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Turbulent Phase Distribution during Lag Synchronization Breakage

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Abstract—The distribution of turbulent phases in a time realization of dynamical systems in the regime of intermittent lag synchronization has been studied. A method of determining the duration of laminar and turbulent phases by means of the wavelet transform is proposed. © *2005 Pleiades Publishing, Inc.*

The synchronization of chaotic oscillations is a basic phenomenon in the nature. Investigations into this phenomenon are of considerable importance, since it is encountered in many physical [1, 2], biological [3–5], and other systems.

There are several principal types of synchronization of coupled chaotic oscillators [6, 7]. Let us briefly mention some of these types. In the regime of complete (full) synchronization (CS), the vectors characterizing the states of the interacting systems obey the relation $\mathbf{x}_1(t) = \mathbf{x}_2(t)$: this equation is characteristic of the identical chaotic oscillators. If the control parameters of the coupled systems are slightly different, the state vectors are close, $|\mathbf{x}_1(t)–\mathbf{x}_2(t)| \approx 0$, but still differ from each other. Another type of synchronization of coupled chaotic oscillators with slightly different control parameters is the so-called lag synchronization (LS) [10, 11]. In this case, the oscillations in one system follow those in the pother system with a certain time lag: $\mathbf{x}_2(t + \tau) \approx \mathbf{x}_1(t)$, where τ is the delay time. As the parameter of coupling between the two systems is increased, the delay time decreases and tends to zero, so that the coupled system pass to the CS regime. The phenomenon of phase synchronization (PS) is usually described and analyzed in terms of the phase $\phi(t)$ of a chaotic signal [6, 7, 12]. Then, the PS regime means entrainment of the phases of two chaotic oscillators, whereas their amplitudes remain uncorrelated. The transition from PS to LS proceeds via an intermittent lag synchronization (ILS) regime [11, 13]. In the analysis of this process, it is expedient to study a signal representing the difference $\mathbf{x}_2(t + \tau) – \mathbf{x}_1(t)$, which tends to zero in the LS regime. In the ILS state, this signal appears as a random sequence of regular (laminar) phases separated by irregular (turbulent) outbursts. Under the laminar phase, we imply the interval of time in which $|\mathbf{x}_2(t + \tau) – \mathbf{x}_1(t)| \approx 0$. Thus, the differential signal is subject to sharp variations of large amplitude. When the coupling parameter is increased, the number of such irregular outbursts decreases. This intermittent transient regime is referred to as the on–off intermittency [14].

In the analysis of intermittency, an important aspect is related to the diagnostics of laminar and turbulent phases of motion. There are effective methods of separating laminar phases (see, e.g., [13]), whereas the diagnostics of turbulent phases meets certain difficulties. Previously, we proposed [15] to analyze intermittency using wavelet transformation [16], which offers an effective approach to analysis of the behavior of nonlinear systems featuring complicated dynamics (see also [17, 18]). The continuous wavelet transform of a chaotic signal is defined as

$$W(t, s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} x(t') \psi^*\left(\frac{t-t'}{s}\right) dt', \qquad (1)$$

where $x(t)$ is a time realization, $\psi(\eta)$ is the base wavelet function (the asterisk denotes complex conjugation), and s is the analyzed time scale. For the base wavelet, we suggested to used the Morlet wavelet $\psi(\eta) = \pi^{-1/4} e^{j\omega_0\eta} e^{-\eta^2/2}$ representing a rapidly decaying harmonic function with $\omega_0 = 2\pi$. It was shown that the wavelet surface $|W(s, t_0)|$ has significantly different structures for the laminar and turbulent phases of motion [15]. Thus, an analysis of the wavelet surface structure can provide for a sufficiently reliable separation of the different phases of motion.

Let us apply the approach based on the continuous wavelet transform to the analysis of two mutually (two-way) coupled nonidentical Rössler oscillators [14]:

$$\begin{aligned} \dot{x}_{1,2} &= -\omega_{1,2} y_{1,2} - z_{1,2} + \varepsilon(x_{2,1} - x_{1,2}), \\ \dot{y}_{1,2} &= \omega_{1,2} x_{1,2} + a y_{1,2}, \\ \dot{z}_{1,2} &= f + z_{1,2}(x_{1,2} - c), \end{aligned} \qquad (2)$$

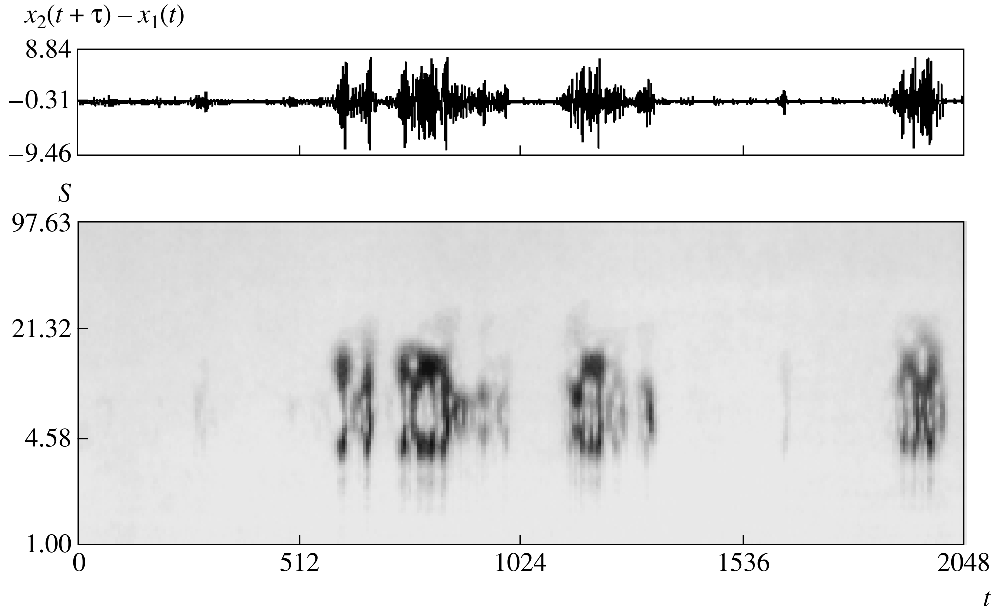

Fig. 1. The regime of intermittent lag synchronization between two coupled Rössler systems (2) with $\varepsilon = 0.13$ ($\tau = 0.32$): (a) the typical time series of the differential signal $\mathbf{x}_2(t + \tau) - \mathbf{x}_1(t)$; (b) the corresponding projection of the wavelet surface modulus $|W(t, s)|$.

where ε is the coupling parameter, $\omega_1 = 0.99$, $\omega_2 = 0.095$, $a = 0.165$, $f = 0.2$, and $c = 10$. As the ε value increases, the coupled systems sequentially exhibit the PS, LS, and CS regimes. In the interval $0.1 < \varepsilon < 0.15$ (corresponding to the passage from PS to LS regime) the coupled systems occur in the ILS state. For arbitrary systems, the delay time τ is usually determined using the similarity function (see, e.g., [11, 13, 14]). However, recently we have demonstrated [19] that τ for the coupled systems under consideration is proportional to a phase shift between the main spectral components of the Fourier spectrum and depends on the coupling parameter according to the power law $\tau = k\varepsilon^{-1}$. For the values of parameters specified above, $k = 0.0418$. In this study, the delay time was determined using the established power relation.

Figure 1 shows the typical time series of the signal $\mathbf{x}_2(t + \tau) - \mathbf{x}_1(t)$ and the corresponding projection of the wavelet surface modulus $|W(t, s)|$ for $\varepsilon = 0.13$ ($\tau = 03.2$). The dark regions correspond to maxima of the wavelet surface. As can be seen, the projection of the wavelet surface modulus clearly reveals the pattern of laminar and turbulent phases in the given time series. In the regime of chaotic dynamics, the maxima of the wavelet surface correspond to the outbursts of oscillations with various time scales. Regions corresponding to the turbulent phases are clearly localized on the time scale. In the laminar periods, the wavelet surface structure remains virtually unchanged and its amplitude is minimal.

Figure 2 shows an average energy distribution with respect to the time scale for a time series duration of $N = 2^{15}$ dimensionless time units. This curve exhibits two clear maxima corresponding to the time scales $s_1 = 4.4$ and $s_2 = 9$. These values determine the average structure of the wavelet surface in the turbulent phase. We propose the following procedure for determining the duration of laminar and turbulent phases. First, the wavelet transformation $|W(t, s)|_{s = s_1, s_2}$ is constructed for the two time scales (s_1 and s_2) corresponding to the maxima of the averaged energy distribution $\langle E(s) \rangle$ and a threshold value Δ_i is selected for each wavelet curve $|W(t, s_i)|$ ($i = 1, 2$). As was noted above, the wavelet surface amplitude reaches maximum in the turbulent phases. It might seem natural to consider the motion as laminar when the given wavelet curve $|W(t, s_i)|$ is above the threshold and as turbulent, above this level. However, situations are possible where the amplitude of the wavelet curve $|W(t, s_1)|$ is at minimum (i.e., is below the threshold Δ_1), whereas the $|W(t, s_2)|$ amplitude is greater than Δ_2 and vice versa, so that the time scale s_2 is predominating at a given moment of time. This possibility is related to the fact that the excitation and suppression of oscillations takes place for these very time scales over the entire turbulent phase. Therefore, exact determination of the laminar and turbulent phases requires the analysis to be performed for both time scales. Accordingly, the laminar phase is assigned only to the states in which both wavelet curves $|W(t, s_i)|$ are below the corresponding thresholds Δ_i. It should be noted that, if the system under consideration is characterized by a single maximum in the energy distribution over the time scales, the problem simplifies and the wavelet transformation can be considered for this single fixed scale.

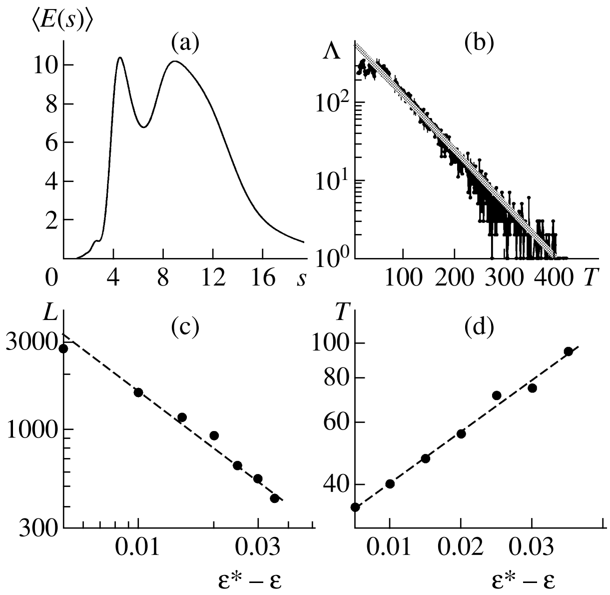

Fig. 2. The results of wavelet analysis of the regime of intermittent lag synchronization between two coupled Rössler systems (2): (a) an average energy distribution with respect to the time scale for $\varepsilon = 0.1$ ($\Delta_1 = 2$, $\Delta_2 = 1.5$); (b) a statistical distribution Λ (plotted on a logarithmic scale) of the turbulent phase duration T for $\varepsilon = 0.1$ (solid line corresponds to the exponential dependence with an exponent of –0.015); (c) a double logarithmic plot of the average of the laminar phase duration L versus critical parameter ε–ε^* near the LS regime threshold $\varepsilon^* \approx 0.15$ (dashed line corresponds to the power law with an exponent of –1); (d) a semilogarithmic plot of the average turbulent phase duration T versus critical parameter ε–ε^* near the LS regime threshold $\varepsilon^* \approx 0.15$ (dashed line corresponds to an exponential law).

Using the method described above, wee have obtained the distributions of laminar and turbulent phases in course of breakage of the LS regime for various values of the coupling parameter and determined the dependence of the average durations of these phases on the coupling parameter. Figure 2b shows a statistical distribution [Lambda] (plotted on a logarithmic scale) of the turbulent phase duration T for $\varepsilon = 0.1$. The number of analyzed phases was about 30000 of the characteristic intervals. As can be seen, the distribution obeys the exponential law with an exponent of –0.015. An analogous distribution for the laminar phases obeys the power law with an exponent of –3/2 (this value is typical of the on–off intermittency) [13].

We have also constructed plots of the average duration of the laminar and turbulent phases versus the critical parameter ε–ε^*, where $\varepsilon^* \approx 0.15$ is a threshold coupling parameter for establishment of the LS regime (Figs. 2c and 2d). These characteristics were determined for 600–2700 phases (at a fixed length of the time series). When the coupling parameter increases in the interval $0.11 < \varepsilon < 0.15$, the number of turbulent (and laminar) phases sharply decreases, the laminar phases become longer, and the turbulent phases become shorter. Figure 2c presents a plot of the laminar phase duration L versus critical parameter ε–ε^* in a double logarithmic scale. As can be seen, the average L value decreases according to the power law $(\varepsilon^*–\varepsilon)^{-1}$ typical of the on–off intermittency. Figure 2d presents a plot of the turbulent phase duration T (in the logarithmic scale) versus ε–ε^*. As can be seen, the average T value grows according to the exponential law with increasing difference ε^*–ε. It should be emphasized that the proposed method (in contrast to that described in [13]) readily provides data on the turbulent phase durations.

In conclusion, we have developed an effective method based on the wavelet transformation for an analysis of the LS regime breakage in a system of two coupled chaotic oscillators. The results obtained for the average laminar phase duration well agree with the data published previously [13]. It is established that the turbulent phase duration obeys exponential dependence on the deviation of the coupling parameter from the LS threshold value. The proposed wavelet analysis procedure has a quite universal character with respect to the description of intermittency and, hence, it can be used for the separation and analysis of laminar and turbulent phases in the time series of various dynamical systems exhibiting the phenomenon of intermittency.

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