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Experimental Study of the Generalized Synchronization of Chaotic Oscillations in the Presence of Noise

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Abstract—The influence of an external source of noise on the position of the boundary of generalized synchronization (GS) of two unidirectionally coupled chaotic oscillators has been experimentally studied for the first time. It is established that the GS threshold remains unchanged in a broad range of the external noise intensity, i.e., the GS regime is stable with respect to external noise.

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Synchronization of chaotic oscillations in dynamical systems is among the phenomena of nonlinear dynamics that are most extensively studied in recent years [1, 2]. One of the interesting types of synchronous dynamics in chaotic systems is the regime of generalized synchronization (GS) [3–5]. Another important issue is the effect of noise on the synchronization of chaos. The number of reported investigations of the influence of noise on the GS of chaotic oscillations is still small [6–8], although this regime can be successfully used, particularly in systems of hidden data transmission [9]. Therefore, the problem of stability of the generalized chaotic synchronization with respect to external noises is of considerable importance.

This Letter presents the results of the first experimental investigation of the effect of external noise on the threshold of GS in a system of two unidirectionally, dissipatively coupled chaotic oscillators implemented in radio-engineering circuits.

As is known, a system of unidirectionally coupled chaotic oscillators occurs in the regime of GS provided that, upon the termination of the transient process, the states $\mathbf{x}$ and $\mathbf{y}$ of the coupled subsystems are related as $\mathbf{x} = F(\mathbf{y})$, where F is a certain function[3, 4]. A convenient procedure for the diagnostics of the GS regime is offered by the auxiliary system method [10], which is based on the introduction of an additional system $\mathbf{v}$ that is identical to the slave (response) system $\mathbf{x}$ and occurs under the action of the same master (drive) signal. Then, in the absence of synchronization, the dynamics of the response ($\mathbf{x}$) and auxiliary ($\mathbf{v}$) systems are different, whereas the presence of a synchronizing chaotic signal leads to the establishment of identical dynamics whereby $\mathbf{x}(t) = \mathbf{v}(t)$.

In order to obtain some theoretical estimates, let us consider the behavior of two unidirectionally coupled dynamical systems with slightly different parameters in the presence of an external noise $\xi(t)$:

$$\dot{\mathbf{y}}(t) = \mathbf{H}(\mathbf{y}(t), \mathbf{g}_d),$$

$$\dot{\mathbf{x}}(t) = \mathbf{H}(\mathbf{x}(t), \mathbf{g}_r) + \varepsilon\mathbf{A}(\mathbf{y}(t) - \mathbf{x}(t) + D\xi(t)),$$

where $\mathbf{y} = (y_1, y_2, y_3)$ and $\mathbf{x} = (x_1, x_2, x_3)$ are the state vectors of the master (drive) and slave (response) systems, respectively; $\mathbf{H}$ is the vector field of these systems; $\mathbf{g}_d$ and $\mathbf{g}_r$ are the vectors of parameters of the drive and response systems, respectively; $\mathbf{A} = \{\delta_{ij}\}$ is the coupling matrix ($\delta_{ii} = 0$ and $\delta_{ij} = 0$ or 1); ε is the coupling parameter; and D is the noise intensity.

In order to elucidate mechanisms leading to the appearance of GS in the system under consideration, let us use the modified system approach [4]. According to this, the response system is considered as a modified system with the additional dissipation $-\varepsilon\mathbf{A}\mathbf{x}(t)$,

$$\dot{\mathbf{x}}_m(t) = \mathbf{H}'(\mathbf{x}_m(t), \mathbf{g}_r, \varepsilon) = \mathbf{H}(\mathbf{x}(t), \mathbf{g}_r) - \varepsilon\mathbf{A}\mathbf{x}(t),$$

which occurs under the external action $\varepsilon\mathbf{A}(\mathbf{y}(t) + D\xi(t)]$. The GS regime in the system under consideration can be considered as resulting from the two simultaneous interrelated processes of increase in the (i) dissipation in the modified system and (ii) amplitude of the external (chaotic and noise) signal. These processes are related via parameter ε and cannot take place separately in the response system. However, increasing dissipation in the modified system leads to a simplification of its behavior, which is manifested by the passage from chaotic to periodic oscillations. On the contrary, the external action tends to complicate the behavior of the modified system by imparting it the dynamics of the drive system. The GS regime can only be established provided that the intrinsic chaotic dynamics in the response system is suppressed due to the dissipation. Thus, the stability of the GS regime is determined primarily by the properties of the modified system. Therefore, a threshold for the onset of the GS

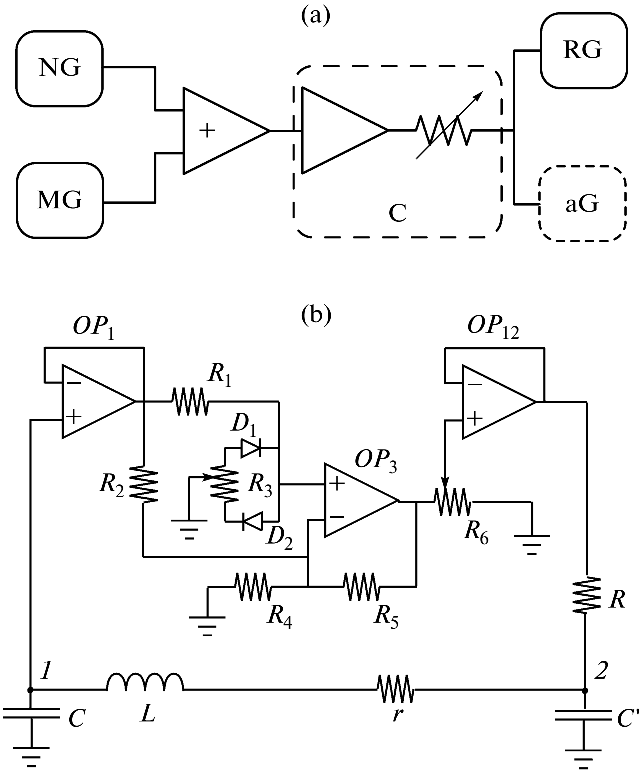

Fig. 1. (a) General schematic diagram of the experimental setup: (MG) master generator; (SG) slave generator; (aG) auxiliary generator; (NG) noise generator; (C) coupling device; (+) inverting adder; (b) circuit diagram of the base generator: (OP_1, OP_2) TLo82; (OP_3) K174UD7; (D_1, D_2) 1N4148; R_1, R_2 = 7.2 kΩ; R_3 = 100 Ω; R_4 = 180 kΩ; R_5 = 12 kΩ; R_6 = 4.7 kΩ; (RC', rLC) low-frequency filters; r = 56 Ω; R = 630 Ω; C = 330 nF; C' = 150 nF; L = 3.3 mH; output signals were measured at points *1* (input of the slave generator) and *2*.

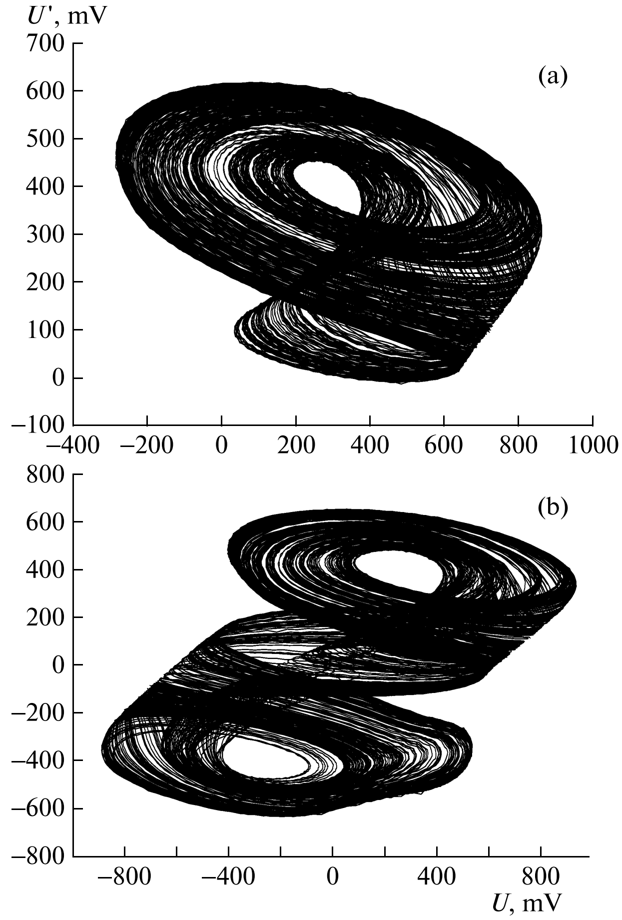

Fig. 2. Phase portraits of characteristic oscillation regimes in the radio circuit under consideration with (a) a ribbon attractor (R_6 = 2 kΩ) and (b) double scroll attractor (R_6 = 1.5 kΩ).

regime must not strongly depend on the intensity of noise $D\xi(t)$ that acts on the unidirectionally coupled chaotic systems. Once the noise does not significantly alter the characteristics of the modified system, it will not strongly influence the GS threshold either.

We have experimentally studied the effect of external noise on the threshold of GS onset in a system of two unidirectionally coupled chaos generators using a setup schematically depicted in Fig. 1a. The base element was a chaotic oscillator described in [11, 12], a circuit diagram of which is presented in Fig. 1b. This circuit comprises a nonlinear amplifier and a linear feedback chain involving two low-frequency filters. The characteristic oscillation frequency was $f \approx 8$ kHz. The GS was studied in a system of two unidirectionally coupled generators, where the coupling coefficient was defined as $\varepsilon = 1/R\sqrt{L/C}$. The base oscillator was connected to a DAC–ADC board L-783 (LCard) built in a personal computer, which was used to monitor the voltage dynamics on C and C' capacitors. The synchronizing signal was formed from the signal generated in the circuit, which was digitized by the ADC and then reproduced via the DAC. This synchronizing signal was fed into the system under consideration via a unidirectional dissipative coupling chain (C in Fig. 1a). The external noise signal represented a δ-correlated noise.

The radio-engineering circuit under consideration can generate chaos in the two main regimes, which are characterized by attractors of the ribbon (Fig. 2a) and double scroll (Fig. 2b) types. We have experimentally studied three cases: (i) both master and slave subsystems generated chaos in the regimes with attractors of the ribbon type; (ii) same with both attractors of the double scroll type; and (iii) with a ribbon attractor in the master subsystem and a double scroll attractor in the slave system. In each case, we first determined the values of the coupling parameter corresponding to the onset of GS in the absence of noise. The length of the analyzed time series was 3 s, which corresponded to $N \approx 3 \times 10^4$ characteristic oscillation periods [13, 14]. It was established that the indicated combinations of regimes in the drive and response subsystems, the

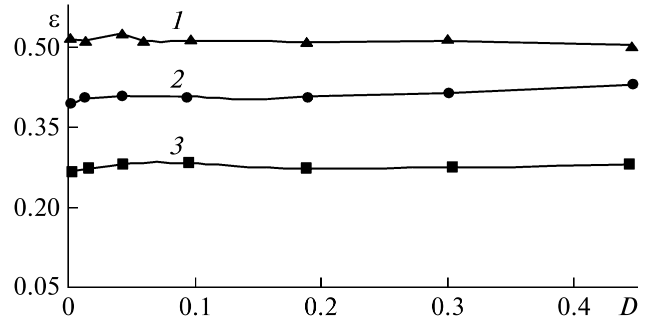

Fig. 3. Plots of coupling parameter ε corresponding to GS onset versus relative noise intensity (noise to signal ratio) D for unidirectionally coupled generators occurring in regimes with various attractors: (i) both attractors of ribbon type; (ii) both attractors of double scroll type; (iii) ribbon attractor in master subsystem and double scroll attractor in the slave system.

boundary of the GS domain corresponded to (i) $\varepsilon = 0.52$, (ii) $\varepsilon = 0.28$, and (iii) $\varepsilon = 0.38$, respectively.

In order to determine the effect of external noise on the GS threshold, the experiments were repeated with various levels of noise introduced into the coupling device. The noise signal intensity was evaluated as the ratio of the noise power P_N to the power P_{MG} of oscillations in the master generator: $D = P_N/P_{MG}$. Figure 3 shows plots of the coupling parameter ε corresponding to the GS onset versus relative noise intensity D in the three cases studied. As can be seen, the boundary of the GS domain (expressed in units of the coupling parameter) remains approximately constant when the noise intensity varies in a broad range. At the same time, the GS threshold significantly depends on the regime (i.e., parameters) of the drive and response system, in agreement with the results of a theoretical analysis of the position of the GS domain boundary in the system without noise [5]. It should also be noted that the further increase in the noise intensity D leads to breakage of the chaotic oscillation regime and the transition to a stationary state.

In conclusion, we have experimentally studied for the first time the effect of external noise on the position of the boundary of the domain of GS for two unidirectionally coupled chaotic oscillators implemented in radio engineering circuits. It is established that the GS threshold remains unchanged in a broad range of the external noise intensity.

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