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SOLITONS AND CHAOS

Experimental Study of the Time-Scale Synchronization in the Presence of Noise

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Abstract—The time-scale synchronization of chaotic oscillations in two dissipatively coupled radio-frequency chaotic oscillators has been experimentally studied. The effect of noise on the efficiency of chaotic synchronization diagnostics is analyzed and a high stability of time-scale synchronization to noise in the coupling channel between the oscillators is shown.

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1. INTRODUCTION

Synchronization of chaotic oscillations is a fundamental phenomenon in the nonlinear theory of oscillations and waves; it manifests itself in both physical [1, 2] and biological [3, 4] systems. The applications of chaotic synchronization in telecommunication systems [5], systems for analyzing the behavior of living organisms (treatment and interpretation of neurophysiological data [6, 7] and data on the cardiovascular system [4, 8, 9]), and in the dynamics of chemical oscillators [10] are of interest.

Several main synchronization types have been selected for coupled chaotic oscillators, which were studied both theoretically and experimentally: phase, complete, and generalized synchronizations [1]. Phase synchronization is considered most frequently [11]; it occurs for two coupled chaotic oscillators when the difference in the instantaneous phases $\phi(t)$ of chaotic signals $x_{1,2}(t)$ remains time-limited: $|\phi_1(t) - \phi_2(t)| \leqslant \text{const}$.

There is no universal way of introducing an instantaneous phase of chaotic signal. For good dynamic systems with a relatively simple topology of chaotic attractor, whose Fourier spectrum contains one fundamental frequency component f_0, several methods exist, which make it possible to determine the instantaneous phase $\phi(t)$ of chaotic signal. For systems with a phase-incoherent attractor, phase synchronization often leads to incorrect results [12]. It is generally difficult to diagnose the phase synchronization of such systems. This situation is typical of noise systems (where the presence of nondeterministic signal leads to loss of attractor phase coherence) and systems characterized by several time scales of dynamics.

A new approach to introducing the concept of chaotic synchronization of coupled chaotic oscillators—time-scale synchronization—was proposed in [13–15]. To describe the behavior of complex chaotic dynamic systems, whose Fourier spectrum does not contain a pronounced component, it was proposed to analyze their behavior on different time scales. It was numerically shown that this method is efficient for analyzing systems with both several characteristic time scales and a high noise level. In this paper, we report the results of experimental studying the time-scale synchronization of chaotic oscillations by the example of a system of dissipatively and unidirectionally coupled radio-frequency chaotic oscillators.

2. TIME-SCALE SYNCHRONIZATION

Let us briefly recall the basic concept of time-scale synchronization. For a signal $g(t)$ under study, which is assumed to be generated by an oscillator system (this can be a model dynamic system or an experimental time series of a physical or biological system), time scales s (which correspond to frequencies f of the Fourier spectrum) and associated phases $\phi_s(t)$ are introduced. The latter means that some continuous set of phases $\phi_s(t)$ of chaotic signal, which characterize the behavior of the signal time scales, is considered for it.

This continuous set of phases is introduced by applying a continuous wavelet transformation of the time series $g(t)$ with a complex basis [16]. This transformation is implemented using the convolution

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$$W_{(s,t_0)} = \int_{-\infty}^{+\infty} x(t)\,\psi^*_{s,t_0}(t)\,dt, \qquad (1)$$

where $\psi_{s,t_0}(t)$ is a wavelet function, derived from the mother wavelet ψ_0:

$$\psi_{s,t_0}(t) = \frac{1}{\sqrt{s}}\,\psi_0\left(\frac{t-t_0}{s}\right). \qquad (2)$$

The time scale s determines the wavelet $\psi_{s,t_0}(t)$ width, t_0 is the time shift of the wavelet function along the time axis, and the asterisk indicates complex conjugation. The Morlet wavelet

$$\psi_0(\eta) = \frac{1}{\sqrt[4]{\pi}}\exp(j\Omega_0\eta)\exp\left(-\frac{\eta^2}{2}\right) \qquad (3)$$

is used as the mother wavelet.

Choosing the wavelet parameter Ω_0 to be 2π, one provides the relation $s=1/f$ between the time scale s of the wavelet transformation and the Fourier-transformation frequency f.

The wavelet surface

$$W(s,t_0) = \left|W(s,t_0)\right|\exp(j\varphi_{s,t_0}) \qquad (4)$$

characterizes the behavior of the system on each time scale s at any instant t_0. The quantity $|W(s,t_0)|$ characterizes the intensity of the corresponding time scale s at the instant t_0. Thus, the phase is naturally determined for each time scale s:

$$\varphi_{s(t)} = \varphi(s,t_0) = \arg W(s,t_0). \qquad (5)$$

In other words, the behavior of each time scale s can be characterized using the associated phase $\phi_s(t)$.

If there is an range of time scales $[s_1; s_2]$, such that the phase locking condition

$$\left|\phi_{s_1}(t) - \phi_{s_2}(t)\right| \leqslant \text{const} \qquad (6)$$

is satisfied for any scale $s\in[s_1; s_2]$, and the fraction of the wavelet spectrum energy falling in this range is nonzero,

$$E_{\text{synch}} = \frac{1}{T}\int_{s_1}^{s_2}\int_0^T W^2(s,t_0)\,dt_0\,ds > 0, \qquad (7)$$

the scales $s\in[s_1; s_2]$ are synchronized and chaotic oscillators are in the time-scale synchronization regime [14, 15].

In relation (6) $\phi_{s_{1,2}}(t)$ are continuous phases of the first and second oscillators, corresponding to the synchronized time scales s.

If the interacting oscillators are in the chaotic synchronization regime, the realizations $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$ necessarily contain some range of time scales Δs where phases are synchronized. To reveal time-scale synchronization, one must check the validity of conditions (6) and (7).

3. SCHEMATIC OF EXPERIMENT

A chaotic oscillator with 1.5 degrees of freedom is very often used in experimental studies of chaotic synchronization as a model object [17]. It was applied in a number of experiments aimed at verifying various chaotic synchronization phenomena (complete, phase, and generalized synchronizations and effects at the chaotic synchronization threshold). Hence, it is a convenient object for experimental study of nonautonomous chaotic dynamics.

The electric circuit of the experimental setup for implementing unidirectional coupling between two chaotic signal oscillators is shown in Fig. 1. Each oscillator is a nonlinear amplifier with a linear feedback. The feedback circuit consists of two low-pass filters of the first and second orders.

The output signals $U_1(t)$ and $U_2(t)$ from the driving and response oscillators were recorded, respectively, at points *1* and *2* of the circuit for further analysis. The parameters of the elements of each oscillator under study are given in the caption to Fig. 1. Note that the capacitance $C1$ in the second-order filter of the feedback circuit of the driving oscillator was taken to be 100 nF to provide a small mismatch between the fundamental generation frequencies of the driving and response systems. As a result the generation

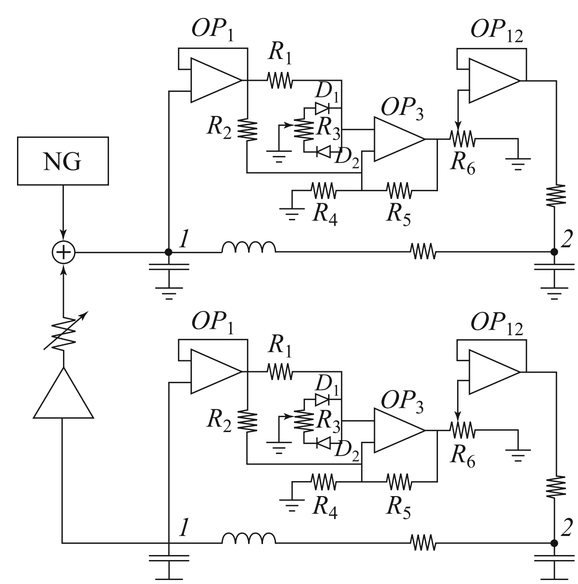

Fig. 1. Schematic of the experimental setup: operational amplifiers OP_1 and OP_2 (TL082); operational amplifier OP_3 (K174UD7); diodes D_1 and D_2 (1N4148); the resistances are $R_1=R_2=7.2\,\text{k}\Omega$, $R_3=100\,\Omega$, $R_4=180\,\text{k}\Omega$, $R_5=12\,\text{k}\Omega$, and $R_6=4.7\,\text{k}\Omega$; RC'' and rLC are low-pass filters with the element values $r=56\,\Omega$, $R=630\,\Omega$, $C=330\,\text{nF}$, $C''=150\,\text{nF}$, and $L=3.3\,\text{mH}$; the output signal is picked off points *1* and *2* (point *1* is the response oscillator input); and NG is the noise oscillator.

frequencies of the driving and response oscillators were $f_1 = 8.9$ kHz and $f_2 = 8.6$ kHz.

The unidirectional dissipative coupling was implemented using a coupling device in the form of step variable controlled resistance, connected successively to a voltage follower. The synchronizing signal was picked off the second-order filter output in the driving oscillator (point *1* in the driving oscillator circuit in Fig. 1) and applied through the coupling device at the output of the second-order filter of the response oscillator. The coupling coefficient was determined by the coupling resistance: $\varepsilon = 1/R(L/C)^{1/2}$. A δ-correlated noise signal, produced by a functional oscillator Agilent-33220A (NG in Fig. 1(a)), was additively introduced into the coupling device.

The time realizations $U_1(t)$ and $U_2(t)$ were digitized using a 12-bit ADC L-Card L783, incorporated in a personal computer, and numerically processed. A realization containing approximately 10^5 oscillation periods was recorded for each value of the coupling parameter ε from the interval [0.1, 5], which was sufficient for reliable diagnostics of chaotic synchronization regimes.

4. RESULTS OF EXPERIMENTAL STUDY OF TIME-SCALE SYNCHRONIZATION IN THE PRESENCE OF NOISE

Let us consider the results of experimental study of time-scale synchronization in the system under consideration with an increase in the level of noises mixed into the coupling channel between the oscillators. The experiment was carried out as follows: in the absence of noise we determined the synchronous dynamics threshold, then a noise signal was introduced into the system, and the effect of its intensity (which was determined as the signal-to-noise ratio (SNR)) on the diagnostics was investigated.

An analysis of the experimental data revealed that the coupling parameter value $\varepsilon = 0.3$ corresponded to the time-scale synchronization threshold. To illustrate this, Fig. 2 shows the dependence of the boundaries of synchronous scale ranges on the coupling parameter (line *1* corresponds to the absence of noise). It can clearly be seen that synchronization arises at $\varepsilon = 0.3$ on the time scale $s_0 \cong 0.12$ ms, which corresponds to the fundamental frequency in the Fourier spectrum of generation power. Furthermore, with an increase in the coupling parameter, the range of synchronous scales is rapidly expanded and synchronous scales near $s_1 \cong 0.212$ ms and $s_2 \cong 0.25$ ms arise, which correspond to two other independent frequencies in the oscillation spectrum of the oscillator. However, the energy corresponding to

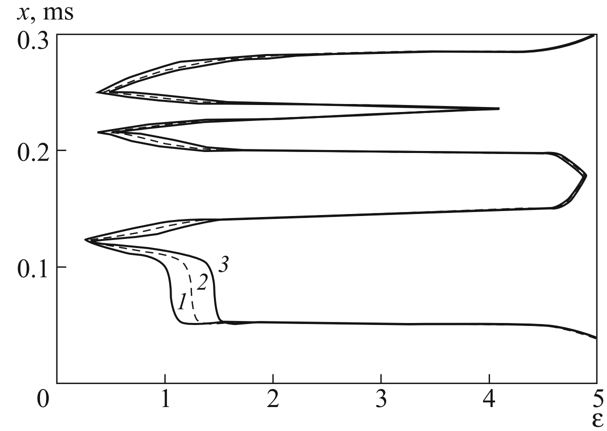

Fig. 2. Dependences of the synchronous scale range on the coupling parameter ε: (*1*) external source noise is absent and (*2*, *3*) SNR = 5 and 1, respectively.

these scales is much lower. Nevertheless, time-scale synchronization allows one to efficiently select oscillation synchronization at these frequencies. Note that the other method of chaotic synchronization analysis (phase synchronization) makes it possible to analyze oscillation synchronization only at the fundamental frequency, corresponding to the fundamental scale s_0.

At the coupling parameter $\varepsilon = 4.8$ complete synchronization (i.e., synchronization of all scales) occurred in the system, which corresponded to complete coincidence of oscillations in each coupled oscillator.

The introduction of noise with increasing intensity into the system barely affected the synchronization threshold. This can be seen well in Fig. 2 (lines *2* and *3*), which shows the dependences of the synchronous-scale boundaries with an increase in the coupling parameter for two SNR values. The boundaries of the ranges hardly change their shape, somewhat narrowing and rising only at small coupling coefficients. The introduction of a noise signal with SNR < 10 into the system impeded observation of phase synchronization in the range of coupling parameters $\varepsilon \in 0.3$–0.6, which indicated loss of phase coherence for the response attractor. Thus, the analysis of the data obtained suggests an important conclusion about time-scale synchronization stability.

To illustrate this result, we analyzed the synchronization measure, defined as the fraction of energy corresponding to synchronized time scales [12]:

$$\gamma = \int_{s_1}^{s_2} \left\langle W^2(s, t_0) \right\rangle_t ds, \quad (8)$$

where $\langle ... \rangle_t$ means time averaging. An increase in the synchronization measure indicates that the number

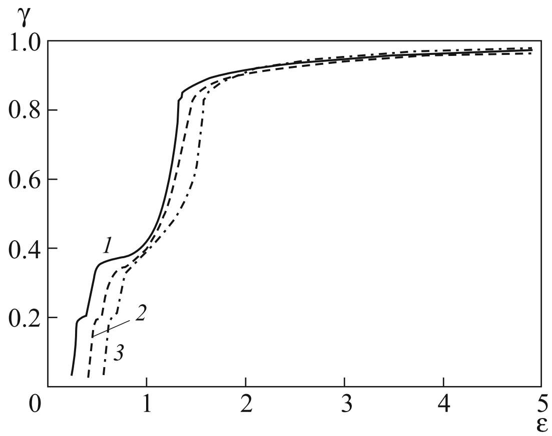

Fig. 3. Dependences of the synchronization measure of chaotic oscillators on the coupling parameter ε: (*1*) external source noise is absent and (*2*, *3*) SNR = 5 and 1, respectively.

of synchronized time scales increases with the coupling coefficient. The values $\gamma = 0$ and 1 correspond, respectively, to asynchronous dynamics and complete chaotic synchronization.

The dependences of the synchronization measure on the coupling parameter for different noise intensities are shown in Fig. 3. It is clearly seen that these dependences are qualitatively and quantitatively similar. With an increase in the noise intensity the synchronization threshold shifts to larger coupling parameters. A comparison of Figs. 2 and 3 suggests that at $\varepsilon > 2$ almost all scales are synchronized and the normalized energy (8) corresponding to these scales tends to unity. Note that at large coupling parameters noise does not affect the diagnostics of time-scale synchronization.

5. CONCLUSIONS

The effect of noise on the diagnostics of time-scale synchronization was experimentally investigated for the first time by the example of two unidirectionally coupled radio-frequency oscillators of chaotic oscillations. It is shown that time-scale synchronization has a high stability to external noise mixed into the coupling channel between the oscillators. Thus, this method is promising for diagnostics of different synchronous oscillation regimes in the presence of intense noise and interferences in physiology, neurophysiology, and systems for the secure data transmission.

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