Influence of feedback on the complex dynamics of an electron beam with a virtual cathode in a virtode

A. E. Khramov

"Kolledzh State Educational-Scientific Center, Saratov State University
(Submitted October 17, 1997)
Pis'ma Zh. Tekh. Fiz. 24, 51–57 (March 12, 1998)

Results are presented of a numerical simulation of the nonlinear dynamics of a relativistic electron beam with a virtual cathode in a drift relativistic-electron-beam virator system with and without external feedback. © 1998 American Institute of Physics. [S1063-7850(98)01003-9]

As a result of the development of efficient sources of high-current relativistic electron beams (REBs) with currents exceeding 10–100 kA and acceleration energies of 0.1–10 MeV, the power of almost all classical microwave devices has increased sharply. However, the success achieved in acceleration technology cannot be fully utilized in these devices because the beam current is limited. As the current approaches a certain level, the efficiency of the interaction between the electrons and the electromagnetic fields deteriorates. Supercritical-current generators, or virators,1 do not have this shortcoming. They use the oscillations of a so-called virtual cathode, a region in the interaction space whose potential is close to that of the cathode, for phase selection of the electrons. At present, the use of virators to generate superpower microwave radiation pulses must be considered its most important application. However, it has been shown that virtual-cathode devices may be controlled by external microwave signal, allowing them to be used as modules of phased-array antennas.

Here, results are reported of investigations of the nonlinear dynamics of an electron beam with a virtual cathode in a drift REB virator system with and without external feedback delay.

The system studied is a section of a cylindrical waveguide closed on both sides by conducting walls. A single-velocity electron beam is injected into the system. A characteristic feature of the system is the external feedback, which was achieved by kinematic modulation of the electrons entering the drift space by an electromagnetic signal derived from the interaction space and influencing the flux with a delay \( \tau \).

A description of the nonlinear and nonsteady-state processes in this system was constructed using a self-consistent system of Maxwell equations and the Vlasov transport equation, which was solved numerically using a conservative difference scheme.23 The main parameters on which the behavior of this system depends are the ratio of the beam current to the critical vacuum current, denoted by \( \alpha \), and the feedback parameters: the delay time \( \tau \) and the coordinate of the point from which the feedback signal is taken.

It is well known from both numerical simulations and experiments that an electron beam with a virtual cathode exhibits complex irregular dynamics. In Ref. 4 an electrostatic model was used to show that an electron beam with a virtual cathode in a Pierce diode may demonstrate different types of behavior, including dynamic chaos. In the present study, the investigations were carried out using a virator model which allows for nonpotential effects which limit the propagation velocity of the electromagnetic waves in the system.5

Figures 1a–1c give the power spectra, attractor projections, and time series of the beam current from the region of formation of the virtual cathode for various values of \( \alpha \). Note that the time series was taken after the transition process had damped away.

For low supercriticality \( \alpha (\alpha<2) \) periodic relaxation oscillations are established in the system (Fig. 1a, \( \alpha=1.5 \)). The power spectrum contains narrow peaks which are multiple harmonics of the fundamental frequency \( \omega_0=2\omega_p \), where \( \omega_0=2\pi/T_0 \) (\( T_0 \) is the characteristic scale of the oscillations which corresponds to the total transit time of the electrons reflected by the virtual cathode) and \( \omega_p \) is the plasma frequency of the electron beam. The attractor projection corresponds to a single-stage limit cycle. As the parameter \( \alpha \) increases (2<\( \alpha<5 \)), the periodic oscillations disappear (Fig. 1b, \( \alpha=3.7 \)). The spectrum contains a noise pedestal on which are superposed a well-defined fundamental-frequency peak and peaks of its harmonics which merge into the noise pedestal as \( \alpha \) increases. It can be seen from the phase portrait that a chaotic attractor appears at the base of a single unstable limit cycle which corresponds to an attractive limit set for the periodic motion. In addition to the chaotic behavior of the amplitude, the time series reveals random phase shifts of the oscillations. This behavior suggests that in this range of variation of \( \alpha \), a rotational form of chaos is present in the system whose image in phase space is an inhomogeneous attractor. For \( \alpha>5 \) (Fig. 1c), the spectrum is extremely noisy. No clearly defined peaks can be identified, and the structure of the attractor is fairly complex, consisting of a set of unstable periodic orbits.

All these observations suggest that the system formed by an electron beam with a virtual cathode and an electromagnetic field exhibits complex nonlinear dynamics. To refine the characteristics of the complex behavior of the system, the fractal dimension of the reconstructed attractor is now determined. Figures 2a–2c give the slope of the correlation.

1063-7850/98/24(3)/189 © 1998 American Institute of Physics
integral^6,7 as a function of $\varepsilon$ in a log-log plot for various values of the embedding space dimension $m$.

For $\alpha<2$ (Fig. 2a, $\alpha=1.5$), the dimension of the limit set is an integer, which corresponds to a regular manifold. As the supercriticality increases, the dimension becomes a fraction. For $2<\alpha<5$ (Fig. 2b, $\alpha=3.7$) the dimension of the limit set saturates with an embedding space dimension of four. The correlation dimension of the attractor $D_c$ increases monotonically with increasing $\alpha$. The saturation of the attractor dimension for $m<5$ suggests that the system formed by a virtual-cathode electron beam and an electromagnetic field possesses 1.5–2 degrees of freedom. The nature of the spectrum, and the fact that the fractal dimension is finite and fractional confirm that when the beam current exceeds the critical vacuum current in the range $2<\alpha<5$, dynamic chaos is established in the vircator. For the supercriticality $\alpha>5$ (Fig. 2c, $\alpha=12$), it can be seen that the attractor dimension does not saturate for values of the embedding space dimension $m<9$. The duration of the time series does not allow accurate estimates of the attractor dimension for $m>9$. This behavior of $D_c$ leaves the problem of determining the chaotic regime for $\alpha>5$ unresolved.

The problem of controlling the behavior characteristics of a vircator system has recently become important. Thus, an REB generator with supercritical current and controlled feedback, called a vortic, is of particular interest^8. It was demonstrated experimentally that the feedback has a strong influence on the characteristics of the device.

An analysis is made of the case when the feedback signal is taken from the virtual cathode region. Figures 1d–1f and 2d–2f show the oscillation characteristics in a system with fixed $\alpha$ for various values of $\tau$: 20 (d), 40 (e), and 60 (f).

The investigations reveal that the dynamics of the system is determined by the relation between the characteristic scale $T_0$ of the oscillations in the system and the feedback delay time $\tau$. Incorporation of feedback with a short delay time ($\tau<T_0/2$ complicates the system dynamics: a noise pedestal appears in the spectrum and the embedding space dimension is $m>10$ (Figs. 1d and 2d). This is caused by the
excitation of a many degrees of freedom in the system. For $\tau > T/2$ ordering of the oscillations is observed until regular motion is established in the system. The form of the phase portrait, the dimension of the limit set, and the spectral composition of the radiation all vary as a function of $\tau$. In this case, the characteristic oscillation frequency varies in the range $(0.7 - 1.5)\omega_0$, where $\omega_0$ is the free oscillation frequency of the virtual cathode. Note that variation of the delay time of the feedback signal can also be used to control the amplitude of the virtual cathode oscillations and thus the level of the generated power in the system. There is an optimum delay time $\tau$ for which the electron interaction power $P_e$ in the system has a maximum. The ratio $(P_e)_{\text{max}}/(P_e)_{\text{max}} \sim 0.5$ is obtained and the ratio of $(P_e)_{\text{max}}$ to the free oscillation power of the virtual cathode is of the order of 0.65.

Figure 1g shows the oscillation characteristics of the system for the case where feedback is introduced in the collector region. The oscillations are highly irregular, the power spectrum is noislike, and the attractor has a comparatively uniform structure.


Translated by R. M. Durham