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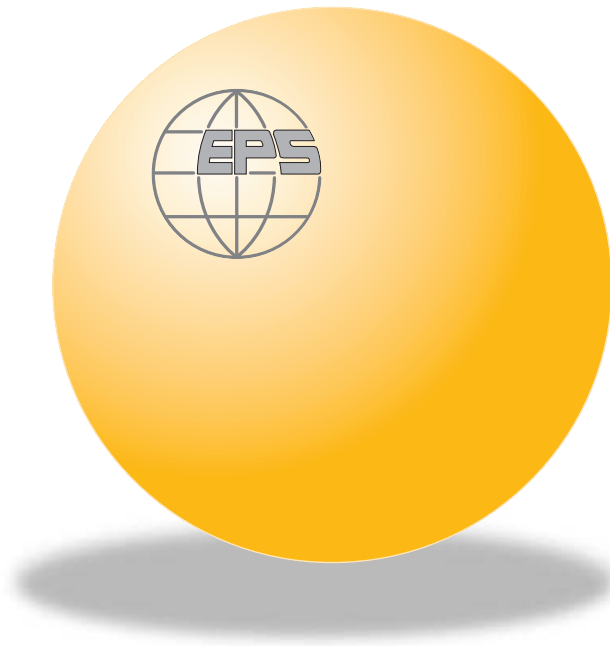
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Intermittent generalized synchronization in unidirectionally coupled chaotic oscillators

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Abstract. – A new type of behavior of unidirectionally coupled chaotic oscillators near the transition to generalized synchronization has been detected. It has been shown that the appearance of generalized synchronization is preceded by intermittent behavior: close to the threshold parameter value the coupled chaotic systems demonstrate generalized synchronization most of the time, but there are time intervals during which the synchronized oscillations are interrupted by non-synchronous bursts. This type of system behavior has been called intermittent generalized synchronization (IGS) by analogy with intermittent lag synchronization (ILS) (*Phys. Rev. E*, **62** (2000) 7497).

Synchronization [1,2] of interacting chaotic oscillators is one of the fundamental phenomena of nonlinear dynamics. Recently, several types of chaotic synchronization have been observed in coupled nonlinear oscillators. These are phase synchronization (PS) [3–5], lag synchronization (LS) [6–8], complete synchronization (CS) [9–12] and generalized synchronization (GS) [13–15]. It is important to note that GS may also take place in the non-oscillatory chaotic systems (see [16] for details). All synchronization types are interrelated (see, for details, [7, 13, 17]), but the relationship between them is not completely clarified yet. In recent works [18–21], it has been shown that all these synchronization types may be considered from a common point of view as different manifestations of one universal phenomenon.

It has also been found that onsets of phase and lag synchronization types are preceded by intermittent behavior. Close to the threshold parameter value, the coupled chaotic systems demonstrate synchronized dynamics most of the time, but there are time intervals during which the synchronized oscillations are interrupted by a non-synchronous behavior. These pre-transitional intermittencies have been described in [6, 22, 23] for the case of lag synchronization and in [24–26] for phase synchronization, respectively.

Due to the existence of a unifying framework of coupled chaotic oscillators synchronization one can also expect intermittent behavior at the threshold of the appearance of generalized synchronization. In this work we consider the behavior of coupled chaotic oscillators close to the coupling parameter value corresponding to the onset of the generalized-synchronization regime. As will be shown below, the appearance of generalized synchronization is also preceded

by intermittent behavior in the same way as the phase synchronization and lag synchronization regimes do. This type of system behavior we called *intermittent generalized synchronization* (IGS) by analogy with intermittent lag synchronization (see [22]).

The *generalized-synchronization* (GS) [13–15] regime introduced for the drive $\mathbf{x}(t)$ and response $\mathbf{u}(t)$ systems means that there is some functional relation between unidirectionally coupled chaotic oscillators, *i.e.* $\mathbf{u}(t) = \mathbf{F}[\mathbf{x}(t)]$. This functional relation $\mathbf{F}[\cdot]$ can be very complicated, but there are several methods to detect the synchronized behavior of coupled chaotic oscillators, *e.g.*, the method of nearest neighbors [13, 27]. In our work we have used the auxiliary system approach proposed in [28]. In this case, we consider the dynamics of the drive

$$\dot{\mathbf{x}}(t) = \mathbf{H}(\mathbf{x}(t)) \quad (1)$$

and response

$$\dot{\mathbf{u}}(t) = \mathbf{G}(\mathbf{u}(t), \mathbf{g}, \mathbf{x}(t)) \quad (2)$$

systems. The vector \mathbf{g} characterizes the coupling of the drive $\mathbf{x}(t)$ and response $\mathbf{u}(t)$ systems. At the same time, we also consider the dynamics of the auxiliary system

$$\dot{\mathbf{v}}(t) = \mathbf{G}(\mathbf{v}(t), \mathbf{g}, \mathbf{x}(t)), \quad (3)$$

which is identical to the response system $\mathbf{u}(t)$ but starts with other initial conditions, *i.e.*, $\mathbf{u}(t_0) \neq \mathbf{v}(t_0)$. In the absence of generalized synchronization between the drive $\mathbf{x}(t)$ and response $\mathbf{u}(t)$ systems, the phase trajectories of the response $\mathbf{u}(t)$ and auxiliary $\mathbf{v}(t)$ systems share the same chaotic attractor but are otherwise unrelated. In the case of generalized synchronization, the orbits of the response $\mathbf{u}(t)$ and auxiliary $\mathbf{v}(t)$ systems become identical after the transient dies out due to the generalized-synchronization relations $\mathbf{u}(t) = \mathbf{F}[\mathbf{x}(t)]$ and $\mathbf{v}(t) = \mathbf{F}[\mathbf{x}(t)]$. It is obvious that in the case of generalized synchronization the condition $\mathbf{u}(t) = \mathbf{v}(t)$ should be satisfied and the identity of response and auxiliary systems is a much simpler criterion to test the presence of generalized synchronization rather than the detection of the unknown functional relationship $\mathbf{F}[\cdot]$.

For the coupling parameter values close to the onset of generalized synchronization, the drive and response systems demonstrate the regime of intermittent generalized synchronization. In this case, the generalized-synchronization regime between the drive and response systems has been observed most of the time and at that time the vector states of the response $\mathbf{u}(t)$ and auxiliary $\mathbf{v}(t)$ systems coincide with each other. Nevertheless, there are time intervals when the drive and response systems behave unrelately. Therefore, during these time intervals the vector states of the response $\mathbf{u}(t)$ and auxiliary $\mathbf{v}(t)$ systems differ from each other. If we plot the difference between the vector states of the response and auxiliary systems $|\mathbf{u}(t) - \mathbf{v}(t)|$ *vs.* time variable t , then a graph will be obtained where the laminar phases corresponding to the generalized-synchronous behavior alternate with the turbulent ones when the generalized synchronization is not detected. In other words, in the case of IGS for some time intervals one can detect the presence of the functional relation $\mathbf{F}[\cdot]$ between the drive and response systems, whereas for the other time intervals such functional relation does not exist.

Let us consider the dynamics of two unidirectionally coupled Rössler systems:

$$\begin{aligned} \dot{x}_1 &= -\omega_1 x_2 - x_3, & \dot{u}_1 &= -\omega_2 u_2 - u_3 + \varepsilon(x_1 - u_1), \\ \dot{x}_2 &= \omega_1 x_1 + ax_2, & \dot{u}_2 &= \omega_2 u_1 + au_2, \\ \dot{x}_3 &= p + x_3(x_1 - c), & \dot{u}_3 &= p + u_3(u_1 - c), \end{aligned} \quad (4)$$

where $\mathbf{x} = (x_1, x_2, x_3)^T$ and $\mathbf{u} = (u_1, u_2, u_3)^T$ are the vector states of the drive and response systems, respectively. The control parameter values have been chosen as $\omega_1 = 0.99$, $\omega_2 = 0.95$,

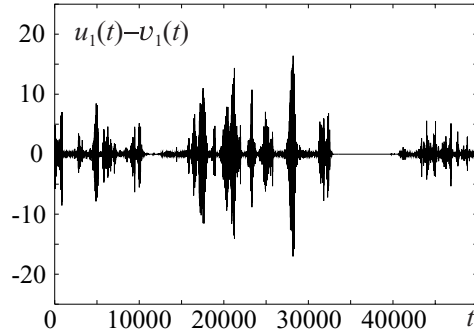


Fig. 1 – The dependence of the difference between the coordinates of the response and auxiliary systems $u_1(t) - v_1(t)$ on time. The coupling parameter between the drive and response systems has been selected as $\varepsilon = 0.106$.

$a = 0.15$, $p = 0.2$, $c = 10$; the parameter ε characterizes the coupling strength between the considered systems. At the same time, the behavior of the auxiliary system $\mathbf{v}(t)$ described also by eq. (4) should be considered in order to detect the presence of the generalized-synchronization regime. As follows from our investigation, PS arises at $\varepsilon_{PS} \approx 0.116$, GS takes place at $\varepsilon_c \approx 0.11$, and LS can be observed for $\varepsilon \geq \varepsilon_{LS} \approx 0.36$. So, GS for the coupling strength exceeding slightly the critical value ε_c is the weak synchronization [15,16] and, therefore, the relationship $\mathbf{F}[\cdot]$ demonstrates fractal properties. For $\varepsilon \geq \varepsilon_{LS}$, GS becomes the strong synchronization as well as $\mathbf{F}[\cdot]$ begins being smooth. In this case, the behavior of the drive and response systems becomes identical but they are shifted in time *vs.* each other. Close to the onset of the GS, the regime of IGS has been detected. The difference between the coordinates of the response and auxiliary Rössler systems is shown in fig. 1.

The time intervals where the difference $u_1(t) - v_1(t)$ is close to zero correspond to the laminar phases of coupled oscillator dynamics when the presence of the functional relation $\mathbf{F}[\cdot]$ between the drive and response systems can be detected. Alternatively, irregular bursts indicate the breaking of the relationship $\mathbf{F}[\cdot]$. It is interesting to note that the amplitude of the irregular bursts is large enough and comparable with the size of the chaotic attractor of the Rössler system.

This type of behavior demonstrates the features of on-off intermittency [29,30] which usually appears at the desynchronization of two coupled identical chaotic oscillators. To analyze the statistical properties of IGS, we numerically compute the distribution $N(\tau)$ of the laminar phase length τ . We suppose that the current state is the laminar phase if the difference between the variables $u_1(t)$ and $v_1(t)$ is less than $\Delta = 0.1$, otherwise the current state is assumed to be an irregular burst.

The distribution of the laminar phase length is shown in fig. 2. One can see that this distribution is close to the power law with exponent $n = -3/2$. It should be noticed that this result does not sensitively depend on the value of the threshold Δ .

The other criterium of the on-off intermittency presence is the dependence of the mean laminar phase length $\langle \tau \rangle$ on the deviation of the coupling strength ε from the critical value ε_c . Figure 3 shows the numerically determined mean laminar phase length $\langle \tau \rangle$ *vs.* deviation $(\varepsilon_c - \varepsilon)$. From this figure one can see the universal power law $\langle \tau \rangle \sim (\varepsilon_c - \varepsilon)^k$ with critical exponent $k = -1$. Therefore, the IGS appears to be an on-off intermittency. It is important to note that the ILS is also an on-off intermittency regime (see, for details, [23]).

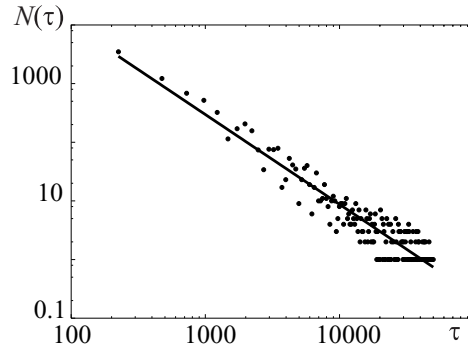


Fig. 2 – The statistical distribution of laminar phases and its approximation $N(\tau) \sim \tau^{-3/2}$ in log-log scale. The coupling parameter between the drive and response systems has been selected as $\varepsilon = 0.106$, the threshold value is $\Delta = 0.1$.

It is important to note that the attractor of the considered Rössler system is coherent for the selected values of the control parameters. At the same time we have observed the IGS presence for the different values of the control parameters when the system behavior is characterized by the funnel attractor (see, *e.g.*, the parameter values pointed out in [5]). The character of the chaotic attractor does not seem to be important for IGS like the parameter mismatch influence: IGS may be observed both for small and large mistuning of the oscillators parameters.

The intermittent generalized-synchronization regime seems to appear in different coupled dynamical systems. To examine this assumption, we have considered the dynamics of two unidirectionally coupled Chua's circuits [31, 32] with piecewise linear characteristics (see fig. 4). The typical property of the considered Chua's circuits is the presence of two distinct characteristic time scales. Therefore, this system may demonstrate periodical, quasi-periodical and chaotic oscillations, depending on the values of the control parameters. The behavior of the considered Chua's circuits in the autonomous regime is chaotic due to the selection of the control parameter values.

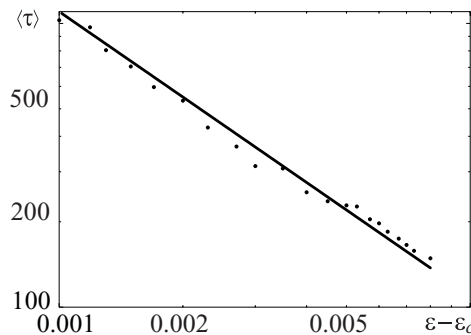


Fig. 3 – The log-log plot of the mean laminar phase length $\langle \tau \rangle$ vs. the deviation $(\varepsilon_c - \varepsilon)$ of the coupling strength from the critical value and its approximation by $\langle \tau \rangle \sim (\varepsilon_c - \varepsilon)^{-1}$. A power law with critical exponent -1 has been observed.

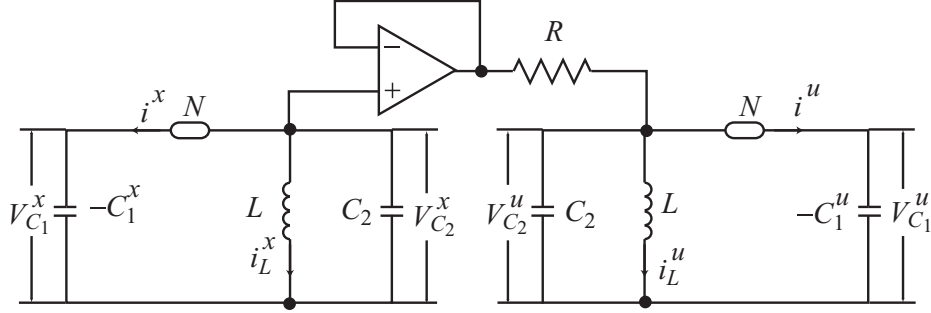


Fig. 4 – Circuit realization of two unidirectionally coupled Chua's oscillators.

The behavior of the drive Chua's circuit is described by

$$\begin{aligned}\dot{x}_1 &= -\frac{\alpha_x}{\gamma} f(x_2 - x_1), \\ \dot{x}_2 &= -\frac{1}{\gamma} (f(x_2 - x_1) + x_3), \\ \dot{x}_3 &= \gamma x_2,\end{aligned}\tag{5}$$

and the equations of the response system are the following:

$$\begin{aligned}\dot{u}_1 &= -\frac{\alpha_u}{\gamma} f(u_2 - u_1), \\ \dot{u}_2 &= -\frac{1}{\gamma} (f(u_2 - u_1) + u_3) + \frac{\varepsilon}{\gamma} (x_2 - u_2), \\ \dot{u}_3 &= \gamma u_2.\end{aligned}\tag{6}$$

The variables $\{x, u\}_1 = V_{C_1^{x,u}}/E$ and $\{x, u\}_2 = V_{C_2^{x,u}}/E$ are the dimensionless voltages on the capacitors $C_1^{x,u}$ and C_2 of the first (drive) and the second (response) oscillators, respectively. The variables $\{x, u\}_3 = i_L^{x,u}/I$ are the dimensionless currents through the inductances L in the drive and response circuits (see fig. 4 for details). The parameters E and I are the normalization factors. $\alpha_{x,u} = C_2/C_1^{x,u}$ and $\gamma = \frac{1}{m_1} \sqrt{C_2/L}$ are dimensionless control parameters; $\xi = t/\sqrt{LC_2}$ is the dimensionless time. The function

$$f(\varsigma) = -\frac{m_0}{m_1} \varsigma + \frac{1}{2} \left(\frac{m_0 + m_1}{m_1} \right) (|\varsigma + 1| - |\varsigma - 1|),\tag{7}$$

is the dimensionless piecewise linear voltage-current characteristic of the nonlinear element N , where m_0 and m_1 are the conductivities of the corresponding branches of the voltage-current characteristic. The ratio m_0/m_1 has been selected as $1/2$. The coupling parameter $\varepsilon = 1/Rm_1$ determines the influence of the drive Chua's circuit on the response one.

In fig. 5 the dependence of the difference between the dimensionless voltages on the capacitors $C_1^{u,v}$ of the response and auxiliary systems on the dimensionless time ξ is shown. One can see the presence of intermittent generalized synchronization between the drive and response Chua's circuits.

Let us briefly discuss the nature of IGS. For this purpose, we consider the extended phase space $\mathbb{D} \oplus \mathbb{R} \oplus \mathbb{A}$ consisting of the phase space of the drive (\mathbb{D}), response (\mathbb{R}) and auxiliary \mathbb{A} systems. We can also consider the subspaces $\mathbb{D} \oplus \mathbb{R}$ and $\mathbb{R} \oplus \mathbb{A}$ of $\mathbb{D} \oplus \mathbb{R} \oplus \mathbb{A}$. The presence

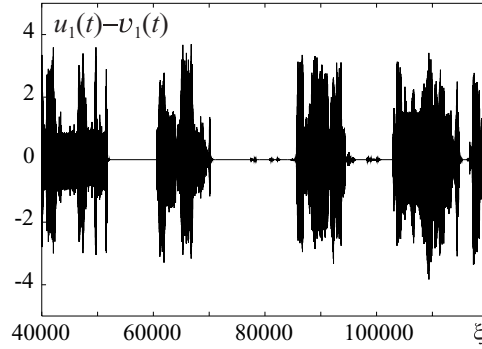


Fig. 5 – The dependence of the difference $u_1(t) - v_1(t)$ between the coordinates of the response and auxiliary Chua's circuits on the dimensionless time ξ . The coupling parameter between the drive and response systems has been selected as $\varepsilon = 0.03105$, the control parameters are $\alpha_x = 2.78$, $\alpha_u = 2.89$, $\gamma = 3.00$.

of GS between the drive and response systems implies the complete synchronization between the response and auxiliary systems. As criterion of GS, one can consider the negativity of the values of the conditional Lyapunov exponents (CLEs) [10, 15] in the $\mathbb{D} \oplus \mathbb{R}$ phase subspace, while the criterion of CS is the negativity of the transversal Lyapunov exponents (TLEs) [2] in the $\mathbb{R} \oplus \mathbb{A}$ subspace. Following the ideas of [15], one can draw the conclusion that CLEs (in $\mathbb{D} \oplus \mathbb{R}$) and TLEs (in $\mathbb{R} \oplus \mathbb{A}$) coincide with each other. Therefore, when GS is destroyed in $\mathbb{D} \oplus \mathbb{R}$ with decreasing coupling parameter, the blowout bifurcation [33, 34] takes place in the $\mathbb{R} \oplus \mathbb{A}$ subspace. At the same time it is well known that on-off intermittency takes place below the coupling parameter value corresponding to the blowout bifurcation. So, the on-off intermittency has to appear in the $\mathbb{D} \oplus \mathbb{R}$ subspace below the threshold of arising GS.

Taking this idea into account, one can draw another conclusion concerning the noise influence. As the noise results in the bubbling in the case of the identical chaotic oscillators for the coupling parameter values above the point of blowout bifurcation, similar effects will be probably observed for GS. It means that the intermittent behavior may be observed in the experiments for the coupling parameter values corresponding to the GS regime due to the noise influence.

In conclusion, we have detected a new type of behavior of unidirectionally coupled chaotic oscillators near to the onset of generalized synchronization. This type of behavior has been called intermittent generalized synchronization. It should be noted that a similar effect has been observed for the unidirectionally coupled time-delayed chaotic oscillators [35]. As the GS phenomenon could also be found in the non-oscillatory chaotic systems (see, for details, the monographic review [16] dealing with synchronization phenomena in chaotic systems), IGS can also be found in such systems below the threshold of arising GS, correspondingly.

The intermittent generalized synchronization seems to be a common phenomenon in nonlinear systems, and two unidirectionally coupled chaotic oscillators usually undergo the transition to generalized synchronization through on-off intermittency. The intermittent behavior of coupled chaotic oscillators close to the onset of PS, LS and GS is likely to be the manifestation of universalities underlying the synchronization phenomenon.

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