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THEORETICAL AND MATHEMATICAL PHYSICS

Synchronization of Spectral Components in Unidirectionally Coupled Systems

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Abstract—The object of investigation is synchronization of spectral components in unidirectionally coupled chaotic systems. It is shown that the behavior of the spectral components in this case is governed by the amount of detuning between interacting systems. Universal laws of spectral component synchronization are discovered.

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INTRODUCTION

Synchronization of chaotic oscillations is today viewed as a fundamental phenomenon in the contemporary theory of oscillations and waves and in nonlinear dynamics [1]. This phenomenon is of great theoretical and applied interest, e.g., in hidden data transmission using deterministic chaotic signals [2–4]; in biological [5, 6], physiological [7–9], and chemical [1–12] problems; and for chaos control including in microwave electronic systems [13–16].

Several types of the synchronous behavior of unidirectionally coupled chaotic systems have been discovered to date, each with its own basic features. These are phase synchronization [17], generalized synchronization [18], lag synchronization [20], and others. In recent years, interaction between these types of synchronous behavior has become an issue of hot discussion. Attempts have been made to study phase and complete chaotic synchronizations in terms of information theory [21, 22]. In a number of works (see, e.g., [23, 24]), different types of synchronizations of coupled chaotic oscillators are treated as different aspects of a general law inherent in coupled nonlinear systems. Koronovskii and Hramov [25–27] suggested that the synchronous behavior of chaotic oscillators be described by considering their behavior on different time scales. Such an approach, called the time scale synchronization, in a natural way generalizes the above types of the synchronous behavior.

Another approach to considering different types of chaotic synchronizations from a unified standpoint is investigation into the synchronization of the spectral components of the Fourier spectra of interacting systems. In works of our research team [28–30], it was shown that different types of synchronizations in mutually coupled systems (phase synchronization, lag synchronization, and complete synchronization) are partial cases of the spectral component synchronization. However, similar investigations into unidirectionally coupled systems have not been conducted so far. Such systems are very suitable, e.g., for analysis of the generalized chaotic synchronization and its relation with other types of synchronizations (specifically, with phase synchronization). It is known that mechanisms responsible for different regimes are also different and depend on the amount of frequency detuning [31–34]. It is therefore of interest to see whether the approach based on the spectral component synchronization is efficient as applied to unidirectionally coupled systems at different values of the frequency detuning. This point is studied in this work.

DIFFERENT TYPES OF CHAOTIC SYNCHRONIZATIONS AND THEIR INTERRELATION IN TERMS OF THE SPECTRAL COMPONENT SYNCHRONIZATION

Consider the behavior of two unidirectionally coupled chaotic systems

$$\begin{aligned} \dot{\mathbf{x}}_d &= \mathbf{H}(\mathbf{x}_d, \mathbf{g}_d), \\ \dot{\mathbf{x}}_r &= \mathbf{G}(\mathbf{x}_r, \mathbf{g}_r) + \varepsilon \mathbf{P}(\mathbf{x}_d, \mathbf{x}_r), \end{aligned} \tag{1}$$

where $\mathbf{x}_{d,r} = (x^1_{d,r}, x^2_{d,r}, x^3_{d,r})$ are the vectors of state of the driving and responding systems, respectively; $\mathbf{H}$ and $\mathbf{G}$ stand for the vector field of the systems; $\mathbf{g}_d$ and $\mathbf{g}_r$ are the vectors of parameters; $\mathbf{P}$ determines unidirectional coupling between the systems; and ε determines the strength of coupling between the systems (coupling parameter).

It is known that, as coupling parameter ε increases, the synchronization conditions may vary in the following sequence: chaotic phase synchronization, generalized synchronization, lag synchronization, and complete synchronization. Note that phase synchronization may arise both before and after generalized synchronization depending on the detuning between control parameters $\mathbf{g}_d$ and $\mathbf{g}_r$ [31, 35].

Under the conditions of chaotic phase synchronization, the phases of interacting systems, when introduced in a conventional way [17], become coupled, while the amplitudes of time realizations for these systems generally remain fully uncoupled. The conditions of generalized synchronization feature a unique functional dependence $\mathbf{F}[\cdot]$ between the states of the driving and responding systems, $\mathbf{x}_r(t) = \mathbf{F}[\mathbf{x}_d(t)]$. This function may be both smooth (strong generalized synchronization) and fractal (weak generalized synchronization) [36]. The conditions of complete chaotic synchronization and lag synchronization are the strong forms of generalized synchronization: in the latter case, the states of interacting systems are identical but shifted in time by interval τ (that is, $\mathbf{x}_r(t) = \mathbf{x}_d(t - \tau)$, while in the former case, the states of the systems are almost coincident ($\mathbf{x}_d(t) \approx \mathbf{x}_r(t)$).

To describe the above types of chaotic synchronizations in unidirectionally coupled systems in terms of the spectral component synchronization, let us consider, by analogy with [29], time realizations generated by the driving and responding chaotic oscillators. As for bidirectionally coupled systems, the Fourier spectra of the driving and responding oscillators under the conditions of lag synchronization,

$$S_{d,r}(f) = \int_{-\infty}^{+\infty} x^j_{d,r}(t)e^{-i2\pi ft}dt \quad (j = 1, 2, 3), \tag{2}$$

are related to each other by the relationship

$$S_r(f) \approx S_d(f)e^{-i2\pi\tau f}.$$

For difference $\Delta\phi_f$ between instantaneous phases corresponding to spectral component f of Fourier spectra $S_{d,r}(f)$, the following relationship is valid:

$$\Delta\phi_f = \phi_{f_d}(t) - \phi_{f_r}(t) = 2\pi\tau f. \tag{3}$$

Thus, as in the case of bidirectionally coupled systems, points corresponding to the phase difference between the spectral components of chaotic oscillators experiencing lag synchronization on plane (f, $\Delta\phi_f$) must align, forming a straight line with slope $k = 2\pi\tau$. If the chaotic systems are identical, the conditions of complete synchronization ($\tau = 0$) will set in; accordingly, the slope of the straight line on plane (f, $\Delta\phi_f$) will equal zero.

When the lag synchronization regime breaks (for example, the parameter of coupling between the oscillators decreases), some of the spectral components of the Fourier spectra will be out of synchronism and so the points on plane (f, $\Delta\phi_f$) will deviate from the straight line. Yet, quite a few questions are to be solved for unidirectionally coupled systems. Specifically, (i) how the conditions of generalized synchronization and phase synchronization will show up in the case of a small and large detuning between the control parameters?, (ii) how differently will the spectral components behave in these two cases?, and (iii) how different relations between the generalized synchronization and phase synchronization can be explained in the case of a small and large detuning? Let us consider particular cases to answer these questions.

CHAOTIC SYNCHRONIZATION IN UNIDIRECTIONALLY COUPLED RESSLER SYSTEMS

To illustrate synchronization between spectral components in unidirectionally coupled systems, consider the behavior of two Ressler systems, which feature phase-coherent chaotic attractors in the autonomous regime [35],

$$\begin{aligned} \dot{x}_d &= -\omega_d y_d - z_d, \\ \dot{y}_d &= \omega_d x_d + a y_d, \\ \dot{z}_d &= p + z_d(x_d - c); \\ \\ \dot{x}_r &= -\omega_r y_r - z_r + \varepsilon(x_d - x_r), \\ \dot{y}_r &= \omega_r x_r + a y_r, \\ \dot{z}_r &= p + z_r(x_r - c). \end{aligned} \tag{4}$$

Here, subscripts "d" and "r" refer to the driving (autonomous) and responding systems, respectively, and $a = 0.15$, $p = 0.2$, and $c = 10$ are control parameters, the numerical values of which were taken from [31]. Parameters $\omega_{d,r}$ are the eigenfrequencies of the driving and responding systems, respectively. For the responding system, this parameter is set constant, $\omega_r = 0.95$; for the driving system, it will be varied in the range $\omega_d = 0.80$–1.12 so as to detune the interacting oscillators.

Figure 1 outlines the domains of phase synchronization, generalized synchronization, and lag synchronization for system (4) on the parameter plane (ω_d, ε). It is easy to see that the conditions of lag synchronization always set in after generalized synchronization and phase synchronization, while a relation between the last two depends on the detuning of the interacting systems. To gain insight into the synchronous regimes on the "detuning–strength of coupling" parameter plane, let us see how the spectral components synchronize separately for a small and large detuning.

Large Detuning of Control Parameters

Consider first the case of a large frequency detuning; that is, parameter ω_d is set equal to 0.99. As follows from Fig. 1, the phase synchronization follows the generalized synchronization for the given values of the control parameters (see also [35]). At the same time, since the detuning is large, synchronization (more specifically, time-scale synchronization) arises before the phase synchronization [37]. In addition

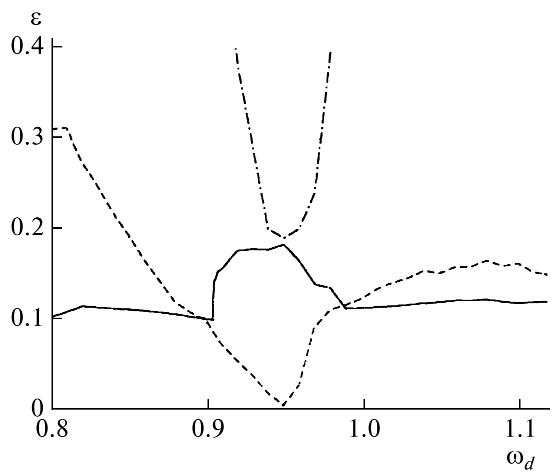

Fig. 1. Domains of phase synchronization (dashed line), generalized synchronization (continuous line), and lag synchronization (dash-and-dot line) in unidirectionally coupled Ressler systems (4) on parameter plane (ω_d, ε).

(see above), the lag synchronization takes place if coupling parameter ε is fairly high. The values of the coupling parameter corresponding to the beginning of the synchronous conditions are the following: $\varepsilon_{TSS} = 0.1$ for the time-scale synchronization, $\varepsilon_{GS} = 0.112$ for the generalized synchronization, $\varepsilon_{PS} = 0.118$ for the phase synchronization, and $\varepsilon_{LS} \approx 0.50$ for the lag synchronization.

Let us see how the changeover of synchronization conditions shows up in terms of Fourier spectra. As a quantitative parameter characterizing the degree of synchronization, we take the number of spectral components of Fourier spectra $S_{d,r}(f)$ that are in synchronism [29],

$$\sigma = \frac{1}{N}\sum_{j=1}^{N}(\Delta\phi_{f_j} - 2\pi\tau f_j)^2. \quad (5)$$

Here, summation is over all the spectral components of Fourier spectra $S_{d,r}(f)$ and time lag τ is defined as a time shift between the Fourier spectral components accounting for a major part of the oscillation energy. Under the conditions of complete synchronization and lag synchronization, σ must tend to zero. If these conditions are violated because of a decrease in the coupling parameter, σ will grow with increasing the number of the spectral components of Fourier spectra $S_{d,r}(f)$ that are out of synchronism.

Figure 2a plots σ against the coupling parameter. It is seen that σ tends to zero at high ε, as in the case of bidirectional coupling, which means that the lag synchronization regime is established in system of oscillators (4). However, the run of the σ–ε dependence somewhat differs from that for the bidirectional coupling [29, 30]. At $\varepsilon < 0.2$ (arrow in Fig. 2a), σ drops sharply, while at $\varepsilon \geq 0.2$, it decreases gradually. The reason for such behavior is elucidated in Figs. 2b–2g,

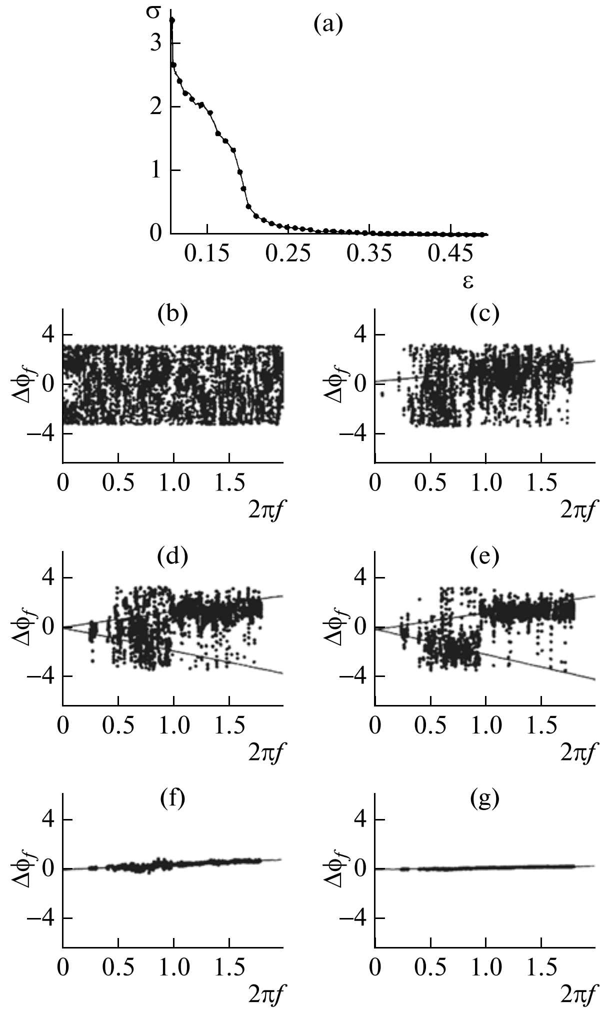

Fig. 2. (a) σ vs. coupling parameter ε and (b–g) phase difference $\Delta\phi_f$ for different spectral components f of Fourier spectra $S_{d,r}(f)$ of coupled Ressler systems (4) for the case of a large eigenfrequency detuning and different values of coupling parameter ε. (b) Asynchronous dynamics, ε = 0.02; (c) time-scale synchronization, ε = 0.1; (d) generalized synchronization, ε = 0.112; (e) phase synchronization, ε = 0.118; (f) ε = 0.25; and (g) lag synchronization, ε = 0.52.Straight lines on panels "c–g" correspond to synchronized spectral components.

which show the values of phase differences between spectral components of the Fourier spectra for unidirectionally coupled Ressler oscillators (4). Before synchronization (ε = 0.02), all spectral components are out of synchronism; that is, all points on plane (f, $\Delta\phi_f$) are distributed randomly (Fig. 2b). This situation is the same as in the case of bidirectional coupling. With an increase in the coupling parameter, the spectral components successively synchronize. At ε = 0.1 (Fig. 2c), one spectral component synchronizes (the basic spectral component of the driving oscillator, which shows up in the appearance of a "phase concentration" near this frequency). At this instant, the time-scale synchronization occurs.

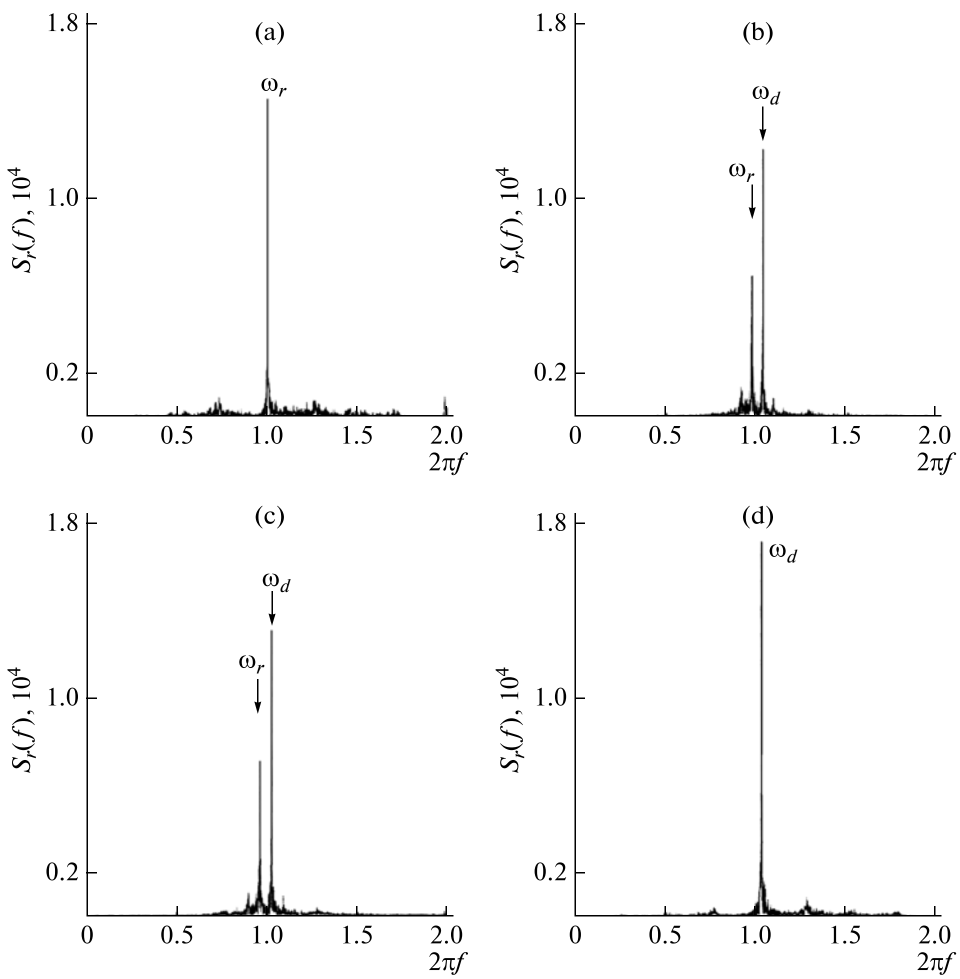

Fig. 3. Fourier spectra of the responding Ressler oscillator from (4) at ε = (a) 0, (b) 0.112, (c) 0.118, and $d = 0.200$. The spectral components at the eigenfrequencies of the driving, ω_d, and responding, ω_r, systems are shown by arrows.

The occurrence of the generalized synchronization at the given values of the control parameters gives rise to two synchronized spectral components at the fundamental frequencies of the driving and responding oscillators (for details, see [33]). It is seen from Fig. 2d (here, $\varepsilon = 0.112$, at which the generalized synchronization is established in system of oscillators (4)) that the spectral components in this case align, forming two straight lines with slopes corresponding to time shifts

$$\tau_{d,r} = \Delta\phi_f(\omega_{d,r})/\omega_{d,r}$$

between the fundamental spectral components of the driving and responding oscillators.

The Fourier spectrum of the responding system in this case contains two clear-cut spectral components at the frequencies mentioned above (Fig. 3b; cf. Fig. 3a, where the same spectrum is shown in the autonomous regime, when $\varepsilon = 0$). When the phase synchronization sets in ($\varepsilon = 0.118$, Fig. 2e), the Fourier spectrum of the responding system still contains two fundamental components but their intensity changes (Fig. 3c). In this case, the phase synchronization makes the chaotic attractor of the responding system phase-coherent (phase coherency is lost at $\varepsilon \approx 0.11$ [32, 34]). A further increase in the coupling parameter "suppresses" the spectral component at the "eigenfrequency" of the responding system. Thus, only the clear-cut component at the fundamental frequency of the driving system is left in the Fourier spectrum (Fig. 2d). Such a situation happens at $\varepsilon \geq 0.2$, which explains the change in the run of the $\sigma(\varepsilon)$ dependence. Now, the spectral components are expected to align, and an increase in the coupling parameter results in their successive synchronization (by analogy with bidirectional coupling). Thus, Fig. 2f ($\varepsilon = 0.25$) refers to the conditions of fairly strong phase synchronization when most spectral components are synchronized and Fig. 2g ($\varepsilon = 0.52$) illustrates the conditions of lag synchronization, in which case all spectral components are synchronized.

Small Detuning of Control Parameters

Consider now the case when the control parameters of interacting oscillators (4) are detuned insignificantly. Parameter ω_d of the driving system is set equal to 0.97, with the values of the remaining control parameters being the same. We will study the transition from the asynchronous state to the state of lag synchronization in system of oscillators (4).

The variation of the spectral components with coupling parameter ε is presented in Fig. 4. By analogy with Fig. 2, Fig. 4a plots σ versus the coupling intensity and this curve suggests that the lag synchronization arises in the system at $\varepsilon_{LS} \approx 0.24$. This value is in good agreement with the lag synchronization boundary shown in Fig. 1. Also, it follows from Fig. 4 that here the phase synchronization sets in at $\varepsilon_{PS} = 0.092$, while the generalized synchronization takes place at $\varepsilon \geq \varepsilon_{GS} = 0.139$; that is, the phase synchronization arises before the generalized synchronization.

When the eigenfrequencies are detuned only slightly, the behavior of the spectral components is qualitatively akin to the case of bidirectional coupling between the oscillators [29, 30]. With asynchronous dynamics present in the system ($\varepsilon = 0.01$), all spectral components (points on plane $(f, \Delta\phi_f)$) are randomly distributed within the interval $[-\pi, \pi]$ (see Fig. 4b). At $\varepsilon = 0.092$ (Fig. 4c), the eigenfrequencies are locked (phase concentration on plane $(f, \Delta\phi_f)$) and the phase synchronization sets in (in this case, the appearance thresholds of the phase synchronization and time-scale synchronization nearly coincide [37]). At $\varepsilon = 0.14$, the generalized synchronization occurs. It should be noted that here the mechanisms responsible for the generalized synchronization somewhat differ: the generalized synchronization is due to the synchronization of the fundamental frequency of the driving system with its subharmonics [33]. Since the intensities of subharmonics are low, the phase locking at these frequencies is difficult to detect (Fig. 4d); consequently, the generalized synchronization shows up in almost the same way as the phase synchronization in terms of spectral components (cf. Figs. 4c, 4d). At $\varepsilon = 0.24$, the lag synchronization arises, as follows from the alignment of the spectral components on plane $(f, \Delta\phi_f)$ (Fig. 4e).

Thus, the changeover from one type of synchronization to another in unidirectionally coupled chaotic systems also results in synchronization of their Fourier spectral components. However, the spectral components behave identically only in the cases of the asynchronous dynamics and lag synchronization, since their underlying mechanisms are the same both at different detunings of the eigenfrequencies and at different types of couplings between the systems. In terms of Fourier spectra, the appearance of the generalized synchronization and phase synchronization in unidirectionally coupled systems depends on the detuning between the eigenfrequencies of interacting systems. If the detuning is small, the phase synchronization arises via locking of fundamental spectral components. In this case, only one spectral component turns out to be synchronized. As the degree of coupling between the oscillators grows, the subharmonics of the fundamental frequency also synchronize and the generalized synchronization regime sets in.

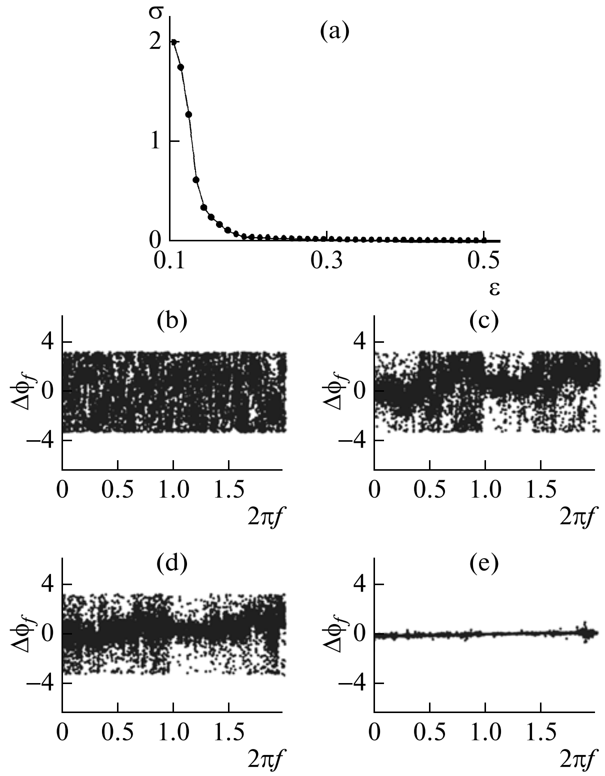

Fig. 4. (a) σ vs. coupling parameter ε and (b–e) phase difference $\Delta\phi_f$ for different spectral components f of Fourier spectra $S_{d,r}(f)$ of coupled Ressler systems (4) for the case of a small eigenfrequency detuning and different values of coupling parameter ε. (b) Asynchronous dynamics, $\varepsilon = 0.01$; (c) phase synchronization, $\varepsilon = 0.092$; (d) generalized synchronization, $\varepsilon = 0.14$; and (e) lag synchronization, $\varepsilon = 0.30$.

In the case of a large detuning, the synchronization of one spectral component means that the time-scale synchronization regime is established. When the second spectral component synchronizes, the generalized synchronization occurs. In this case, the Fourier spectrum of the responding oscillator contains two clear-cut components and all points on plane $(f, \Delta\phi_f)$ align, forming two straight lines, the slopes of which correspond to time shifts between these components. The presence of two clear-cut components in the Fourier spectrum of the responding system indicates that its attractor is phase-incoherent. Under such conditions, the phase synchronization cannot be detected. When the coupling parameter grows, the intensity of the spectral component at the frequency of the responding system drops and its chaotic attractor becomes phase-coherent again. In this case, the phase synchronization can again be detected by conventional methods, which means the establishment of the phase synchronization at a large detuning [32, 34].

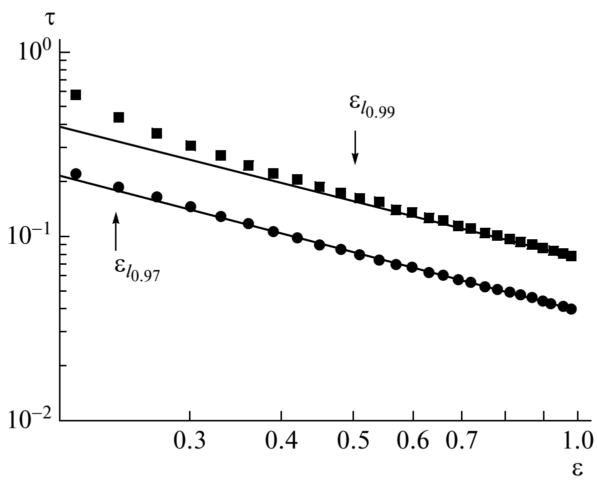

Fig. 5. Time shift τ between the main components of the Fourier spectra for unidirectionally coupled Ressler systems (4) vs. coupling parameter ε in the case of (■) high ($\omega_d = 0.99$) and (●) small ($\omega_d = 0.97$) detunings of the eigenfrequencies. Continuous curves correspond to power laws $\tau = k\varepsilon^n$ with $n = -1$ ($k = 0.04$ at $\varepsilon = 0.97$ and $k = 0.08$ at $\varepsilon = 0.99$). The values of the coupling parameter, $\varepsilon_{l_{0.99}} = 0.5$ and $\varepsilon_{l_{0.97}} = 0.24$, that correspond to the lag synchronization threshold are shown by arrows. It is seen that a universal power law takes place upon the lag synchronization conditions are established.

UNIVERSAL LAWS IN SPECTRAL COMPONENT SYNCHRONIZATION

Let us synopsize universal laws encountered in spectral component synchronization. It was shown both analytically and numerically [28] that, in the case of lag synchronization, the time shift between frequency components (this shift is the same for all frequencies) depends on the coupling parameter by the law $\tau \propto \varepsilon^{-1}$, this law being valid both for unidirectionally and bidirectionally coupled systems. To corroborate the universality of this power law, we will show that it works both at small and at large detunings of the eigenfrequencies.

Figure 5 plots the time shift between synchronized spectral components versus coupling parameter ε for large and small eigenfrequency detunings (for their values, see above) and respective approximations. It is seen that the universal power law is observed in both cases after the lag synchronization sets in.

CONCLUSIONS

We studied spectral component synchronization in unidirectionally coupled chaotic systems. It is shown that different types of chaotic synchronizations can be viewed as particular cases of spectral component synchronization, the behavior of spectral components under the conditions of lag synchronization and complete synchronization being independent of the frequency detuning between the systems. In these cases, the dependence of the time shift between the frequency components of the interacting systems on the coupling parameter follows a power law.

At large and small frequency detunings, the phase synchronization and the generalized synchronization show up in a different way, since their underlying mechanisms are different. If the frequency detuning between the interacting systems is small, the phase synchronization arises because of the synchronization of only the spectral component at the eigenfrequency of the driving system. The generalized synchronization sets in when several spectral components at the eigenfrequency and its subharmonics synchronize. Since the generalized synchronization is due to the synchronization of a large number of spectral components, it must follow the phase synchronization.

When the detuning is large, the fundamental frequencies are not locked but two spectral components at the fundamental frequency of the driving system and at the eigenfrequency of the responding one may synchronize. In this case, the generalized synchronization sets in and the phase synchronization becomes impossible to detect because the chaotic attractor loses phase coherence. An increase in the strength of coupling between the systems suppresses the spectral component at the frequency of the responding system and the attractor becomes phase-coherent again; as a result, the phase synchronization arises. Therefore, when the detuning of the eigenfrequencies is large, the phase synchronization follows the generalized synchronization.

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