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Nonautonomous Noise-Induced Synchronization

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Abstract—Two uncoupled Van der Pol oscillators driven by a common external harmonic signal and common white noise have been considered as a sample system. It was shown that the length of the time interval preceding the synchronous mode arising is inversely proportional to the value of the noise intensity.

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INTRODUCTION

Recently, it is has been popular to study phenomenon of synchronization in chaotic systems of various kinds. The consideration of synchronization is realized in such areas as biophysics, sociology, physiology, radio physics, hidden information transfer, and in many others [1–3]. It is an important and actual direction of theoretical and experimental research. Now, various types of synchronous chaotic behavior are emphasized, and each type has its singularities and diagnostics methods: generalized synchronization [4], phase synchronization [5], time-lagged synchronization [6], interleaved synchronization with delaying [7], full synchronization [8], synchronization of time scales [9–11], etc. Synchronization is the one of the most important fundamental nonlinear phenomenon.

It is important to note that all real objects showing synchronous behavior to a variable extent are subject to the influence of noise. It is known that noise has a destructive influence upon the synchronous dynamics of dynamic systems. On the other hand, there are examples when noise can exert a constructive influence. In particular, it is known that such a phenomenon as synchronization [12, 13] induced by noise exists; it was first described in [14]. The given type of synchronous behavior is exhibited at the changing of determined and random behavior and shows that the influence of noise on uncoupled self-sustained oscillation systems can promote the establishment of identical system behavior that was originally inconsistent under the initial conditions. Noise-induced synchronization is a mode when, under the influence of a random signal on two identical controlled parameters that are decoupled from each other with oscillations starting from various initial conditions, a synchronization appears in the behavior of the systems; i.e., after a defined time called the transient time (which in dynamic systems can be long-term enough [15,16]), these systems begin to work identically. Determining the presence of noise-induced synchronization is possible by means of the maximally constrained Lyapunov exponent or by realizing the direct comparison of the states of the considered systems [12].

As noted above, both considered phenomena are well enough studied for various dynamic systems and described in the works of many authors. At the same time, the question about what happens at the changing of these two processes, namely, concerning the onset of noise-induced synchronization in nonautonomous systems (in the case when a nonautonomous system without noise demonstrates an asynchronous mode), has not been considered earlier. The purpose of the present work consists of the investigation of the phenomenon of noise-induced synchronization in nonautonomous dynamic systems using the example of uncoupled Van der Pol oscillators under exterior harmonic action in the presence of noise.

THE INVESTIGATED SYSTEM

The considered system represents two uncoupled nonautonomous Van der Pol oscillators that are located under identical exterior harmonic action in the presence of noise:

$$\begin{aligned} \ddot{x}_1 - (\lambda - x_1^2)\dot{x}_1 + \omega x_1 &= A\sin(\Omega t) + \xi(t), \\ \ddot{x}_2 - (\lambda - x_2^2)\dot{x}_2 + \omega x_2 &= A\sin(\Omega t) + \xi(t), \end{aligned} \quad (1)$$

where $x_{1,2}$ is the state of first and second oscillators, respectively; ω is the natural frequency of each subsystem; and Ω is the frequency of the exterior harmonic action, where $\omega = 1.0$ and $\Omega = 0.98$. In expression (1), $\xi(t)$ represents the δ-correlated white noise ($\langle\xi(t)\rangle = 0$, $\langle\xi(t)\xi(\tau)\rangle = D\delta(t-\tau)$), where D is the intensity of the noise. The values of the remaining parameters of system (1) were chosen as follows: $\lambda = 0.1$; the amplitude of the exterior harmonic signal $A = 0.022$. The initial conditions for the oscillators were given as nonidentical conditions.

It is known that, at a enough high amplitude of the exterior signal, nonautonomous oscillators (1) in the absence of noise ($D = 0$) will show synchronous behavior [5]. In the case of synchronization, it is obvious that the action of the same noise ($D \neq 0$) on two identical oscillators showing synchronous dynamics does not leads to the qualitative modification of the situation and the states of both oscillators (1) will still coincide with each other at each instant.

In the case when the amplitude of the exterior action A is chosen in such a manner that the oscillators are out of the locking range, the situation is the reverse. Under such conditions, the considered oscillators (1) do not show synchronous dynamics, and the highest conventional Lyapunov exponent is equal to zero. Adding of identical noise changes the character of the considered systems' behavior. In particular, it is known that the zero conventional Lyapunov exponent depends on the intensity of the noise. In [17], it is shown that the highest conventional Lyapunov exponent Λ for the oscillator (1) becomes negative with the amplitude of the exterior action lying below the boundary of the synchronous mode onset A_c detected in the case of the absence of noise $D = 0$ (at the specified values of the control parameters $A_c = 0.0239$). As the zero value of Λ serves as a criterion for the noise-induced synchronization, it is possible to expect that the given phenomenon will be observed here. Therefore, by fixing the value of the amplitude of the exterior action A at an acceptable level (below the boundary A_c), it is possible during the modification of the intensity of the noise to discover the presence of the noise-induced synchronization in the investigated nonautonomous dynamic system.

Numerical modeling results of the behavior of two nonautonomous Van der Pol oscillators (1) that were realized by Euler's method with an integration time step of $h = 0.0005$ will be described in the following section. Modeling of a random signal $\xi(t)$ was realized as is described in [18].

THE NONAUTONOMOUS NOISE-INDUCED MODE OF SYNCHRONIZATION IN A DYNAMIC SYSTEM

The investigation of the onset of a nonautonomous noise-induced mode in a dynamic system was realized using the example of system (1). The behavior of two noncommutated nonautonomous Van der Pol oscillators was considered under exterior harmonic action in the presence of various intensity noise. As one of the diagnostic techniques of the mode transition of the in-sync state in dynamic systems is the direct comparison of the systems' states, the time realizations of the considered oscillators $x_1(t)$ and $x_2(t)$ were compared with each other (Fig. 1). The initial conditions were chosen in a random way, and the transient by the duration $T = 10^5$ was mapped out.

In Fig. 1à, the plane (x_1, x_2) illustrating the states of two nonautonomous uncoupled oscillators at identical instants t is shown at zero noise intensity ($D = 0$), when the oscillators $x_1(t)$ and $x_2(t)$ show asynchronous behavior and, accordingly, $x_1(t) \neq x_2(t)$. It is known that, for the establishment of the fact of synchronization with the help of such a graph, it is necessary that it represents a straight line [12]. It is easy to note in this case that a linear dependence of the state of one oscillator on the other is not observed.

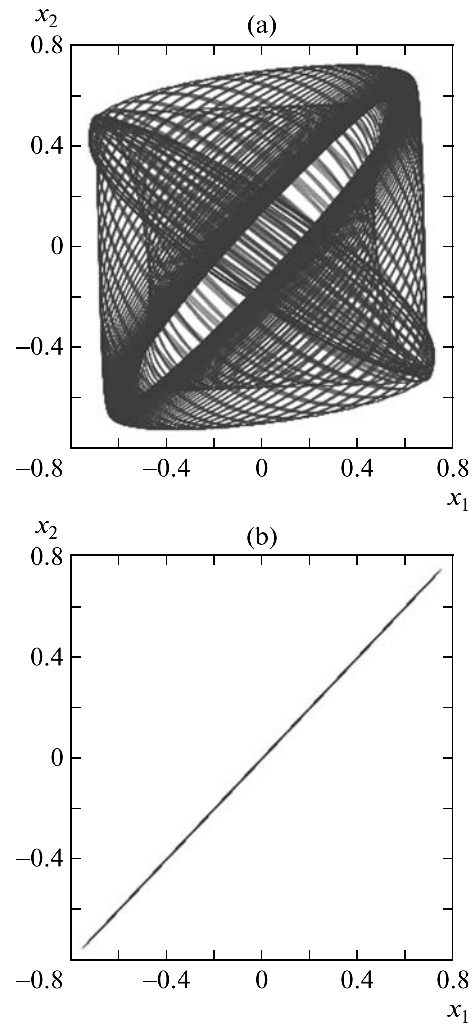

Fig. 1. (a) States of the first (x_1) and second (x_2) uncoupled nonautonomous Van der Pol oscillators (1) at certain instants at zero noise intensity ($D = 0$). (b) A case of states of the first (x_1) and second (x_2) uncoupled nonautonomous Van der Pol oscillators (1) at certain instants at non-zero noise intensity ($D = 0.5$).

At zero noise intensity in system (1), an absolutely different qualitative pattern is observed. Figure 1b illustrates the states of two investigated generators at identical instants but already under the influence of noise (in this case, $D = 0.5$). Obviously, the available mapping represents practically a straight line. From this, it is possible to draw the conclusion that nonautonomous uncoupled Van der Pol oscillators under such conditions, i.e., not only under the influence of an exterior harmonic signal but also in the presence of

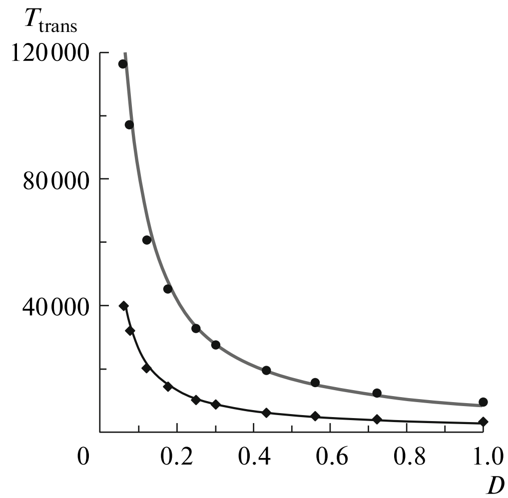

Fig. 2. Dependences of the mean transition time of synchronous (T_{trans}) oscillations of Van der Pol oscillators (1) from the noise intensity (D) at zero amplitude of the exterior signal (●) and at $A < A_c$, $A = 0.022$ (◆) approximated by power functions with exponents (–1) (clear and dark lines, respectively).

noise of nonzero intensity, show a synchronous type of behavior called by analogy with [13] *nonautonomous noise-induced synchronization*. Thus, it is important to note that this phenomenon is observed only in the presence of random exterior action, while the separate influence of a periodic signal does not lead to identical dynamics of the considered oscillators. At the same time, it is known [19, 20] that, if the system is acted on by noise and the amplitude of the exterior harmonic signal $A = 0$, then it is also possible to observe synchronous behavior of oscillators (1); however, the transient-process time appears much more similar to the stabilization time of the synchronization transition for the situation when the system is under the influence of both harmonic and stochastic signals.

It is obvious that the transition time of the synchronous oscillations of the oscillators is different depending on the intensity of the noise acting on them. In Fig. 2, the points denote the calculated dependences of the mean transient-process time leading to the synchronous dynamics for the intensity of the noise D in two cases: at zero amplitude of the exterior harmonic signal (●) and at the amplitude of the exterior nonzero signal $A = 0.022$ and at a smaller critical value A_c (◆). The averaging was done for $N = 100\,000$ realizations.

From the figure, it is obvious that, at first, with the increase of the intensity of the noise there occurs the reduction of the time interval in which the synchronous mode transition takes place, and, secondly, it is illustrated that, in the absence of exterior harmonic action, i.e., at $A = 0$, the mean transition-process time has much more duration T_{trans} at the same D but with ($0 < A < A_c$). Also, in the drawing, it is shown how the duration of the synchronous oscillations' transition depends on the intensity of the noise D. On the plane $T_{trans}(D)$, it is obvious that the received dependences are well enough approximated by the graphs of the power function with an exponent (–1) (the clear and dark lines for $A = 0$ and $A = 0.02$, respectively). Thus, the mean transition-process time is inversely proportional to the intensity of the stochastic signal acting on the investigated system of oscillators (1).

CONCLUSIONS

For the first time, the phenomenon of nonautonomous noise-induced synchronization was investigated using the example of a system of two uncoupled Van der Pol oscillators under exterior harmonic action in the presence of noise. The initiation of this phenomenon is possible under the simultaneous action of an exterior signal and noise and under the separate influence on the system of only a stochastic signal. The duration of the synchronous mode transition is considered at various noise intensities. It is shown that the mean duration of the transition depends both on D (inversely) and on the mode of action on the investigated system (harmonious and stochastic simultaneously or only stochastic). It is possible to expect that the received results have significant generality and that similar behavior will be observed for a wide class of nonlinear systems.

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