ISSN 1063-7850, Technical Physics Letters, 2006, Vol. 32, No. 6, pp. 501–504. © Pleiades Publishing, Inc., 2006.
Original Russian Text © A.A. Koronovskiĭ, V.I. Ponomarenko, M.D. Prokhorov, A.E. Hramov, 2006, published in Pis'ma v Zhurnal Tekhnicheskoĭ Fiziki, 2006, Vol. 32, No. 11, pp. 81–88.

Detecting Synchronization of Self-Sustained Oscillators Using Wavelet Analysis of Univariate Data for Variable External Drive Frequency

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Received December 20, 2005

Abstract—A new method based on the continuous wavelet transform of univariate data is proposed for detecting the synchronization of a self-sustained oscillator under external drive action with linear frequency modulation. The efficacy of the proposed method is demonstrated in application to a model van der Pol oscillator and experimental physiological data.

PACS numbers: 05.45.xt

DOI: 10.1134/S1063785006060150

As is well known, the interaction between nonlinear oscillatory systems of different natures—including those featuring chaotic behavior—can lead to their synchronization [1]. In recent years, much effort was devoted to the investigation of synchronization in living organisms whose activity involves the interaction of a large number of complicated rhythmic processes [2, 3]. A convincing example of such interaction between various physiological rhythms is offered by the functioning of the cardiovascular system (CVS) in humans, where the possible synchronization of the main rhythms has been recently discovered [4–6]. It was also established that the system of arterial pressure regulation can be considered as a self-sustained oscillator under an external drive action, the latter being represented by breathing [6, 7].

Recently, we have developed [8–10] a new method based on the continuous wavelet transform, which can be used to detect the synchronization of a self-sustained oscillator under the action of an external drive with linear frequency modulation and to distinguish this situation from the case of spurious synchronization, whereby an external signal is superimposed on the self-sustained oscillations and the signals are simply mixed without a change in the oscillator frequency. The efficacy of the proposed method was demonstrated for a modified van der Pol oscillator and for a physiological experiment on heart rhythm synchronization by breathing at a rate linearly varied with time. In order to distinguish the synchronization from spurious superposition, we used the time series of the external driving action (in the physiological experiment, it was breathing) and the oscillator response (*R–R* intervals extracted from ECG records). It was established, in particular, that CVS oscillations can be synchronized at a frequency of about 0.1 Hz (Mayer waves), and the corresponding regime of synchronization for the variable drive frequency was determined.

In application to the analysis of physiological data, it is of special interest to use univariate data for detecting the epochs of synchronization between different rhythms [11–14]. This Letter describes a modification of the method developed previously in order to detect the synchronization by wavelet analysis of a scalar time series (e.g., the *R–R* intervals in physiological experiments).

We propose to analyze the synchronization using an approach known as time scale synchronization, which is based on the introduction of a set of phases $\varphi_s(t)$ corresponding to the time scales s of the time series under consideration. This set is determined using a continuous wavelet transform $W(s, t) = |W_s(t)|\exp[\varphi_s(t)]$ with a complex basis (for more details, see [15–17]). The base wavelet is chosen in the form of the Morlet wavelet $\psi_0(\eta) = (1/\sqrt[4]{\pi})\exp(j\sigma\eta)\exp(-\eta^2/2)$ [18], while selection of the wavelet parameter $\sigma = 2\pi$ provides for the relation $s = 1/f$ between the time scale s of the wavelet transform and the frequency f of the Fourier transform.

Following [8–10], we consider a theoretical model representing the asymmetric van der Pol oscillator under an external periodic drive action, which is described by the following equation:

$$\ddot{x} - \mu(1 - \alpha x - x^2)\dot{x} + \Omega^2 x = K\sin(\omega_L t), \qquad (1)$$

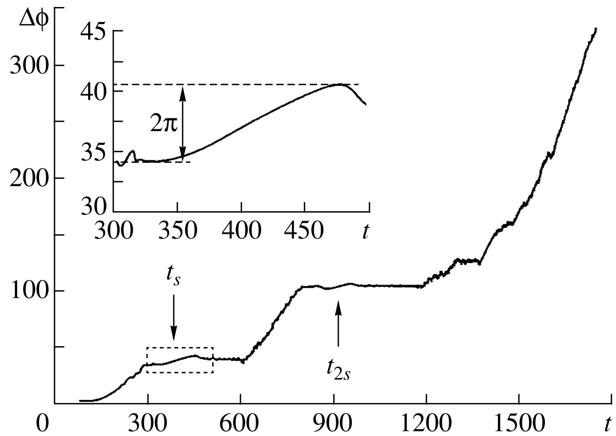

Fig. 1. Dynamics of the phase difference $\Delta\phi(t)$ between the first and second harmonics in the $x(t)$ signal of the asymmetric van der Pol oscillator under external action with an amplitude of $K = 0.2$ and the linearly increasing frequency $\omega_L/2\pi$. The inset shows the framed fragment on a greater scale. See the text for explanations.

with the parameters $\mu = 1.0$ and $\Omega = 0.24\pi$. Here, K is the amplitude of the external drive signal, ω_L is the external drive frequency that varies with time as $\omega_L = 2\pi[0.03 + (0.2 - 0.03t)/T]$, t is the current time, and $T = 1800$ is the maximum time. The parameters in Eq. (1) are selected so as to provide a convenient comparison to the results of physiological experiments. The case of $\alpha = 0$ corresponds to the classical van der Pol oscillator, which is characterized by a symmetric limit cycle. For the analysis of synchronization between processes in the CVS, we consider a model of the modified van der Pol oscillator with a quadratic nonlinearity ($\alpha = 1$). In this case, the power spectrum contains both even and odd harmonics of the fundamental frequency f_0, which is typical of real systems.

The main idea of the method proposed for detecting the synchronization using univariate data (scalar time series) consists in an analysis of the time variation of the phase difference $\Delta\phi(t) = \varphi_s(t) - \varphi_{s2}(t)/2$ corresponding to the first ($\varphi_s(t)$) and second ($\varphi_{s2}(t)$) harmonics of the signal. The external drive frequency linearly increases with the time and sequentially passes through the synchronization regimes 1 : 1, 1 : 2, …, 1 : n and so on. In the 1 : 1 regime, the motion from one synchronization wedge to another is accompanied by variation of the signal shape, which is evidence for a change in the phase difference between the two harmonics. If the amplitudes of these harmonics are sufficiently large to provide for the correct phase determination by means of the wavelet transform, the phase difference between the first and second harmonics in the absence of synchronization will remain unchanged. In the presence of synchronization, a change in the external signal frequency will cause a corresponding variation in the phase difference $\Delta\phi(t)$. Analogous situations are observed in the other 1 : n synchronization regimes.

In order to introduce the phases $\varphi_s(t)$ and $\varphi_{s2}(t)$ of the first and second harmonics, respectively, we use the aforementioned approach based on the wavelet transform of the signal. The phase difference $\Delta\phi(t)$ is constructed along the variable time scale $s_L(t) = 2\pi/\omega_L(t)$ that corresponds to the linearly varying frequency $\omega_L(t)$ of the external signal.

Figure 1 shows the results of calculations of the time variation of the phase difference $\Delta\phi(t)$ between the first and second harmonics of the $x(t)$ signal in the asymmetric van der Pol oscillator. As can be seen, the phase exhibits monotonic growth by 2π (clearly revealed in the inset to Fig. 1) in the vicinity of the time $t = t_s$ (indicated by the arrow), at which the external signal frequency is close to the frequency of self-sustained oscillations $\omega_L(t_s) \approx \Omega$. This phase change is indicative of the synchronization. It should be noted that an analogous feature in the phase dynamics is observed at $t = t_{2s}$ (as also indicated by the arrow in Fig. 1), where the external signal frequency is close to the second harmonic of the van der Pol oscillator: $\omega_L(t_{2s}) \approx 2\Omega$.

Thus, the presence of regions in which $\Delta\phi(t)$ exhibits a monotonic change by a multiple of π at the moments of time where the external signal frequency is close to the fundamental frequency (or its harmonics) is evidence of synchronization of the self-sustained oscillator.

It should be noted that the presence of an external signal with variable frequency is of principal importance for detecting the epochs of synchronization using the phase difference between the first and second harmonics. Indeed, if the external signal had a constant frequency, the phase difference $\Delta\phi(t)$ would not change with time. An analogous behavior would also be observed in the absence of synchronization.

Now let us proceed with an analysis of the physiological time series generated by the CVS and the respiratory system in humans. We have tested seven healthy male volunteers aged from 20 to 34. All participants had an average level of physical activity. The ECG and breathing patterns were simultaneously recorded for each participant in a sitting position. The data were digitized at a sampling frequency of 250 Hz with 16-bit resolution and fed into a computer for data storage and processing.

Each participant performed an experiment with breathing according to a preset rhythm with frequency f_b varying from 0.05 to 0.3 Hz. The rhythm was set by 0.5-s-long acoustic signals, whereby the participant took a breath with each arrival of the signal. The test results [8–10] showed that the breathing and CVS

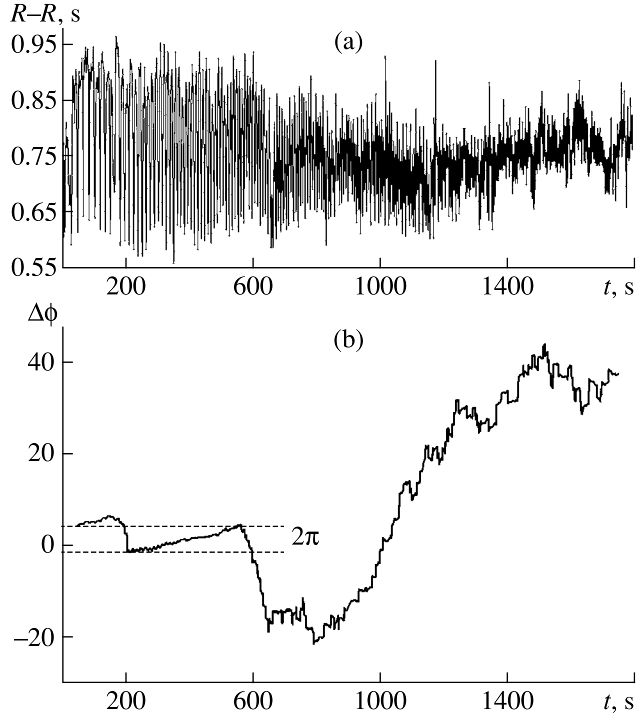

Fig. 2. The typical results of a physiological experiment: (a) a time series of the *R–R* intervals measured during breathing at a linearly varying frequency; (b) dynamics of the phase difference $\Delta\phi(t)$ between the first and second harmonics of the linearly varying breathing rate $f_b(t)$, constructed for a sequence of *R–R* intervals.

rhythms were synchronized for the time intervals within $t_s \in$ [200 s, 600 s], which corresponds to the external drive frequencies in the interval $f_L \in$ (0.078 Hz, 0.13 Hz).

By extracting a sequence of *R–R* intervals (representing time intervals T_i between two sequential *R* peaks) from ECG records, we obtain information on the heart rhythm variability. Figure 2a shows a typical time series of the *R–R* intervals observed in the experiment on breathing with a linearly increasing rate. Note that, since the period of time T_i between counts in the sequence of *R–R* intervals was not constant, we developed and used a special method for the continuous wavelet transform of time series with nonequidistant time counts.

Figure 2b shows the time variation of the phase difference between the time scale $s_L(t) = 1/f_L(t)$ corresponding to the breathing with a linearly increasing frequency and the second harmonic $s_{2L}(t) = 1/(2f_L(t))$ in the sequence of *R–R* intervals. As can be seen, the average phase difference $\Delta\varphi(t)$ exhibits an almost linear variation (indicated by the dashed lines) in the interval from 200 to 600 s. This behavior is evidence for a synchronization in the corresponding frequency region. Outside this region, the phase difference exhibits rapid variations, which is indicative of the absence of synchronization regimes. From this result, we can conclude that the dynamics of breathing (as manifested in the *R–R* signal time series) in the interval from 200 to 600 s influences the internal rhythm of the Mayer wave with a frequency of 0.1 Hz. Outside this interval, the *R–R* signal merely reflects the breathing rhythm, which does not interact with that of the Mayer waves. Analogous results were obtained upon the wavelet analysis of the experimental time series for all test participants.

Thus, the results of numerical calculations and physiological experiments show that it is possible to detect the synchronization of a self-sustained oscillator with an external signal, by using the wavelet analysis of a single univariate (scalar) time series characterizing the response of the oscillator to the external action with a variable frequency. A necessary condition for the application of this method is the possibility to vary the frequency of the external action. The proposed method can also be used in other experiments on physiological systems for the diagnostics of synchronization using univariate data, provided that the experimental conditions allow the external drive frequency to be varied.

Acknowledgments. This study was supported in part by the Russian Foundation for Basic Research (project nos. 05-02-16305 and 05-02-16273) and the US Civilian Research and Development Foundation for the Independent States of the Former Soviet Union (CRDF award No. REC-006). A.E.H. and A.A.K. are grateful to the "Dynasty" Noncommercial Program Foundation for support; M.D.P. gratefully acknowledges the support from INTAS (grant no. 03-55-920).

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Translated by P. Pozdeev