INFLUENCE OF EXTERNAL ACTION ON CHAOTIC DYNAMICS OF VIRTUAL CATHODE OSCILLATIONS

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ABSTRACT

Results of the numerical simulation of the electron beam modulation influence on virtual cathode oscillation are presented. It is shown that system demonstrates wide diversity of nonlinear oscillation including synchronisation by an external action. Physical processes in the beam are considered. We show that chaotic behaviour is the result of the beam stratification on several groups of electrons with different lifetime. Phase relations for different synchronisation regimes are analysed.

1. Introduction

In the present paper we investigate a simplest model describing the synchronisation of virtual cathode oscillation (VCO) by an external action. We consider a one-dimensional, electrostatic, short-circuited diode model. In our case the external action leads to velocity modulation of an electron beam without a charge density modulation in the injection plane.

The behaviour of the beam in a diode without velocity modulation is determined by the dimensionless parameter related to current $\alpha=\omega_p L/v_0$, where $\omega_p$ is the beam plasma frequency, $L$ is the distance between grids and $v_0$ is the nonperturbated velocity of the beam. Velocity of the beam is described by the temporal dependence: $v\left(x=0\right) = v_0 \left(1+m \sin 2\pi ft\right)$ on the input of system. Here $m$ and $f$ are the modulation depth and frequency, respectively. As provided (see, for example [1]), instability appears in the system for $\alpha>4/3$. The instability leads to formation of the region called virtual cathode (VC) with the potential approximately equal to the acceleration potential. VC oscillates in the time and the space.

The modulated electron beam in the regime with VC may be treated as a simplest model of the high power microwave devices in that a resonant cavity is located between the beam source and the interaction space — vircator-klystron [2]. Besides such a system may be considered as a nonautonomous oscillator with an external signal that operates on the formation of the beam, for example virtod [3].

In this work we discuss a impact of beam modulation especially on chaotic VCO.

2. System behaviour

The effect of electron beam modulation on VCO dynamics was investigated with the aid of PIC-simulation [4]. In Fig.1 bifurcation diagram on the parameter plane ($ff_{vc}, m$) is presented. Reconstructed phase portraits, power spectra and time series of electric field oscillations for different regimes are shown in Fig.2. On this figure we demonstrate the characteristics for domains in the parameter space marked by (a), (b) and so on in the notation in Fig.1 respectively. In Fig.1 regular regimes are marked by ratio of two numbers ($a:b$). Here $a$ and $b$ are a number of attractor trajectory turns remaining in the plane and a
number of turns going out of space respectively. The total number of \((a+b)\) gives the full number of trajectory turns in the phase space. Low-perturbated VCO (Fig.2a) demonstrates the chaotic behaviour. The power spectrum contains wide peaks \(f_{vc}, 2f_{vc}\) and \(3f_{vc}\) on the high noise base. The attractor section demonstrates a complex heterogeneous structure. \(m\) increase leads to the appearance of the chaotic motion with two characteristic frequencies \((f\) and \(f_{vc}\)). For \(ff_{vc} < 0.6\) we have been observing quasi-periodic and torus-chaos regimes. In these cases poincare section is either a smooth curve or an invariance non-smooth curve. On the background of these regimes there are many narrow domains of a regular oscillation with the frequencies of an external action. The synchronisation beaks are surrounded by wide regions of chaotic motion which we call ribbon chaos, because attractors are narrow ribbons in the phase space. They appear on the base of unstable limit cycle coinciding with the attractor for the regular regime. Power spectrum consists of sharp peaks of frequency \(f\) and its harmonics \(2f, 3f \ldots\) (1-ribbon chaos; Fig.2d) and sometimes its subharmonic \(f/2\) (2-ribbon chaos; Fig.2e).

The difference between chaotic regimes called as "chaos-1" and "chaos-2" is as follows: in the second regime (Fig.2f) phase portraits have complex double-turned structure with a slow motion loop and rapid motion ribbon, there are sharp spikes on the frequencies \(f\) in the spectrum. Chaotic oscillation in the first regime (Fig.2c) is characterised by more structureless phase portrait, the spectrum contains two peaks of \(f\) and \(f_{vc}\) on the high noise base.

Dimension of the reconstructed attractors [5] was estimated for the different types of chaotic behaviour. In Fig.3 dependencies of local slope of correlation integrals \(D\) from observation scale \(\varepsilon\) [6] are shown for different values of embedded dimensions \(d\). In the case of weak chaos attractor dimensions are saturated for small values of embedded dimensions (Fig.3a; ribbon chaos). For strong chaos the correlation integral \(D\) loses a clearly expressed slope, saturated values increase (compare embedded value of \(d\) on Fig.3a and Fig.3b).

To anticipate a little, large values of dimensions are justified by the interaction between large numbers of structures in the beam, much as this was shown in [7].

3. Physical processes

Physical processes in our system were analysed with the help of the space-time diagrams of the beam in the interaction space that are shown in Fig.4. Each line on the diagrams represents the trajectory of single charged particle.
Figure 2: Power spectra, reconstructed phase portraits and time series for different regimes
Chaotic dynamics of an unneutralised electron beam is conditioned by the internal distributed feedback. It is provided by the electrons remaining in the region of VC and influencing over the dynamics of VC on the next period of oscillation. This is evident from Fig.4a, which are unique to the low-perturbated VCO. In effect the electron beam with VC is stratificated on several groups of electrons with different lifetimes $\tau$. The distribution function of electrons over the lifetimes in the interaction space $\Phi(\tau)$ best illustrates this point of view. As evident from Fig.5 showing $\Phi(\tau)$ for the chaotic motion (solid line) the distribution function of electrons over the lifetimes have a few humps. Each peak of $\Phi(\tau)$ corresponds to the group of electrons with their characteristic lifetime and trajectory in the phase space. The high sharp peaks consist of reflected particles to the injection plane with different transit time from input grid to VC and back VC to input grid. The such form of $\Phi(\tau)$ proves that a few "VC's" are organised on each period of oscillations in the chaotic regime. As was shown [7], each bunch (VC) in the interaction space is inner structure of the beam. Thus phenomenon of the beam stratification may be considered as appearance of several oscillation patterns in the beam.

To sum up, several patterns exist in the beam on each period of VCO and strongly chaotic oscillation is a result of patterns interaction. The internal feedback in the beam is provided by the third long-living group of electrons. It is to be noted that $\Phi(\tau)$ has the
Figure 5: Distribution function of charged particles over the lifetimes

The suppression of this beam stratification carried out with the aid of velocity modulation provides existence of regular and almost regular regimes (Fig.4b). This is evident from form of $\Phi(\tau)$ (dotted line in Fig.5) which consists of two fundamental peaks corresponding to the transited particles and the reflected particles. Complex structure of $\Phi(\tau)$ is being depleted by the rejection of parts of electrons to the right boundary of the system, as indicated in Fig.4b.

4. Phase locking

In the last years the utilisation of the vircators as modules of the phased array has attracted many researchers. In [9] the system of two coupling virtods is suggested. Therefore the phase relation between an external action (the modulation) and the oscillation in our system is of special interest. In Fig.6 dependencies of phase difference $\Delta\phi$ between an external action operating on the formation of beam and VCO versus dimensionless time

Figure 6: Phase difference between an external action (the modulation) and VCO
shown for two domains of synchronisation. The curve 1 corresponds to the ratio of \( f/f_{vc} \approx 0.5 \). The phase fluctuates at about +80° as viewed in figure. For \( f/f_{vc} \approx 0.95 \) the phase is locked at about +5° (curve 2 in Fig.6).

5. Conclusions

The study of dynamics of considered model has provided evidence that a nonautonomous VCO can demonstrate different types of nonlinear behaviour including synchronisation by the beam modulation at injection. Strongly chaotic behaviour is the result of interaction between several groups of charged particles with the different lifetime. Suppression of these patterns leads to the appearance of regular regimes.

Understanding of physical processes in the unneutralised beam provides the possibility of VCO control in the vircator system with the aid of influence on the distribution of electrons over the lifetimes by the preliminary velocity modulation of the beam.

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References


