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Transition to Phase Synchronization in a System with Periodic Dynamics under the Action of a Chaotic Signal

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Abstract—Intermittent behavior has been studied near the boundary of phase synchronization in an autooscil-
1 latory system with periodic dynamics under the action of an external chaotic signal. It is shown that the intermittency obeys the same scenario as that observed in the case of interaction between two coupled chaotic oscillators.

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Intermittent behavior is a universal phenomenon that is encountered in systems of various natures. In
1 particular, intermittency is among the classical scenarios of the transition from periodic oscillations to chaos [1–3]. Intermittent signals represent the alternating sequences of regular motion (laminar phases) and chaotic outbursts (turbulent phases). As the control parameter is increased, the chaotic outbursts are more frequently repeated and eventually the motion becomes fully chaotic. Depending on the character of the loss of stability by a periodic regime (determined by the limit
2 1 cycle multiplicators), intermittency is classified into types I–III [4].

The intermittent behavior can also be observed near the boundaries of various chaotic synchronization regimes, which corresponds to the intermittent phase synchronization (PS) [5, 6], intermittent generalized synchronization [7], and intermittent delay synchronization [8]. There is a classification of such intermittent regimes. The passage to generalized and delay synchronization is referred to as the on–off type [7, 8], while the passage to PS is classified (depending on the detun-
1 ing of control parameters) as the intermittency of type
1 I, the so-called eyelet intermittency [5, 9], and the ring
1 intermittency [6].

In recent years, much attention has been devoted to the investigation of intermittent behavior near the PS boundary (see, e.g., [5, 6. 9–11]. For the diagnostics of PS [12–14], it is a common practice to introduce the phases $\varphi_{1,2}(t)$ of the signals of two interacting oscillators (or an oscillator and an external signal), monitor their difference, and detect the PS provided that the phase locking condition is satisfied:

$$|\Delta\varphi(t)| = |\varphi_2(t) - \varphi_1(t)| < \text{const}. \quad (1)$$

In the case of intermittent behavior near the PS boundary, the time series of the phase difference are characterized by the presence of synchronous intervals (laminar phases), which are interrupted by sudden jumps in which the phase difference abruptly changes by 2π (turbulent phases).

In determining the scenarios of transition to PS, an important role belongs to the statistics of laminar phase durations. In the case of small frequency detunings, this statistics is different for the systems with periodic and chaotic dynamics. In particular, for the coupled or induced oscillations of systems exhibiting periodic behavior, the transition to PS proceeds via a saddle-nodal bifurcation, and the mean laminar phase difference as a function of the supercriticality parameter obeys a power law with an exponent of –1/2 that corresponds to the intermittency of type I [4, 11]: 1

$$\langle l \rangle \sim |P - P_c|^{-1/2}, \quad (2)$$

where P is the coupling strength (or the external signal frequency) and P_c is its critical value that corresponds to the point of transition to the PS regime.

The situation with chaotic systems is somewhat different: in the case of an external harmonic signal acting upon a chaotic oscillator and for the mutual oscillations of two chaotic systems, a power law corresponding to an intermittency of type I is valid within a limited inter- 1
val of the supercriticality parameter, far from the point of transition to the PS regime. In the vicinity of this point, the laminar phases become very long and their distribution obeys the exponent

$$\langle l \rangle \sim \exp(k|P - P_c|^{-1/2}), \quad (3)$$

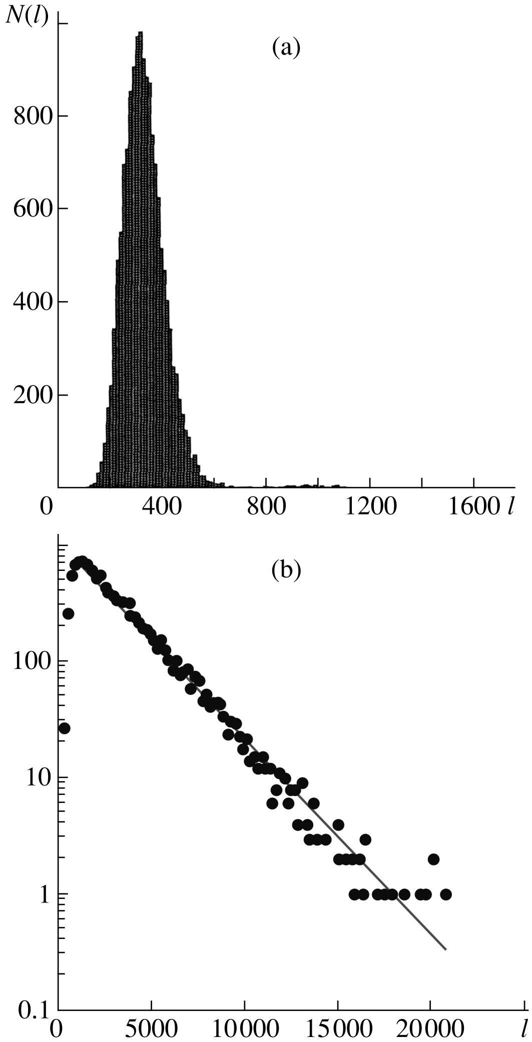

Fig. 1. Distributions of the laminar phase duration $M(l)$ (a) far from and (b) close to the PS threshold. In the latter case, vertical axis is drawn on the logarithmic scale (solid line corresponds to the exponent with the index –0.0004. The distribution is Gaussian bar from the PS domain and exponential near the PS threshold.

where k is a certain constant. This regime is called eye-
1 let intermittency [5].

Thus, the transition to PS in systems demonstrating chaotic behavior—irrespective of the character (periodic or chaotic) of the external signal—is accompanied
1 by a sequence of two intermittency types. As for the
1 systems with periodic dynamics, only the intermittency of type I was observed under the action of an external harmonic signal. At the same time, the effect of an external chaotic signal on the intermittent behavior in such systems has not been studied until now. It was
1 therefore of interest to know which types of intermittency are observed in this case and whether the character of the external signal influence the scenario of transition to a chaotic PS regime, or it is determined entirely by the dynamics of the driven system.

In order to answer these questions, let us study the behavior of the classical van der Pol autooscillator under the external chaotic action of the Rössler system:

$$\begin{aligned} \dot{x} &= \alpha(-\omega y - z), \\ \dot{y} &= \alpha(\omega x + ay), \\ \dot{z} &= \alpha(p + z(x - c)), \\ \ddot{u} - (\lambda - u^2)\dot{u} + u &= \varepsilon(Dy - \dot{u}), \end{aligned} \tag{4}$$

where $a = 0.15$. $p = 0.2$, and $c = 10$ are the control parameters of the Rössler system, $\omega = 0.9689$ is the eigenfrequency of this system, $\lambda = 0.1$ is the only control parameter in the van der Pol autooscillator, α is the parameter introduced in order to control (by analogy with [15]) the characteristic time scale of oscillations in the Rössler system, $D = 0.06634$ is the parameter selected so as to provide that the amplitudes of the Fourier components of the van der Pol autooscillator and the external signal in the autonomous regime would coincide, and ε is the coupling parameter. For the given control parameters and $\alpha = 1$, the main frequency components in the Fourier spectra of the Rössler and van der Pol systems coincide, which corresponds to the zero frequency detuning between these systems.

Let us set $\alpha = 0.99$ (weak frequency detuning between oscillators) and trace the transition to PS in the system under consideration. The coupling parameter corresponding to a threshold for the appearance of PS in system (4) for the indicated control parameters is $\varepsilon_c = 0.023$. For $\varepsilon < \varepsilon_c$, the regime of chaotic PS is broken and the system exhibits intermittency. In order to 1
determine the intermittency type, let us analyze (by 1
analogy with [9]) the statistical characteristics of the laminar phase durations l near ε_c and far from this threshold. Figure 1 shows distributions $N(l)$ of the l values for $\varepsilon = 0.01$ and 0.02. The number of analyzed phases in both cases was 10 000.

Far from the PS domain, the duration of laminar phases is virtually constant. In this case, the distribution is close to Gaussian (Fig. 1a). We can assume that, for a sufficiently weak coupling, system (4) features the intermittency of type I. Near the PS boundary, the lam- 1
inar phases rather strongly vary with the time and become very long (similar to the case of [9]). As can be seen from Fig. 1b, this distribution obeys the exponential law. In this case, the intermittency is of the eyelet 1
type [9].

Now let us consider how the average laminar phase duration $\langle l \rangle$ varies depending on the coupling parameters ε. Figure 2 shows a plot of $\langle l \rangle$ versus supercriticality $\varepsilon^* - \varepsilon$, where $\varepsilon^* = \varepsilon_c$ (the moment of transition to the PS regime) or $\varepsilon^* = \varepsilon_t \approx 0.0185$ (the transition from type I to eyelet intermittency regime, in which the distribu- 1
tion ceases to be Gaussian and the laminar phases become irregular). For $\varepsilon^* \lesssim \varepsilon_t$, the plot is described by the power law $\langle l \rangle \sim (\varepsilon_t - \varepsilon)^{-1/2}$ (the curve is constructed in a double logarithmic scale). For $\varepsilon_t \lesssim \varepsilon < \varepsilon_c$, we have $\langle l \rangle \sim \exp[k(\varepsilon_c - \varepsilon)^{-1/2}]$, where k is a constant factor (by

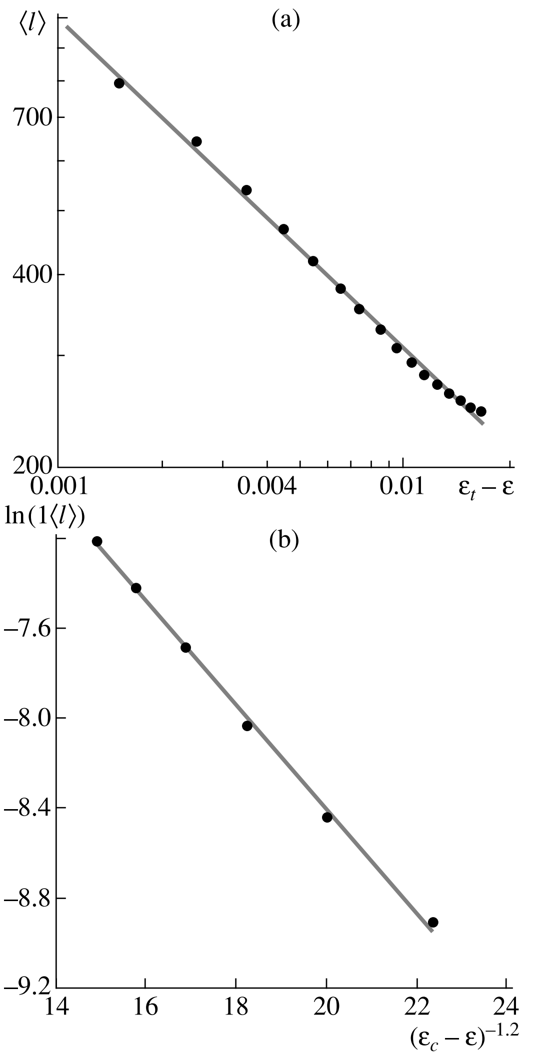

Fig. 2. Distributions of the average laminar phase duration depending on the supercriticality parameter: (a) $\langle l\rangle$ versus $\varepsilon_t - \varepsilon$ (double logarithmic scale); (b) $\ln(l/\langle l\rangle)$ versus $(\varepsilon_c - \varepsilon)^{-1/2}$. Solid lines correspond to the theoretical dependences (a) $<\langle l\rangle \sim (\varepsilon_t - \varepsilon)^{-1/2}$ and for $\varepsilon_t = 0.0185$ and (b) $\langle l\rangle \sim \exp[0.237(\varepsilon_c - \varepsilon)^{-1/2}]$for $\varepsilon_c = 0.023$.

analogy with [11], the curve in Fig. 2b is plotted as $\ln(1/\langle l\rangle)$ versus $(\varepsilon_c - \varepsilon)^{-1/2})$. Analogous dependences are observed for all values within $0.98 \lesssim \alpha \lesssim 1.02$ and, to within the substitution of variables, coincide with laws (2) and (3), respectively.

In conclusion, when an external chaotic signal acts on a system with periodic dynamics, the PS regime is preceded by the intermittent behavior. Similar to the case of two coupled chaotic systems, different types of intermittency are observed for small frequency detunings near the PS threshold and far from it. An analysis of the statistical characteristics of the laminar phase duration shows that, by analogy with the cases studied previously, type I intermittency is observed far from the point of transition to the PS regime, whereas the eyelet type intermittency is observed near this threshold. 1

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REFERENCES

1. P. Manneville and Y. Pomeau, Phys. Lett. A **75**, 1 (1979).
2. Y. Pomeau and P. Manneville, Commun. Math. Phys. **74**, 189 (1980).
3. P. Berge, Y. Pomeau, and C. Vidal, *L'Ordre Dans Le Chaos* (Hermann, Paris, 1988).
4. H. G. Schuster, *Deterministic Chaos* (Physik, Weinheim, 1984; Mir, Moscow, 1988).
5. A. S. Pikovsky, G. V. Osipov, M. G. Rosenblum, et al., Phys. Rev. Lett. **79**, 47 (1997).
6. A. E. Hramov, A. A. Koronovskiĭ, M. K. Kurovskaya, and S. Boccaletti, Phys. Rev. Lett. **97**, 114101 (2006).
7. A. E. Hramov and A. A. Koronovskiĭ, Europhys. Lett. **70**, 169 (2005).
8. S. Boccaletti and D. L. Valladares, Phys. Rev. E **62**, 7497 (2000).
9. K. J. Lee, Y. Kwak, and T. K. Lim, Phys. Rev. Lett. **81**, 321 (1998).
10. E. Rosa, E. Ott, and M. H. Hess, Phys. Rev. Lett. **80**, 1642 (1998).
11. S. Boccaletti, E. Allaria, R. Meucci, and F. T. Arecchi, Phys. Rev. Lett. **89**, 194101 (2002).
12. A. Pikovsky, M. Rosenblum, and J. Kurths, *Synchronization—A Universal Concept in Nonlinear Sciences* (Cambridge Univ. Press, Cambridge, 2001; Tekhnosfera, Moscow, 2003).
13. G. V. Osipov, A. S. Pikovsky, M. G. Rosenblum, and J. Kurths, Phys. Rev. E **55**, 2353 (1997).
14. A. A. Koronovskiĭ, M. K. Kurovskaya, O. I. Moskalenko, A. E. Hramov, Zh. Tekh. Fiz. **77** (1), 21 (2007) [Tech. Phys. **52**, 19 (2007)].
15. K. Pyragas, Phys. Rev. E **54**, R4508 (1996).

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SPELL: 1. intermittency, 2. multiplicators