values of $\Delta\varphi_{min}$ and $\Delta\varphi_{max}$ (in the given example, $\Delta\varphi_{min} \approx -3$ and $\Delta\varphi_{max} \approx 0$). Note that, as was pointed out above, all fluctuations in the phase difference (which hinder the exact detection of the beginning and end of the turbulent intervals) in this case are also localized within the interval $[\Delta\varphi_{min}, \Delta\varphi_{max}]$ corresponding to the laminar behavior. This eliminates difficulties in the determination of boundaries of the intervals of synchronous and asynchronous dynamics.

It should be emphasized that the proposed approach is based on a direct analysis of the instantaneous phases and does not employ additional transformations (such as the averaging over a sliding window, continuous wavelet transform, etc.) for the phase difference $\Delta\varphi(t)$, which could significantly increase the computation time.

The proposed method was verified by determining the distributions of laminar interval durations τ_1 for the coupled subsystems (1) and (2) with various values of the coupling parameter ε. As can be seen from the data presented in Fig. 2, the distributions $N(\tau_1)$ of laminar intervals determined as described above obey the exponential law, in complete agreement with the well-known rule established previously [2, 7].

We have also used the proposed method to separate turbulent intervals for subsystems (1) and (2) with the same coupling parameter. The resulting distributions of the durations τ_2 of these intervals are presented in Fig. 3, from which it is seen that the $N(\tau_2)$ profiles are close to Gasussian distributions.

In conclusion, a new method is proposed for separating the intervals of laminar and turbulent behavior in the time series of interacting chaotic oscillators occurring near the boundary of the phase synchronization regime. This method requires no additional transformations of the analyzed quantity, which significantly simplifies the procedure of detecting laminar and turbulent intervals in the behavior of coupled systems under consideration. The results, obtained using the proposed method, are in very good agreement with theoretical estimations.

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