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Generalized Synchronization in a System of Coupled Klystron Chaotic Oscillators

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Abstract—The phenomenon of generalized chaotic synchronization has been studied in a system of two unidirectionally coupled chaotic oscillators modeling two-resonator klystron autooscillators. A mechanism explaining the observed behavior is presented.

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The synchronization of chaotic oscillations in dynamical systems is among the basic phenomena extensively studied in recent years [1, 2]. According to the commonly accepted classification, there are several types of synchronous behavior distinguished in the analysis of coupled chaotic dynamical systems with small numbers of the degrees of freedom [2–7]. Among these, much attention has been devoted to the phenomenon of generalized synchronization (GS) of chaotic systems [3, 8]. The GS has been studied in detail in model systems, while a small number of such investigations have been applied to real physical objects. Of special interest among the real physical systems of this type are those of electron nature, which operate in complicated regimes and are widely used as high-power microwave radiation sources [9, 10].

The devices and systems of microwave electronics and radio physics constitute the basis of any information and telecommunication systems and are widely employed for data communication and processing, technological control, and basic research. Knowledge of the main mechanisms, which makes possible the synchronization of oscillations and the control of chaos in radioelectronic systems is very important from the standpoint of both basic science and various practical applications such as data transmission (including hidden communications) by chaotic signals [11, 12], remote sensing by chaotic signals, electronic countermeasures [13], and many others. In this context, it was also of interest to apply the GS concepts to experimental results obtained for klystron autooscillators with delayed feedback [14].

This Letter presents the results of a theoretical analysis of the GS phenomenon in a system of two-resonator klystron microwave chaotic oscillators with delayed feedback.

Let us consider a model system of unidirectionally coupled two-resonator klystrons with delayed feedback. The behavior of the drive and response autooscillators in this system can be described by the following system of equations [15, 16]:

$$\begin{aligned} \dot{F}_1^d(\tau) + \gamma F_1^d(\tau) &= \gamma F_2^d(\tau - \Delta\tau), \\ \dot{F}_2^d(\tau) + \gamma F_2^d(\tau) &= -2i\alpha e^{-i\psi} J_1(|F_1^d(\tau)|) \frac{F_1^d(\tau)}{|F_1^d(\tau)|}, \end{aligned} \tag{1}$$

$$\begin{aligned} \dot{F}_1^r(\tau) + \gamma F_1^r(\tau) &= \gamma (F_2^r(\tau - \Delta\tau) + \varepsilon F_2^d(\tau)), \\ \dot{F}_2^r(\tau) + \gamma F_2^r(\tau) &= -2i\alpha e^{-i\psi} J_1(|F_1^r(\tau)|) \frac{F_1^r(\tau)}{|F_1^r(\tau)|}, \end{aligned} \tag{2}$$

where the superscripts "d" and "r" refer to the drive and response systems, respectively; $F_1(\tau)$ and $F_2(\tau)$ are the normalized slowly varying complex amplitudes of the voltage oscillations in the input and output resonators, respectively; $J_1(\cdot)$ is the first-order Bessel function of the first kind; τ is the dimensionless time; $\Delta\tau = 1$ is the dimensionless delay time in the feedback chain; α is the parameter of resonator excitation (equivalent to a product of the gain and feedback coefficient); ψ is the total phase incursion for the time of signal travel in the feedback chain; γ is the decay parameter; and ε is the coupling parameter, which characterizes the signal decay in the circuit coupling the drive and response autooscillators. The values of control parameters were selected as follows: $\alpha = 10.9$; $\gamma = 1.0$; $\psi = 0.4875\pi$ [16]. With these control parameters, the model klystron autooscillators exhibited chaotic behavior in the autonomous regime ($\varepsilon = 0$). Since Eqs. (1) and (2) involve the delay, this system of equations was numerically solved using a single-step Euler method (with a time step of t = 0.001).

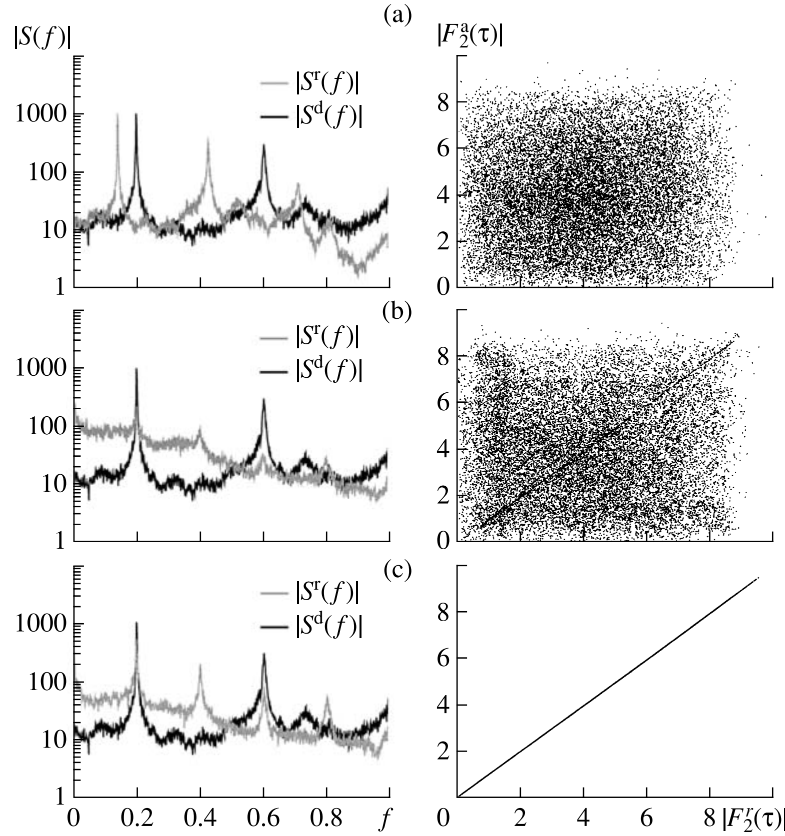

Fig. 1. Frequency spectra *S*(*f*) constructed using the time series of the drive ($F_2^d(\tau)$) and response ($F_2^r(\tau)$) oscillators and the corresponding patterns on the ($|F_2^r(\tau)|$, $|F_2^a(\tau)|$) plane for various values of the coupling parameter ε: (a) 0.0; (b) 0.89; (c) 0.93.

The state of a klystron autooscillator will be characterized by the output signal amplitude $F_2(\tau)$, representing a dimensionless complex amplitude of voltage oscillations at the output resonator exit. The GS regime implies that the output signal amplitudes of the drive ($F_2^d(\tau)$) and response ($F_2^r(\tau)$) oscillators are related by a certain function $G[\cdot]$, so that the two signals obey the functional relation $F_2^r(\tau) = G[F_2^d(\tau)]$ after termination of the transient process. A convenient approach to the diagnostics of GS between the drive and response systems under consideration is offered by the auxiliary system method [17]. According to this method, an additional response system (auxiliary autooscillator) identical to the original response system is introduced, for which the initial conditions are set different from those corresponding to the initial state of the response autooscillator. Let $F_2^a(\tau)$ denote the amplitude of the output voltage oscillations for the auxiliary autooscillator. If the GS between the interacting autooscillators is absent, the states of the response and auxiliary systems, $F_2^r(\tau)$ and $F_2^a(\tau)$, are always different. In contrast, when the GS takes place, the relations $F_2^r(\tau) = G[F_2^d(\tau)]$ and $F_2^a(\tau) = G[F_2^d(\tau)]$ are valid and, hence, the states of the response and auxiliary systems upon termination of the transient process must be identical: $F_2^r(\tau) \equiv F_2^a(\tau)$. Thus, the identity of states (i.e., of the amplitudes of voltage oscillations at the output resonators) of the response and auxiliary systems upon termination of the transient process (which can be rather lengthy [18]) is a criterion of the existence of GS between the drive and response systems. This is most clearly illustrated on the ($|F_2^r(\tau)|$, $|F_2^a(\tau)|$) plane, where a straight line representing the diagonal of this plane will be obtained in the GS regime.

For the GS diagnostics, a chaotic signal from the drive autooscillator was simultaneously fed to the response autooscillator (which demonstrated chaos in the autonomous regime) and to the auxiliary system. During the experiment, the coupling parameter ε was varied and the relationship between the response and auxiliary autooscillators in this system was monitored. It was established that, for ε ≈ 0.92, the states of the response and auxiliary systems are identical. As was

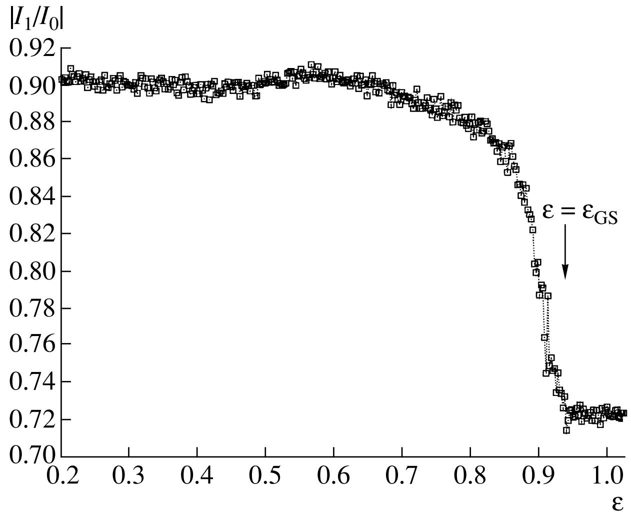

Fig. 2. A plot of the time-averaged relative amplitude of the first harmonic of the electron bunch current I_1/I_0 in the response klystron autooscillator versus coupling parameter ε. The arrow indicates the value of $\varepsilon = \varepsilon_{GS}$ for which the GS regime is established.

noted above, this condition is a criterion of the onset of GS in the initial system [17].

Figure 1 (left panels) shows the spectra constructed using the time series of the drive and response autooscillators and (right panels) the corresponding patterns on the $(|F_2^r(\tau)|, |F_2^a(\tau)|)$ plane for various values of the coupling parameter ε. Figure 1a illustrates the state with ε = 0.0, which corresponds to autonomous operation of both drive and response autooscillators. Figure 1b shows the spectra and $((|F_2^r(\tau)|, |F_2^a(\tau)|))$ pattern for ε = 0.89. As can be seen, the states of the drive and response autooscillators in this case are still different, but components corresponding to the frequencies of drive system appear in the spectrum of the response system. In other words, the regime of frequency synchronization between drive and response autooscillators is observed. For ε ≈ 0.93 (Fig. 1c), the states of the drive and response autooscillators are equivalent, which corresponds to the GS regime.

Let us consider possible mechanisms of the generalized chaotic synchronization of klystron chaotic oscillators. The observed phenomenon can be explained as follows. Previously, it was established [19] that a mechanism leading to the GS in unidirectionally coupled model systems is related to the suppression of intrinsic chaotic oscillations in the response system. Then, a question arises as to how the intrinsic chaotic dynamics can be suppressed in a klystron oscillator. As is known, the characteristics of klystron amplifiers are determined by the first harmonic of the electron bunch current [20, 21]. This feature is related to the fact that autooscillatory systems (such as klystrons) used in microwave electronics are characterized by pronounced resonance properties: since the bandwidth is very narrow, the output resonators can interact only with one harmonic component of the bunch current, namely, with the fundamental harmonic. Using relations of the theory of cascade electron bunching in the drift space, it is possible to obtain an expression for the first harmonic of the current within the framework of the model under consideration [15]:

$$I_1 = 2iI_0J_1\left(\frac{|F_1(\tau)|}{2}\right)e^{-i(\theta_0 - \arg(F_1(\tau)))}, \quad (3)$$

where I_0 is the electron beam current, θ_0 is the undisturbed flight angle in the drift space between the first and second resonator, and $\arg[F_1(\tau)]$ is the oscillation phase in the first resonator.

Figure 2 shows a plot of the I_1/I_0 ratio versus coupling parameter ε, where square symbols represent the I_1/I_0 values numerically calculated for various ε. As can be seen, ε ≈ 0.9 corresponds to a sharp change in the amplitude of the first harmonic. Therefore, we can ascertain that the action of the drive autooscillator on the response autooscillator in the case of ε ≈ 0.92 leads to breakage of the intrinsic chaotic oscillations in the latter system. Indeed, the results of experiments showed that, for this level of the normalized amplitude of the first harmonic of the bunch current ($I_1/I_0 \approx$ 0.72–0.73), the klystron can generate only a signal of single frequency. The chaotic generation is possible for $I_1/I_0 \approx$ 0.87–0.90.

Thus, the mechanism of GS establishment in the system under consideration is as follows. When a chaotic signal is fed from the drive to the response autooscillator, the regime of frequency synchronization is first established between the two autooscillators (Fig. 1b). If the autooscillators were operating in a periodic regime, the GS phenomenon (manifested by the coinciding amplitudes of output signals in the drive and response autooscillators) would be observed as well. However, the unsuppressed intrinsic dynamics of the response (and auxiliary) autooscillator does not allow a single-valued relationship to be established between the output signal amplitudes of the drive and response systems [19, 22]. As soon as the intrinsic chaotic dynamics in the response (and auxiliary) autooscillator is suppressed (Fig. 2), the GS regime is observed (Fig. 1c).

In conclusion, we have theoretically demonstrated the possibility of GS in a system of unidirectionally coupled klystron chaotic autooscillators. It should be noted that the obtained theoretical results have been confirmed by the investigation of GS in a system of unidirectionally coupled five-resonator klystron oscillators with delayed feedback. These experimental results will be considered in detail and compared to theory in a subsequent expanded publication.

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