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Spatiotemporal Chaos Synchronization in Beam–Plasma Systems with Supercritical Current

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Abstract—It is established that coupled beam–plasma systems with supercritical current can feature the phenomenon of chaotic synchronization. As the coupling between subsystems increases, a distributed beam–plasma system exhibits the transition from asynchronous behavior via phase synchronization to the state of complete chaotic synchronization. The phenomenon of chaotic synchronization has been studied using a method developed previously based on the introduction of a continuous manifold of phases of the chaotic signal. *© 2005 Pleiades Publishing, Inc.*

In recent years, the phenomenon of synchronization of chaotic dynamical systems with small numbers of the degrees of freedom has been extensively studied [1]. According to modern classification, there are several types of chaotic synchronization, including generalized [2], phase [1], lag [3], and complete synchronization [4]. The interest in these phenomena is related, in particular, to the possibility of data transmission by means of chaotic oscillations [5]. The generalized synchronization implies that there exists a certain function $\mathbf{F}[\mathbf{x}_1(t)]$ relating the states of chaotic oscillators such that $\mathbf{x}_2(t) = \mathbf{F}[\mathbf{x}_1(t)]$, where $\mathbf{x}_{1,2}$ are the state vectors of the two coupled systems. The phase synchronization is described in terms of the phase $\phi(t)$ of a chaotic signal and implies that the phases of the chaotic signals are entrained, while their amplitudes remain uncorrelated and appear chaotic. By the lag synchronization, we imply a regime in which the dynamics of one subsystem is characterized by a certain delay time τ relative to another: $\mathbf{x}_1(t) \approx \mathbf{x}_2(t - \tau)$. Finally, the complete synchronization means fully identical dynamics of two chaotic oscillators: $\mathbf{x}_1(t) \approx \mathbf{x}_2(t)$.

Previously, it has been shown [6, 7] that the generalized, phase, lag, and complete synchronization regimes are closely related, being essentially various manifestations of the same type of synchronous dynamics of coupled oscillators, referred to as the time scale synchronization. The character of a particular regime (featuring phase, lag, or complete synchronization) is determined by the number of synchronized time scales s introduced by means of a continuous wavelet transform [8]. Since the time scale s is related to a certain frequency, the synchronization of chaotic oscillations implies the appearance of phase coupling between the components ω of the Fourier spectra $S(\omega)$ [9].

As was noted above, the main results in investigations of the chaotic synchronization were obtained for the dynamical systems with a small number of degrees of freedom. However, it would be also of interest to study this phenomenon in spatially distributed systems featuring chaotic behavior. Most of the investigations in this direction were performed for the phenomenological models of systems comprising lattices of coupled oscillators [10, 11] or standard equations in partial derivatives (e.g., of the Ginzburg–Landau [12, 13] or Kuramoto–Sivashinsky [14] types). Detailed investigations of the chaotic synchronization in beam–plasma systems were very few (see, e.g., [15]). The transitions between different types of chaotic synchronization in such systems were not studied and the analogies with these phenomena known in systems with a small number of degrees of freedom were not revealed so far.

This paper reports on the results of investigation of the chaotic synchronization in coupled beam–plasma systems with supercritical current representing hydrodynamical models of the Pierce diode (see [16, Lecture 4]), which are of considerable interest as the models of plasma systems featuring various types of chaotic behavior [16, 18–23].

In a model Pierce diode, an initially monoenergetic electron beam with a constant space charge density (neutralized by an immobile ion background) moves between two grounded grids. The only control parameter determining the system dynamics is the Pierce parameter $\alpha = \omega_p L/v_0$ representing the unperturbed angle of electron motion (ω_p is the plasma frequency, L is the distance between grids, and v_0 is the initial electron velocity at the system input). For $\alpha > \pi$ [1–3], the so-called Pierce instability is developed in the system that leads to the formation of a virtual cathode and the

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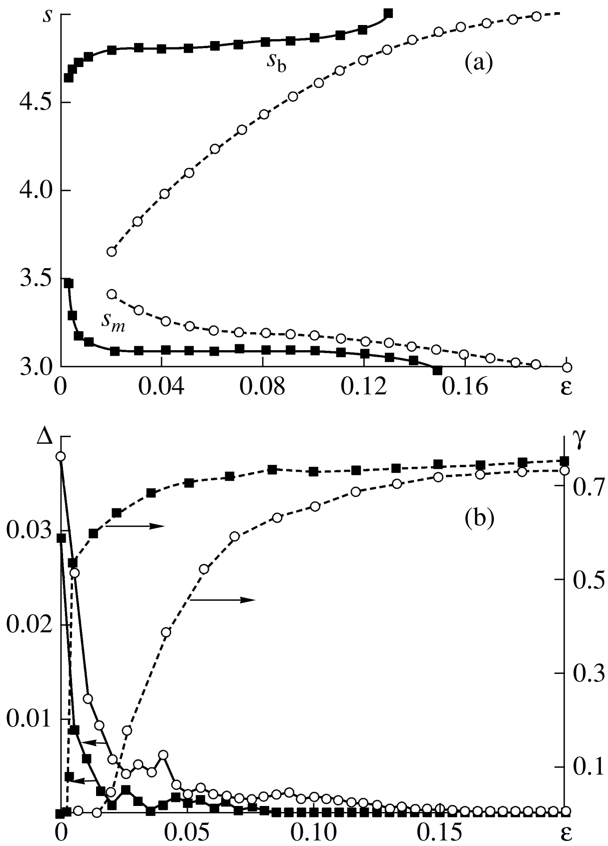

Fig. 1. Plots of (a) the lower (s_m) and upper (s_b) boundaries of the interval of synchronized time scales and (b) the parameter of identity (Δ) and relative energy (γ) of the synchronized spatiotemporal oscillations versus coupling parameter ε for the two coupled Pierce diodes with small ($\alpha_1/\pi = 2.860$, $\alpha_2/\pi = 2.861$] (■)) and large ($\alpha_1/\pi = 2.860$, $\alpha_2/\pi = 2.858$] (○)) detunings.

establishment of a multiflow state in the beam (for detail, see [16, 22]). However, a complete transmission of the electron beam is possible for $\alpha \sim 3\pi$, which makes possible description of the system within the framework of a hydrodynamical approximation This description shows that various types of chaotic oscillations can be observed in this beam–plasma system [16, 18, 20].

Let us consider two coupled Pierce diodes described within the framework of the hydrodynamical approximation by a self-consistent system including the equation of motion, the equation of continuity, and the Poisson equation in dimensionless variables [16]

$$\frac{\partial v_{1,2}}{\partial t} + v_{1,2}\frac{\partial v_{1,2}}{\partial x} = \frac{\partial \varphi_{1,2}}{\partial x}, \qquad (1)$$

$$\frac{\partial \rho_{1,2}}{\partial t} + v_{1,2}\frac{\partial \rho_{1,2}}{\partial x} + \rho_{1,2}\frac{\partial v_{1,2}}{\partial x} = 0, \qquad (2)$$

$$\frac{\partial^2 \varphi_{1,2}}{\partial x^2} = \alpha_{1,2}^2(\rho_{1,2} - 1), \qquad (3)$$

with the boundary conditions

$$v_{1,2}(0, t) = 1, \quad \rho_{1,2}(0, t) = 1, \quad \varphi_{1,2}(0, t) = 0, \qquad (4)$$

where subscripts 1 and 2 refer to the first (master) and second (slave) systems, respectively. Equations (1)–(4) for the hydrodynamical model of the Pierce diode are written in the dimensionless variables of the space charge field potential φ, charge density ρ, electron beam velocity v, spatial coordinate x, and time t (for detail, see [16]).

The coupling between the two systems under consideration is provided through variation of the dimensionless potential at the right-hand boundaries of the systems:

$$\begin{aligned} &\varphi_{1,2}(x = 1.0, t) \\ &= \varepsilon(\rho_{2,1}(x = 1.0, t) - \rho_{1,2}(x = 1.0, t)), \end{aligned} \qquad (5)$$

where ε is the coupling parameter and $\rho_{1,2}(x = 1.0, t)$ are the oscillations of the dimensionless space charge density at the system output. In what follows, we will consider the dynamics of the system at a fixed value of the parameter α_1 ($\alpha_1 = 2.861\pi$) for the variable control parameter α_2.

The results of investigations show that weak detuning between coupled chaotic systems leads to the establishment of a time scale synchronization, which can be determined by introducing a continuous manifold of phases $\phi_s(t)$ of the chaotic signal on various time scales s with the aid of a continuous wavelet transform [6, 7]. The time series of the coupled systems were analyzed in terms of chaotic oscillations in the space charge densities $\rho_{1,2}(x = 0.2, t)$.

The dynamics of the coupled systems under consideration is illustrated in Fig. 1a constructed for $\alpha_1 = 2.860\pi$ and $\alpha_2 = 2.861\pi$, which shows the variation of the range of synchronous scales s_m and s_b with an increase in the coupling parameter ε [6]. As can be seen, time scales on which the system dynamics is synchronized appear at $\varepsilon > 0.0007$. It was demonstrated previously [7] that this state corresponds to phase synchronization of the chaotic oscillations. As the coupling parameter ε grows, the interval of synchronized scales increases and at $\varepsilon \approx 0.08$–0.1 the system dynamics is synchronized virtually in the entire range of time scales. Here, the coupled beam-plasma systems occur in a regime close to the state of lag synchronization with a delay time of $\tau \approx 0.07$. The further growth in ε leads to a decrease in the delay between oscillations and the system tends to the state of complete chaotic synchronization characterized by nearly identical dynamics of each system ($\tau \approx 0$).

At a greater detuning of the parameters of coupled beam-plasma systems, which corresponds to a significantly more complex spectral composition of oscillations in the electron beam, the onset of synchronism between the time scales is observed for greater values of the coupling parameter. Figure 1a also shows the corresponding boundaries [s_m, s_b] in the case of $\alpha_1 = 2.860\pi$ and $\alpha_2 = 2.858\pi$. As ε increases, the coupled systems tend to the state of complete chaotic synchronization.

The degree of identity of the spatiotemporal behavior oscillations in the two distributed systems is conveniently analyzed as dependent on ε in terms of the parameter Δ defined as [13]

$$\Delta = \langle |\rho_1(x, t) - \rho_2(x, t)| + |v_1(x, t) - v_2(x, t)| + |\phi_1(x, t) - \phi_2(x, t)| \rangle, \quad (6)$$

where the angle brackets $\langle \ldots \rangle$ denote averaging over both time and space. The results of this analysis are illustrated in Fig. 1b, which shows that the function Δ(ε) rapidly decreases and tends to zero with increasing ε. As can be seen from Fig. 1b, the value of Δ for a greater detuning between the two beam–plasma systems (case (○)) remains nonzero (yet being sufficiently small for $\varepsilon > 0.17$), in contrast to the case of small detuning (case (■)) where Δ practically vanishes. It is the oscillation regimes with $\Delta(\varepsilon) \approx 0$, which are referred to as the states of complete chaotic synchronization.

An important energy characteristic of the synchronization of coupled chaotic systems is provided by the measure of synchronization introduced previously [6, 7] and defined as the energy fraction γ of the wavelet spectrum corresponding to the synchronized time scales (for detail, see [6]). Figure 1b

Shows the γ(ε) curves for the two sets of control parameters $\alpha_{1, 2}$ considered above. As can be seen, an increase in the coupling parameter is accompanied by the increase in the fraction of energy of the spatiotemporal oscillations corresponding to the synchronized time scales.

Figure 2 shows the boundary of the region of complete chaotic synchronization on the plane of control parameters (α_2, ε) for the constant value of $\alpha_1 = 2.861\pi$. As can be seen, the coupled systems exhibit complete synchronization for any detuning between the control parameters $\alpha_{1, 2}$ for a sufficiently large coupling parameter. This refers both to weakly chaotic oscillations in the case of small detuning and to the well developed chaos in systems with strongly different control parameters. Minimum values of the coupling parameter for which the regime of complete synchronization is possible are naturally observed for weakly decoupled subsystems.

To summarize, we have demonstrated for the first time that coupled beam–plasma systems with supercritical current (coupled Pierce diodes) may feature

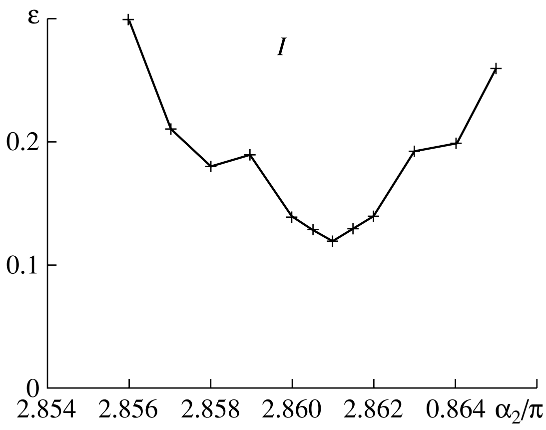

Fig. 2. The boundary of the region (I) of complete chaotic synchronization between two coupled distributed beam–plasma systems (Pierce diodes) on the plane of control parameters (α_2/π, ε) for the constant value of $\alpha_1 = 2.861\pi$.

sequential establishment of different regimes of chaotic synchronization (from phase to complete), which ca be described as various cases of the time scale synchronization [6, 7]. The possibility of establishing the regime of complete synchronization of chaotic spatiotemporal oscillations in beam-plasma systems implies that such autooscillatory media can be used in systems of data transmission in the microwave range.

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