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STATISTICAL, NONLINEAR, AND SOFT MATTER PHYSICS

Noise-Induced Synchronization of Spatiotemporal Chaos in the Ginzburg–Landau Equation

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Abstract—We have studied noise-induced synchronization in a distributed autooscillatory system described by the Ginzburg–Landau equations, which occur in a regime of chaotic spatiotemporal oscillations. A new regime of synchronous behavior, called incomplete noise-induced synchronization (INIS), is revealed, which can arise only in spatially distributed systems. The mechanism leading to the development of INIS in a distributed medium under the action of a distributed source of noise is analytically described. Good coincidence of analytical and numerical results is demonstrated.

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1. INTRODUCTION

In recent years, the phenomenon of synchronization of chaotic oscillations in various dynamical systems has been extensively studied by researchers engaged in various fields [1–3]. These investigations were mostly performed within the qualitative theory of dynamical systems and nonlinear dynamics [3–5] in application to radio engineering [6, 7], biophysics [8–10], physiology [11–13], data transmission theory [14–16], theory of complex networks [17], etc. The main results in the investigation of synchronization of chaotic autooscillations in coupled and nonautonomous systems were obtained in the analysis of systems with small degrees of freedom (flow systems and maps), where various types of synchronous chaotic behavior were revealed. These are phase synchronization (PS) [18–20], generalized synchronization (GS) [21–23], lag synchronization [24], intermittent lag synchronization [25], intermittent GS [26, 27], noise-induced synchronization [28–30], complete synchronization [31–34], and time-scale synchronization [35–37].

Much less attention has been devoted to studying the synchronization of chaotic spatiotemporal oscillations in distributed active media, although such investigations are of considerable basic importance for deeper insight into general laws of the interaction of complex nonlinear systems of various natures (chemical, physical, biological, etc.). Indeed, many manifestations of chaotic synchronization exhibit significant specific features when this phenomenon is analyzed in continuous spatially distributed media. In particular, we have demonstrated [38, 39] that the phenomenon of GS in distributed media described by the Ginzburg–Landau equations acquires principally new features as compared to the case of lumped systems [5, 28–30]. In particular, it was found that distributed systems can exhibit the phenomenon of partial GS [39], which is determined by the spatially distributed character of coupled media. Complex spatiotemporal dynamics in the development of time-scale synchronization under the action of an external chaotic signal was demonstrated [7, 40, 41] in a system of coupled electron–wave media with cubic phase nonlinearity under the conditions of synchronized electron and backward electromagnetic waves [42]. Investigations of chaotic synchronization in distributed media is also of considerable applied significance because the models of such systems are frequently employed in the analysis of processes in coupled systems, as well as in laser, biological, physiological, chemical, and other systems.

The phenomenon of synchronous behavior in spatially distributed systems has been frequently studied using models of autooscillatory media described by the Ginzburg–Landau [2, 38, 43–46] and Kuramoto–Sivashinsky [47] equations and by chains and lattices of coupled oscillators [48], maps [2], networks of chaotic elements [17, 49], and beam–plasma and electron–wave systems [50, 51].

One of the least studied types of the chaotic synchronization of dynamic systems is noise-induced synchronization (NIS), which was originally described by Fahy and Hamann [52]. This type of synchronization is worthy of special attention because it is manifested at the border of deterministic and stochastic behavior [5, 28–30] and demonstrates that the action of noise on an ensemble of dynamical autooscillatory systems can favor the establishment of identical behavior of initially uncorrelated chaotic systems. Elucidation of the mechanisms of such synchronization in various systems is important for better understanding of nonlinear effects in chemical, economic, and living systems [53–57].

By NIS is implied the regime where a random signal acting upon two chaotic systems $\mathbf{u}(t)$ and $\mathbf{v}(t)$ with identical control parameters, starting from different initial conditions $\mathbf{u}(t_0) \neq \mathbf{v}(t_0)$, leads eventually, after termination of a certain transient process, to their identical behavior, which is manifested by the equation $\mathbf{u}(t) = \mathbf{v}(t)$. The diagnostics of NIS can be provided by direct comparison of the $\mathbf{u}(t_0)$ and $\mathbf{v}(t_0)$ states of the two systems. Another method of detecting NIS consists in calculating the maximum Lyapunov exponents of a dynamic system [29, 58] occurring under the action of noise. The maximum Lyapunov exponent characterizes the presence of an instability in a given autooscillatory system under the action of an external noise source. After the establishment of NIS, the maximum Lyapunov exponent becomes negative, which is just what implies the identical dynamics: under the action of noise, such systems "forget" their individual initial conditions and pass to identical states [5]. Once the maximum Lyapunov exponent is positive, there is no synchronism.

This article presents a detailed investigation of NIS in a distributed autooscillatory medium described by the Ginzburg–Landau equation in the presence of an external distributed noise source. For this study, the continuous spatially distributed media described by these equations are chosen because they can be considered standard models for investigating spatiotemporal chaos and the formation of structures in various distributed media [43]; they are frequently used for studying methods of managing chaos [48] and chaotic synchronization [39, 46, 59] in distributed systems. We have discovered a principally new phenomenon, called incomplete noise-induced synchronization (INIS), which is related to the distributed character of an autooscillatory system under the action of an external noise. It will be shown that, in this case, the diagnostics of NIS in distributed media using the maximum Lyapunov exponent requires additional detailed analysis of the spatiotemporal dynamics of the nonautonomous system.

2. DESCRIPTION OF MODEL

Let us consider a mathematical model representing two one-dimensional complex Ginzburg–Landau equations with identical control parameters, which describe the complex fields $u(x, t)$ and $v(x, t)$ under the action of a common source of noise $\zeta(x, t)$:

$$\begin{aligned} u_t &= u - (1 - i\beta)|u|^2 u + (1 + i\alpha)u_{xx} + D\zeta(x, t), \\ v_t &= v - (1 - i\beta)|v|^2 v + (1 + i\alpha)v_{xx} + D\zeta(x, t) \end{aligned} \quad (1)$$

with periodic boundary conditions

$$u(x, t) = u(x + L, t), \quad v(x, t) = v(x + L, t), \quad (2)$$

where L is the spatial period, α and β are the real control parameters, and D is the intensity of a complex noise source ($\zeta = \zeta_1 + i\zeta_2$). We have considered the source of noise with an asymmetric probability density distribution function for the real (ζ_1) and imaginary (ζ_2) parts defined in the unit interval $\zeta_{1,2} \in [0, 1]$:

$$p(\zeta_{1,2}) = \begin{cases} 2\zeta_{1,2}, & 0 \le \zeta_{1,2} \le 1 \\ 0, & \zeta_{1,2} < 0, \quad \zeta_{1,2} > 1. \end{cases} \quad (3)$$

The noise process with the distribution function (3) has been simulated with a scheme that is conventionally used for the generation of a noise process with a ramp distribution [60].

The control parameter β was fixed at $\beta = 4$, while the control parameter α was varied within $\alpha \in [1, 2]$ and the spatial period was chosen to be $L = 40\pi$. As is known, the autonomous distributed system with such control parameters occurs in the state of spatiotemporal chaos [43, 61]. Initial conditions ($u(x, t = 0)$ and $v(x, t = 0)$ for the complex fields were set randomly.

The Ginzburg–Landau equations (1) with an additional stochastic term ζ were numerically solved using the standard computational scheme [62], in which the unknown complex quantity $u(x, t)$ is treated as a complex field on a one-dimensional lattice with spatial period Δx. Introducing the discrete argument as $x_i = i\Delta x$ ($i = 1, \ldots, N$), let us denote the corresponding values of $u(x_i, t)$ by $u_i(t)$. Accordingly, $\zeta_i(t)$ is a random process with the probability density distribution function for the real and imaginary parts obeying conditions (3) and the effective intensity of the spatiotemporal noise in the discrete space being $D = \tilde{D}/\Delta x$ [62]. The Laplacian $\partial^2/\partial x^2$ was represented by a three-point finite-difference approximation [63]. As a result, the stochastic differential equations (1) in partial derivatives are modeled by a discrete one-dimensional lattice using the single-step Euler method [62] with a time step of Δt. The temporal and spatial steps were selected as $\Delta t = 0.0002$ and $\Delta x = L/1024$.

3. NOISE-INDUCED AND INCOMPLETE NOISE-INDUCED SYNCHRONIZATION IN A DISTRIBUTED AUTOOSCILLATORY SYSTEM

Let us consider the dynamics of a system of distributed autooscillatory media described by Eqs. (1) in the regime of spatiotemporal chaos and follow its variation with increasing noise intensity D. For $D = 0$, the systems $u(x, t)$ and $v(x, t)$ occur in the regime of spatiotemporal chaos and their states are different: $v(x, t) \neq u(x, t)$. This is illustrated in Fig. 1a, which shows the evolution of the difference $|u(x, t) - v(x, t)|$ in system (1) in the absence of noise. As the noise intensity increases so that D exceeds a certain threshold, the systems demonstrate identical dynamics (which is chaotic in space and time in each system), which implies the establish-

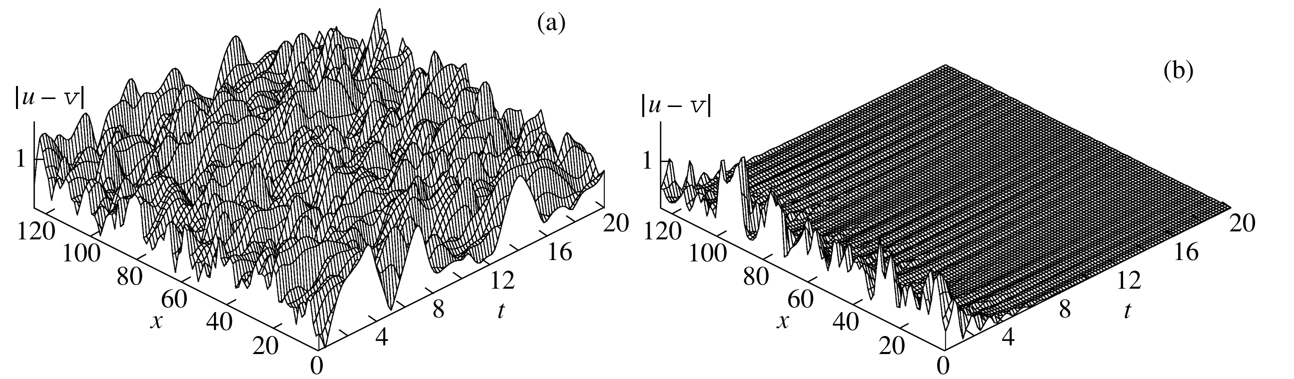

Fig. 1. Evolution of the difference $|u(x, t) - v(x, t)|$ of the states of systems described by the Ginzburg–Landau equations (1) with the control parameters $\alpha = 2$ and $\beta = 4$: (a) in the absence of noise; (b) in the presence of noise with the intensity $D = 3$.

ment of the NIS regime in the given spatially distributed system (Fig. 1b).

In order to detect the NIS regime, it is convenient to consider the difference of states between the two distributed systems under the action of a common source of noise, which is averaged in both time and space:

$$\varepsilon = \frac{1}{TL}\int_{\tau}^{\tau+T}\int_{0}^{L}|u(x,t) - v(x,t)|dxdt. \qquad (4)$$

In the course of numerical modeling, the averaging was performed after a long transient process ($\tau = 200$), during which the noise in both systems was absent. In the NIS regime, the average difference vanishes ($\varepsilon = 0$) and the states of systems u and v at every moment of time are identical.

In addition to the comparison of states of the two identical systems under the action of a common source of noise, we have also studied the behavior of the maximum Lyapunov exponent for a system under the action of noise. The method of calculation of the maximum Lyapunov exponent for Ginzburg–Landau equations (1) was considered in much detail elsewhere [39]. As was noted above, the maximum Lyapunov exponent in the NIS regime must be negative.

Figure 2 shows plots of the maximum Lyapunov exponent λ and the average difference ε of states versus noise amplitude D for system (1) with two values of the control parameter α. For $\alpha = 1$, the noise intensity D at which the maximum Lyapunov exponent crosses the zero level to become negative coincides with that at which the average difference of states defined by Eq. (4) vanishes (Fig. 2, curves *1*). In this case, the NIS threshold is $D_{NIS} \approx 1.5$ (as indicated by the arrow $D_{NIS}^{(1)}$ in Fig. 2) and the situation is essentially the same as that for the well-known transition to NIS in systems with a small number of the degrees of freedom.

For a greater value of the control parameter ($\alpha = 2$), the mechanism of NIS in the system under consideration is quite different (Fig. 2, curves *2*). In this case, the maximum Lyapunov exponent decreases with increasing noise intensity D and, at a certain value ($D_{INIS} \approx 1.53$) vanishes (Fig. 2a, curve *2*), while the average difference of states is still nonzero (Fig. 2b, curve *2*). As the noise intensity D grows further, the ε

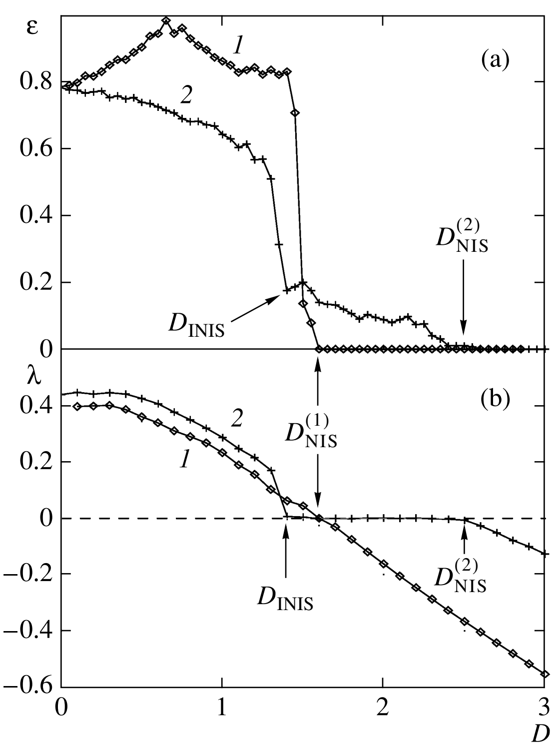

Fig. 2. Plots of the (a) the average difference ε of states defined by Eq. (4) and (b) maximum Lyapunov exponent λ versus noise amplitude D for Ginzburg–Landau equations (1) with different values of the control parameter: $\alpha = 1$ (*1*) and 2 (*2*). Arrows $D_{NIS}^{(1)}$ and $D_{NIS}^{(2)}$ indicate the values of noise intensity corresponding to the appearance of NIS for curves *1* and *2*, respectively; arrow D_{INIS} indicates the INIS onset boundary.

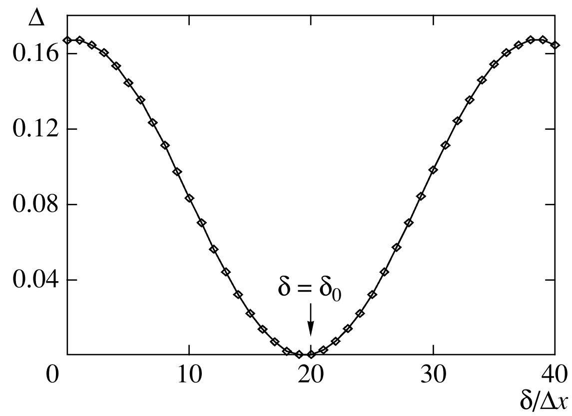

Fig. 3. Plot of the average difference Δ between states $u(x, t)$ and $v(x, t)$ versus the spatial shift δ for Ginzburg–Landau equations (1) with the control parameters $\alpha = 2$, $\beta = 4$ and noise intensity $D = 2$ (numerically calculated using a spatial step of $\Delta x = 40\pi/1024$).

value keeps decreasing to vanish at $D_{\rm NIS}^{(2)} \approx 2.5$, where the Lyapunov exponent becomes negative, which is evidence for the establishment of the NIS regime. Thus, there exists a finite interval of noise intensities ($D_{\rm INIS}$, $D_{\rm NIS}^{(2)}$), in which two identical systems under the action of a common distributed noise behave differently (NIS is absent) despite the fact that the maximum Lyapunov exponent is zero.

Such a behavior of the average difference of states and the maximum Lyapunov exponent with increasing noise intensity is not observed in systems with finite dimensions of the phase space. Apparently, the appearance of this regime is related to the spatially distributed character of the system under consideration and, in the interval of noise intensities where the Lyapunov exponent is zero, we are dealing with a transient regime on the passage from asynchronous chaos to NIS.

Detailed investigation of the system dynamics in the interval of noise intensities $D \in (D_{\rm INIS}, D_{\rm NIS}^{(2)})$ showed that, in this case, the system exhibits some properties inherent to the synchronous regime (in the NIS sense) despite the fact that the maximum Lyapunov exponent is zero and the average difference of states is nonzero. In the case under consideration, the complete coincidence of states (i.e., the NIS regime) can be obtained by spatial shifting of one of the two systems described by the Ginzburg–Landau equations (1) relative to the other by a certain interval δ. In other words, for $v = v(x + \delta, t)$, the average difference of states

$$\Delta(\delta) = \frac{1}{TL}\int_{\tau}^{\tau+T}\int_{0}^{L}|u(x,t) - v(x+\delta,t)|dxdt \qquad (5)$$

is a function of δ. It should be noted that, for the correct calculation of the average difference of states (5), it is necessary to take into account the long transient process that accompanies the emergence of the system at a stable manifold (attractor) in the phase space. In other words, it is necessary to increase the duration τ of the transient process as compared to that in the case described above (in our calculations, $\tau = 2000$). The results of such an analysis are presented in Fig. 3, which shows a plot of the average difference of states Δ versus the spatial shift δ. As can be seen, there exists a spatial shift $\delta = \delta_0$ such that $\Delta = 0$ and, hence the systems exhibit identical behavior. Thus, we observe a behavior resembling the NIS regime. For other values of the spatial shift, the states of the noise-coupled systems in both space and time are different, but the maximum Lyapunov exponent for the given set of control parameters is zero. The special value of the spatial shift $\delta = \delta_0$ strongly depends on the initial conditions.

Taking into account a definite similarity of the new regime observed in the noise-driven spatially distributed system and the NIS phenomenon, this regime characterized by vanishing of the maximum Lyapunov exponent is called incomplete noise-induced synchronization (INIS).

4. MECHANISMS OF NOISE-INDUCED AND INCOMPLETE NOISE-INDUCED SYNCHRONIZATION DEVELOPMENT IN A SPATIALLY DISTRIBUTED SYSTEM

Let us consider factors favoring the development of NIS and INIS regimes in a spatially distributed chaotic system. For this purpose, it is expedient to compare the NIS and GS in the distributed active medium under consideration. There are well-known mechanisms leading to the appearance of GS, which are related to suppression of the intrinsic dynamics of a chaotic oscillator by the external action (see, e.g., [23, 64]). Using particular types of the systems with finite numbers of the degrees of freedom (Rössler, Lorentz, logistic maps, etc.), it was demonstrated [30] that the appearance of NIS and GS regimes is based on the same mechanisms. Scenarios of GS development in distributed systems are similar to those in finite-dimensional systems [38]. For elucidating the mechanisms of the GS development, an approach (called the modified system method) has been proposed [23] based on the replacement of a nonautonomous chaotic system by a system with additional dissipation (modified system) caused by coupling between subsystems or by external action. It can be suggested that the mechanism of NIS development is close to that accounting for the GS and, hence, we can trace the appearance of a synchronous regime in the system under consideration using a variant of the modified system method.

Following [30], let us consider the dynamics of a modified Ginzburg–Landau equation with an additional term $\langle D\xi \rangle$ corresponding to the average magnitude of noise:

$$\frac{\partial u_m}{\partial t} = u_m - (1 - i\beta)|u_m|^2 u_m + (1 + i\alpha)\frac{\partial^2 u_m}{\partial x^2} + \langle D\zeta \rangle. \quad (6)$$

For a noise with the distribution function (3), the average value is

$$\langle D\xi \rangle = D\int_{-\infty}^{\infty} \xi p(\xi) d\xi = 2D\int_0^1 \xi^2 d\xi = \frac{2D}{3}. \quad (7)$$

As is well known, the Ginzburg–Landau equation describes various types of spatiotemporal behavior (including traveling waves, phase turbulence, and spatiotemporal chaos) depending on the intervals of control parameters in the phase space [65–67].

Investigations showed that the additional deterministic term in the modified Ginzburg–Landau equation (6) strongly influences the dynamics of the active medium. In particular, if the D value is sufficiently large, a homogeneous stationary state $u_0 = u_0(x, t) =$ const is established in system (6). In this state, the maximum Lyapunov exponent is negative, which corresponds to an NIS regime in the system. As the noise intensity decreases, the state u_0 loses stability. This corresponds to the breakage of NIS in system (1) under the action of the external source of noise.

However, an analysis of the modified system (6) shows that the loss of stability of a stationary state can proceed, depending on the control parameters, according to different scenarios. In order to reveal these variants, let us analytically investigate the loss of stability of a stationary state u_0 of the modified system. The stationary state can be determined by the following equation (which follows from Eq. (6):

$$u_0 - (1 - i\beta)|u_0|^2 u_0 + 2D/3 = 0. \quad (8)$$

Thus, the equation cannot be solved analytically, but the solution can be obtained numerically using the Newton method [68]. For an analysis of the stability of the stationary state determined by Eq. (8), let us consider a linearized modified Ginzburg–Landau equation (6) in the vicinity of the stationary state. Let $\tilde{u} = \tilde{u}_r + i\tilde{u}_i$ be a small perturbation relative to the stationary state $u_0 = u_r + iu_i$:

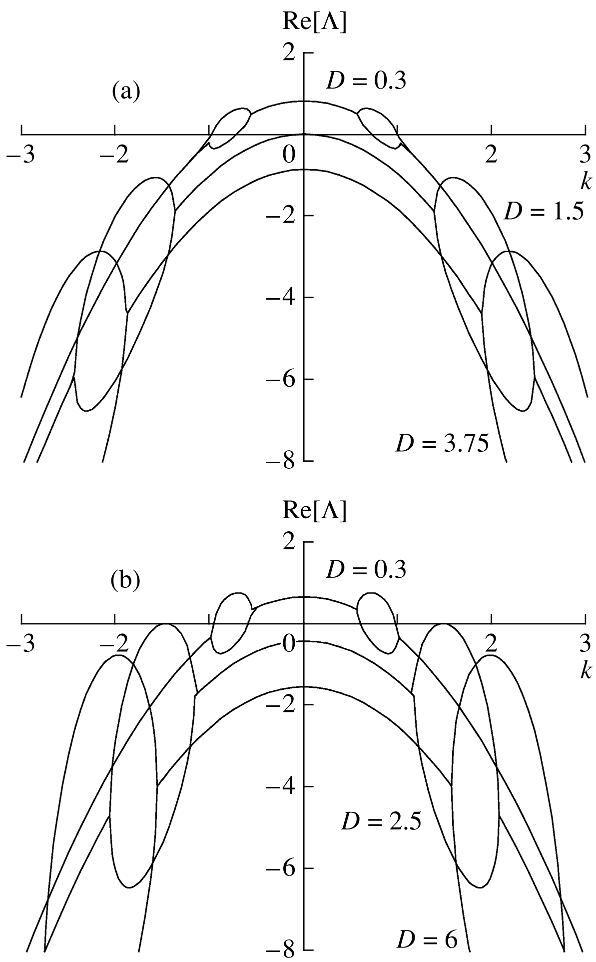

Fig. 4. Plots of the real part of eigenvalues Λ versus wavenumber k for modified Ginzburg–Landau equations with fixed control parameters $\alpha = 1$ (a) and 2 (b) and various noise intensities D.

$$u_m = u_0 + \tilde{u}.$$

Linearizing Eq. (6) and assuming that the small perturbation is exponentially increasing as

$$\tilde{u}_r(x, t) = \hat{u}_r(k)e^{\Lambda t + ikx}, \quad \tilde{u}_i(x, t) = \hat{u}_i(k)e^{\Lambda t + ikx}, \quad (9)$$

we obtain a dispersion relation that determines the stability of the stationary state u_0:

$$\begin{vmatrix} 1 - u_i^2 - 3u_r^2 - 2\beta u_i u_r - k^2 - \Lambda & -(\beta u_r^2 + 3\beta u_i^2 + 2u_r u_i - \alpha k^2) \\ \beta u_i^2 - 2u_i u_r + 3\beta u_r^2 - \alpha k^2 & 2\beta u_r u_i - u_r^2 - 3u_i^2 + 1 - k^2 - \Lambda \end{vmatrix} = 0. \quad (10)$$

Evidently, the stationary state u_0 is stable, provided the condition $\mathrm{Re}\Lambda(k) < 0$ is valid for any k.

Figure 4 shows the plots of $\mathrm{Re}\Lambda(k)$ for sequentially decreasing values of the noise intensity D at fixed val-

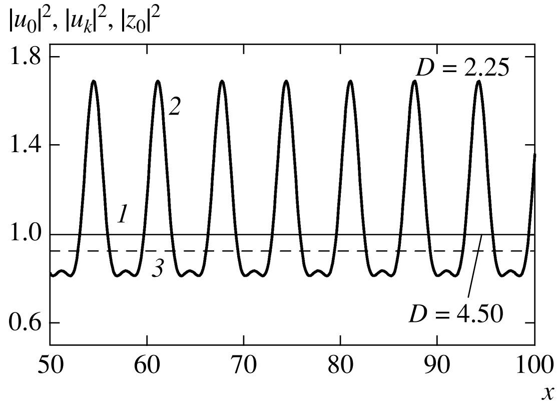

Fig. 5. Profiles of the stationary states $|u_0|^2$ (straight line *1*) and $|u_k(x)|^2$ (curve *2*) observed in system (6) with fixed control parameter $\alpha = 2$ and various values of the noise intensity D. Dashed line *3* represents an unstable stationary state $|z_0|^2$ corresponding to Eq. (8) with $D = 2.25$.

ues of the control parameters ($\alpha = 1, 2$). As can be seen, the stationary state u_0 in the system with $\alpha = 1$ (Fig. 4a) loses stability at $D \approx 1.5$. In this case, the spatial perturbation with a wavenumber of $k = 0$ exhibits exponential growth. As a result, the stationary state u_0 becomes unstable and the system described by Eq. (6) features spatiotemporal chaos. The maximum Lyapunov exponent becomes positive both in the modified Eq. (6) and in the initial Ginzburg–Landau equation (1), so that the NIS regime is not observed in Eq. (1) with the indicated values of the control parameters.

In the case of $\alpha = 2$ (Fig. 4b), stationary state u_0 becomes unstable at $D \approx 2.5$. Here, spatial modes with wavenumbers $k = \pm 0.5$ lose stability via a scenario that is different from that considered above for $\alpha = 1$. When the homogeneous state u_0 loses stability, a state $u_k(x) = u_k(x + l)$ (for $l = 2\pi/k$ as determined by the periodic boundary conditions) appears in the modified Ginzburg–Landau equations with $\alpha = 2$, which is stable in time and periodic in space.

Figure 5 shows characteristic examples of profiles corresponding to such a stationary, spatially periodic state of the system under consideration. Apparently, the maximum Lyapunov exponent for such a stationary state is zero. Thus, the initial system under the action of noise $D\zeta(x, t)$ with the average intensity $\langle D\zeta \rangle$ features a stationary, spatially periodic, noise-perturbed profile structure $u_k(x)$. Therefore, the spatiotemporal dynamics of the state $u_k(x)$, which appears as a complex aperiodic motion, is also characterized by the maximum Lyapunov exponent $\lambda = 0$. Since the two identical systems $u(x, t)$ and $v(x, t)$ under the action of a common source of noise evolve starting from different initial conditions $u(x, 0)$ and $v(x, 0)$, respectively, the periodic spatial structures are not identical, $u_k(x) \neq v_k(x)$. However, owing to the translational symmetry of the initial system with periodic boundary conditions, there is a certain spatial shift δ_0 (also dependent on the initial conditions $u(x, 0)$ and $v(x, 0)$, such that $u_k(x) = v_k(x + \delta_0)$. For this reason, a regime characterized by nonidentical states with vanishing maximum Lyapunov exponent is observed in the Ginzburg–Landau equations (1) with a common source of noise in the interval of $D_{\mathrm{INIS}} < D < D_{\mathrm{NIS}}^{(2)}$. If one of the subsystems is shifted relative to another in space by a certain value of δ_0, which depends on the initial conditions and obeys the relation $u_k(x) = v_k(x + \delta_0)$, the noise-driven systems will behave identically.

The results of analytical investigation showed very good coincidence of the noise intensities D corresponding to the loss of stability of the homogeneous stationary state u_0 (Fig. 4b) and the D_{INIS} value at which the maximum Lyapunov exponent vanishes (Fig. 2b).

5. INCOMPLETE NOISE-INDUCED SYNCHRONIZATION REGIME AND STATISTICAL CHARACTERISTICS OF NOISE

The fact that the appearance of NIS in a distributed autooscillatory system is determined by the average value of the noise signal acting upon the system is well illustrated by the dependence of the development of synchronization on the statistical characteristics of noise in a system described by Eqs. (1). One type of noise that is traditionally used for the investigation of statistical differential equations is normal noise characterized by the Gaussian probability density distribution function. For this reason, it is also important to consider the dynamics of complex Ginzburg–Landau equations (1) under the action of noise ζ with Gaussian distributions of the real (ζ_1) and imaginary (ζ_2) parts of this random variable:

$$p(\zeta_{1,2}) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(\xi - a)^2}{2\sigma^2}\right). \qquad (11)$$

In contrast to the above example with a ramp distribution of noise, where it is impossible to separate the influence of the average noise intensity and dispersion, this can readily be done for the Gaussian distribution function. The noise intensity was fixed at $D = 1$. In this case, the average value of the random process is determined by parameter a, while the noise intensity is determined by dispersion σ. The random process with the required distribution function (11) was achieved using an algorithm described in [68].

We have studied the dependence of the maximum Lyapunov exponent λ on the average value of noise signal with the distribution function (11) for various values of dispersion σ. The results are presented in Fig. 6a, which shows the plots of $\lambda(a)$ for various values of the noise dispersion. According to the results of numerical calculations, the INIS regime is realized in the interval of $a \in [1.0, 1.7]$ (the same as that in the case considered

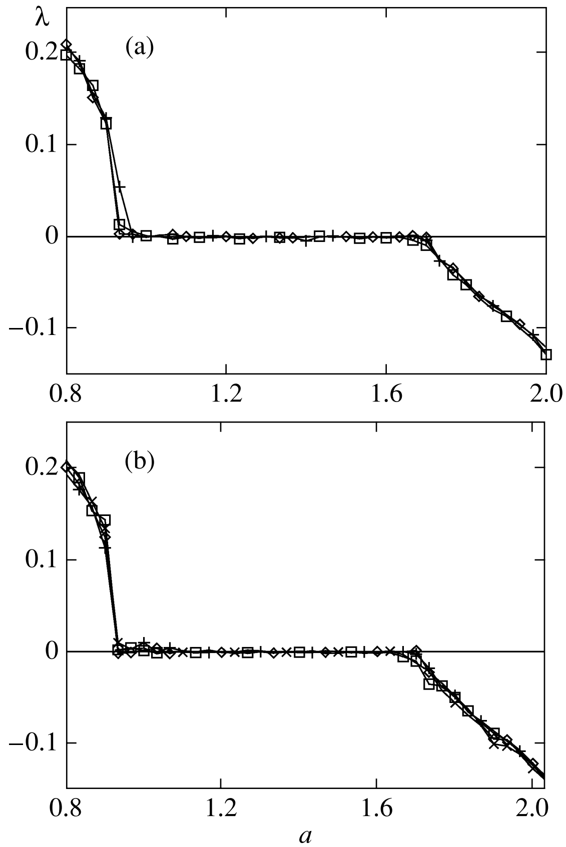

Fig. 6. Plots of the maximum Lyapunov exponent λ versus parameter a for the Ginzburg–Landau equations with $\alpha = 2$, $\beta = 4$ under the action of a source of noise with different statistical characteristics: (a) Gaussian distribution (11) and various values of noise dispersion $\sigma = 0.1$ (◇), 0.2 (+), and 0.5 (□); (b) a uniform distribution with $\sigma = 0.5$ (◇), 1.0 (+), 1.5 (□), and 2.0 (×). As is clearly seen, there is an interval where $\lambda = 0$.

in Section 3) for any value of the dispersion. Therefore, the dispersion (and, hence, the intensity of noise) does not significantly influence the mechanisms of NIS and INIS development in the system under consideration.

Consideration of the spatiotemporal dynamics of two complex Ginzburg–Landau equations under the action of a common noise source with distribution function (11) showed that the states of the subsystems in the interval $a \in [1.0, 1.7]$ were not identical; that is, the NIS regime was absent. Nevertheless, there is a shift δ_0 (determined by the initial conditions) such that the states of the systems in the spatially displaced points coincided: $u(x, t) = v(x + \delta, t)$, which was indicative of the INIS regime.

For the comparison of results obtained using a source of noise with the Gaussian distribution function to the results described in Sections 4 and 5 for the noise with an asymmetric distribution function, it is necessary to take into account that the average value of noise in Section 4 was represented by the mean value $2D/3$ (see Eq. (7)). Therefore, taking the same value for the average intensity of noise with the Gaussian distribution (i.e., substituting $a = 2D/3$), we conclude that the INIS regime is observed at $D \in [1.5, 2.5]$, which is agrees well with the result of analytical considerations in Section 4.

Analogous results were obtained for the noise with a uniform distribution function with an average value of a and the distribution width (dispersion) σ (Fig. 6b). There also exists an interval of the average noise a where $\lambda = 0$ and the INIS regime is achieved.

Thus, based on the results of our investigation, it can be concluded that the development of an INIS regime is determined (i) by the presence of an additional translational degree of freedom and (ii) by the average intensity of a noisy signal acting on the system, while the signal dispersion virtually does play any role in the establishment of INIS.

6. CONCLUSIONS

We have studied for the first time the phenomenon of NIS in a distributed autooscillatory system described by the Ginzburg–Landau equations, which occurs in a regime of chaotic spatiotemporal oscillations. A new regime of synchronous behavior in dynamical system under the action of noise, called incomplete noise-induced synchronization (INIS) is revealed, which can arise only in spatially distributed systems with translational symmetry. The new regime differs from all other types of synchronous behavior known in active media that demonstrate spatiotemporal chaos. INIS can be observed in a broad range of intensities of the external noisy signal, where the maximum Lyapunov exponent vanishes ($\lambda = 0$), while the states of the two identical subsystems under the action of the common source of noise are nonidentical. Nevertheless, there is evidence for the synchronism of oscillations: if the state of one system is shifter relative to another by a certain value (dependent on the initial conditions), the behavior of these systems will be identical and they will exhibit NIS. Theoretical description of this behavior in spatially distributed system has been proposed, which agrees well with the results of numerical calculations. The influence of the type of a noise source on the development of INIS regimes has been studied and it is shown that a significant role in the establishment of this type of synchronism is played by the average value of noise, while the noise signal intensity is almost insignificant.

The results concerning the appearance of INIS, which were obtained for the model described by of complex Ginzburg–Landau equations in the presence of noise with a nonzero average, suggest that this behavior can also be observed in a large variety of analogous active media. Since the influence of a noisy signal source can lead to the formation of structures [69–71], it is expected that INIS can be observed in systems under the action of noise with zero average for some other types of spatial structures (e.g., traveling waves).

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